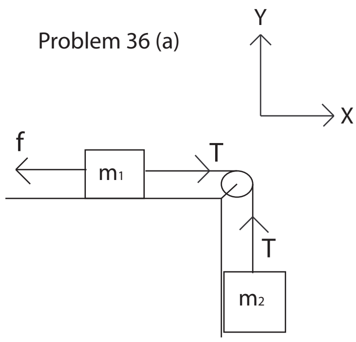
Homework V

Ch. 4 - Problems 36, 37, 58, 65.

Problem 36

(a) The free-body diagram appears below.



Now, if we consider the forces acting on mass m_1, we find:

$$\sum \vec{F}_y \ : \ N - W = 0 \rightarrow N = m_1 g$$
$$\sum \vec{F}_x \ : \ T - \mu_s m_1 g = m_1 a.$$

Careful examination of this equation and comparison to the corresponding equations in the previous problem reveals that the block system only moves if $f_s < T$, where $f_s = \mu_s m_1 g$. If we were to write the force equation for mass m_2,

$$m_2 g - T = m_2 a,$$

and solve this together with the force equations for mass m_1 for the acceleration and tension, we would find,

$$\begin{aligned} m_2 g - \mu_s m_1 g &= (m_1 + m_2) a, \\ a &= \frac{m_2 - \mu_s m_1}{m_1 + m_2} g, \end{aligned}$$

and

$$
\begin{aligned}
T &= m_1(\mu_s g + a), \\
&= m_1 g(\mu_s + \frac{m_2 \mu_s m_1}{m_1 + m_2}), \\
&= m_1 g(\frac{m_1\mu_s + m_2\mu_s + m_2 - m_1\mu_s}{m_1 + m_2}), \\
&= \frac{m_1 m_2}{m_1 + m_2}(1 + \mu_s)g.
\end{aligned}
$$

So, in order for the system to begin moving, we must have

$$
\begin{aligned}
T &> f_s, \\
\frac{m_1 m_2}{m_1 + m_2}(1 + \mu_s)g &> \mu_s m_1 g, \text{or} \\
\frac{m_2}{m_1 + m_2}(1 + \mu_s) &> \mu_s, \\
m_2 + m_2\mu_s &> m_1\mu_s + m_2\mu_s, \\
\mu_s &< \frac{m_2}{m_1}.
\end{aligned}
$$

With m_1 = 10 kg and m_2 = 4.0 kg, this means that $\mu_s < \frac{4.0\ kg}{10\ kg} = 0.40$. Since $\mu_s = 0.50 > 0.40$, the system does not move. Alternately, plugging in the given values we find that a = -0.07g < 0. Clearing, this result cannot be physical, as it tells us that, without an externally applied force, mass m_2 moves upward against the force of gravity. (Although it might be kind of interesting to see, and if you can figure out how to do it, you will go on to fame and fortune.) Nevertheless, our everyday experience tells us this is nonsensical - the only reasonable interpretation being that the system simply does not move. However, it is a good idea to always compare the forces as we did above.

(b) The free-body diagram is the same as in part (a). With a coefficient of kinetic friction $\mu_k = 0.30$ and using the expression for acceleration found above (it carries over perfectly well here, as all we have done is make the substitution $\mu_s \rightarrow \mu_k$), we find

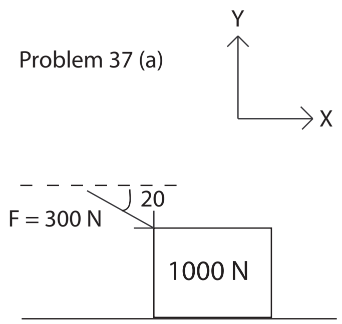
$$
a = \frac{m_2 - \mu_s m_1}{m_1 + m_2}g = \frac{(4.0 - (0.30)(10))kg}{(10 + 4.0)kg}g \approx 0.07g = 0.7\frac{m}{s^2}.
$$

Problem 37

We have the following information:

$$\begin{aligned} F &= 300\ N, \\ \theta &= 20.0^\circ, \\ W &= 1000\ N. \end{aligned}$$

(a) Note that v $\equiv$ constant $\rightarrow$ a = 0. The free-body diagram appears below.



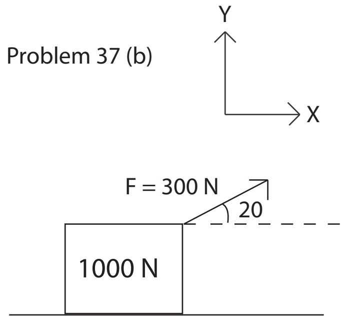
The sum of the forces in the $\hat{\mathbf{y}}$ direction yields

$$\begin{aligned} 0 &= N - W - F\sin(\theta), \\ N &= W + F\sin(\theta), \\ &= 1000 + 300\sin(20), \\ &= 1103\ N. \end{aligned}$$

The sum of the forces in the $\hat{\mathbf{x}}$ direction yields

$$\begin{aligned} 0 &= F\cos(\theta) - \mu_s N, \\ \mu_s &= \frac{F\cos(\theta)}{N}, \\ &= \frac{300\cos(20)}{1000 + 300\sin(20)}, \\ &= 0.256. \end{aligned}$$

(b) The free-body diagram appears below.



