Solution of Quiz 4
September 1, 2000

1. A.
   $R_1, R_2$ and the battery $\varepsilon$ are parallel.
   $R_3, R_4$ and $R_5$ are in series, so the current through them is the same.

2. D.
   If the switch is open, then $R_3, R_4$ and $R_5$ combined are in parallel with $R_2$. Therefore, voltage across $R_2$ is the same as voltage across $R_3, R_4$ and $R_5$ combination.

3. H.
   When the capacitors have reached the steady state, the amount of charge on them becomes constant, so the current through the capacitors is zero. However, current through the resistors is non-zero because there are potential difference across the resistors.

4. E.
   Since $R = \rho L/A$, hence the resistance depends on both the length and the cross-sectional area of the material. The resistivity ($\rho$) depends on the number of free electrons in the material, and so does the resistance.

5. C.
   Components which are parallel to each other have the same voltage, so we should put a voltmeter in parallel with the components if we want to measure the voltage of the components.

6. C.
   Since $R = \rho L/A$, thus decreases the length by half will also decrease the resistance by half.
7. current density $J = \frac{I}{A} = \frac{I}{\pi r^2} = 3.2 \times 10^6 \text{ Am}^{-2}$

drift velocity $v_d = \frac{I}{\eta A c} = 2.4 \times 10^{-4} \text{ m s}^{-1}$

resistance $R = \frac{\rho L}{A} = 2.7 \times 10^{-3} \Omega$

potential difference $\varepsilon = IR = 0.027 V$

electric field $E = \frac{\varepsilon}{L} = 0.054 \text{ V m}^{-1}$

power $P = I^2R = 0.27 W$

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8. \[ I_1 = \frac{\varepsilon_A}{R_1} = 0.8 \text{ A} \]

\[ I_2 = \frac{\varepsilon_B}{R_2} = 1.2 \text{ A} \]

\[ I_4 = \frac{\varepsilon_A}{R_4} = 0.48 \text{ A} \] (1)
To find $I_{35}$, first consider the loop consists of $R_3, R_5, R_4$ and $\varepsilon_B$.

\[
\varepsilon_B - I_{35}(R_3 + R_5) + I_4R_4 = 0
\]

\[
\Rightarrow \varepsilon_B - 4I_{35}(R_3 + R_5) + \varepsilon_A = 0 \quad \text{from (1)}
\]

\[
\Rightarrow I_{35} = \frac{\varepsilon_A + \varepsilon_B}{R_3 + R_5} = 0.6 \, \text{A}
\]

9. (a) When the capacitors are fully charged, voltage across $C_2, C_3$ combination is $\varepsilon$. Since $C_2$ and $C_3$ are in series, so their capacitance equivalence is

\[
C_{2+3} = \frac{C_2C_3}{C_2 + C_3}
\]

\[
\Rightarrow Q_{2+3} = \varepsilon C_{2+3} = \frac{C_2C_3}{C_2 + C_3} \varepsilon
\]

Charge deposits on the capacitor equivalence equals charge deposits on individual $C_2$ and $C_3$, therefore,

\[
\varepsilon_2 = \frac{Q_{2+3}}{C_2} = \frac{C_3}{C_2 + C