The sum of the forces in the $\hat{\mathbf{y}}$ direction yields

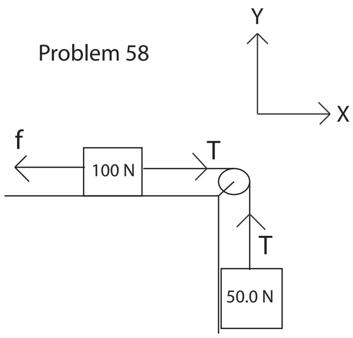
$$\begin{aligned} 0 &= N - W + F\sin(\theta), \\ N &= W - F\sin(\theta), \\ &= 1000 - 300\sin(20), \\ &= 897\ N. \end{aligned}$$

The sum of the forces in the $\hat{\mathbf{x}}$ direction yields

$$\begin{aligned} ma &= \frac{W}{g}a = F\cos(\theta) - \mu_s N, \\ a &= \frac{g}{W}(F\cos(\theta) - \mu_s N), \\ &= \frac{9.8}{1000}(300\cos(20) - (0.256)(1000 - 300\sin(20))), \\ &= 0.511\ \frac{m}{s^2}. \end{aligned}$$

Problem 58

The free-body diagram appears below.



(a) For the mass on the table, the sum of the forces in the $\hat{\mathbf{y}}$ direction yields

$$\begin{aligned} 0 &= N - W_t, \\ N &= W_t = 100\ N. \end{aligned}$$

For the same mass, the sum of the forces in the $\hat{\mathbf{x}}$ direction yields

$$\begin{aligned} 0 &= T - f_s, \\ T &= f_s. \end{aligned}$$

For the hanging mass, the sum of the forces in the $\hat{\mathbf{y}}$ direction yields

$$\begin{aligned} 0 &= W_h - T, \\ T &= W_h, \\ &= 50.0\ N. \end{aligned}$$

Substituting this value above we find

$$f_s = T = 50.0\ N.$$

(b) Recall that the force due to static friction is defined as

$$f_s = \mu_s N.$$

Thus, in order to ensure static equilibrium, the coefficient of static friction must be at least

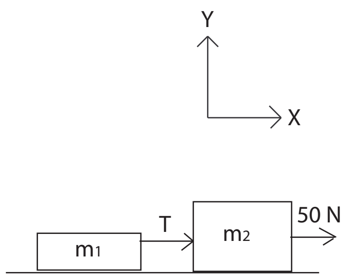
$$\mu_s = \frac{f_s}{N} = \frac{f_s}{W_t} = 0.500.$$

(c) The coefficient of kinetic friction is $\mu_k = 0.250$. Thus, in order for the system to move with constant speed, the total force must be zero. With the definitions found in the previous two parts, this means that

$$\begin{aligned} \mu_k W_t &= W_h, \text{or} \\ W_h &= (0.250)(100) = 25.0\ N. \end{aligned}$$

Problem 65

The free-body diagram appears below.



Problem 65 (a)

We have the following information:

$$\begin{aligned} m_1 &= 10\ kg, \\ m_2 &= 20\ kg, \\ F &= 50\ N. \end{aligned}$$

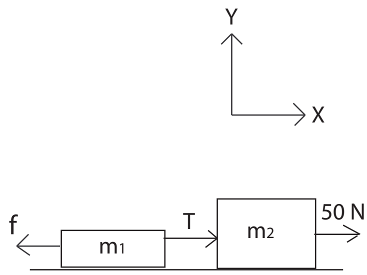
(a) A 10 kg box is attached to a 20 kg box by an ideal string. a 50 N force is applied to the 20 kg box. As can easily be seen in the free-body diagram, the force is actually applied to the system that is both masses. Thus, acceleration of each box is the same and is given by

$$a = \frac{F}{m} = \frac{50N}{(10+20)kg} = 1.7\ \frac{m}{s^2}.$$

The tension in the connecting ideal string is given by

$$T = m_1 a = (10\ kg)(1.67\ \frac{m}{s^2}) = 17\ N.$$

(b) The free-body diagram appears below.



Problem 65 (b)

Let's consider the same scenario, but this time there exists a coefficient of kinetic friction, $\mu_k = 0.10$ between each box and the surface. Then, the force equation for the two box system is

$$\begin{aligned}(m_1 + m_2)a &= 50 - f_k, \\ &= 50 - \mu_k(m_1 + m_2)g, \\ a &= \frac{50 - \mu_k(m_1 + m_2)g}{m_1 + m_2}, \\ &= 0.69\ \frac{m}{s^2}.\end{aligned}$$

The tension can be found from

$$\begin{aligned}m_1 a &= T - f_k, \\ &= T - \mu_k m_1 g, \\ T &= m_1(a + \mu k g), \\ &= 17\ N.\end{aligned}$$

Ch. 7 - 30, 42, 54.

Problem 30

We are given the following information:

$$\begin{aligned} M_\odot &= 1.991E30\ kg, \\ M_{earth} &= 5.98E24\ kg, \\ M_{moon} &= 7.36E22\ kg, \\ R_{SM} &= R_{SE} - R_{EM}, \\ R_{EM} &= 3.84E8\ m, \\ R_{SE} &= 1.496E11\ m, \\ G &= 6.673E-11\ kg^{-1}m^3s^{-2}. \end{aligned}$$

(a) The force exerted by the Sun on the Moon is

$$\begin{aligned} F_{SM} &= -\frac{GM_\odot M_{moon}}{R_{SM}^2}\hat{\mathbf{r}}, \\ &= -4.39E20\ N. \end{aligned}$$

(b) The force exerted by the Earth on the Moon is

$$\begin{aligned} F_{EM} &= \frac{GM_{earth}M_{moon}}{R_{EM}^2}\hat{\mathbf{r}}, \\ &= 1.99E20\ N. \end{aligned}$$

(c) The force exerted by the Sun on the Earth is

$$\begin{aligned} F_{SE} &= -\frac{GM_\odot M_{earth}}{R_{SE}^2}\hat{\mathbf{r}}, \\ &= -3.55E22\ N. \end{aligned}$$

Problem 42

We have the following information:

$$
\begin{aligned}
h &= 20\ m, \\
M_{mars} &= 0.1074\ M_{earth}, \\
R_{mars} &= 0.5282\ R_{earth}.
\end{aligned}
$$

(a) The acceleration due to gravity at the surface of the earth is given by

$$
\begin{aligned}
g_{mars} &= -\frac{GM_{mars}}{R_{mars}^2}\hat{\mathbf{r}}, \\
&= -\frac{G * 0.1074M_{earth}}{(0.5282R_{earth})^2}\hat{\mathbf{r}}, \\
&= 0.3850 * -\frac{GM_{earth}}{R_{earth}^2}\hat{\mathbf{r}}, \\
&= 0.3850g_{earth}, \\
&= 3.77\ \frac{m}{s^2}.
\end{aligned}
$$

(b) The time it takes the instrument package to reach the surface is

$$
\begin{aligned}
t &= \sqrt{\frac{2 * h}{g_{mars}}}, \\
&= \sqrt{\frac{40m}{3.77\frac{m}{s^2}}}, \\
&= 3.26\ s.
\end{aligned}
$$

Problem 54

(a) We have the following information:

$$
\begin{aligned}
l &= 100\ m, \\
r &= 10\ km, \\
m &= 1000\ kg, \\
M &= 100\ M_{\odot}.
\end{aligned}
$$

Now, recall that the gravitational force as we have seen it thus far is given as

$$\tilde{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\mathbf{r}}.$$

However, we can no longer treat the spaceship as a point mass due to the magnitude of the gradient of the local gravitational field. Thus, we may write the differential as

$$d\vec{F} = -\frac{GMdm}{r^2}\hat{\mathbf{r}} = -\frac{GM\lambda dr}{r^2}\hat{\mathbf{r}},$$

where $\lambda = \frac{m}{l} = 10\ \frac{kg}{m}$ is the linear mass density of spaceship.

Then, the total force is given by

$$\begin{aligned}
F &= -G\lambda M\int_R^{R+l}\frac{dr}{r^2}, \\
&= G\lambda M(\frac{1}{r})|_R^{R+l}, \\
&= G\lambda M(\frac{1}{R+l} - \frac{1}{R}), \\
&= -\frac{G\lambda Ml}{R(R+l)}, \\
&= -\frac{GMm}{R^2(1+\frac{l}{R})}, \\
&\approx -\frac{GMm}{R^2}(1-\frac{l}{r}).
\end{aligned}$$

(b) The force per kilogram mass at the two ends of the spaceship is given by

$$\begin{aligned}
\frac{F(R)}{m} &= -\frac{GM}{R^2}, \\
\frac{F(R+l)}{m} &= -\frac{GM}{(R+l)^2}.
\end{aligned}$$

The difference between the two is $\left|\frac{F(R)}{m}\right|$ - $\left|\frac{F(R+l)}{m}\right|$

$$
\begin{aligned}
\Delta &= \frac{GM}{R^2} - \frac{GM}{(R+l)^2}, \\
&= \frac{GM}{R^2(R+l)^2}((R+l)^2 - R^2), \\
&= \frac{GM}{R^2(R+l)^2}(2Rl + l^2), \\
&= 2.62E12\ \frac{N}{kg}.
\end{aligned}
$$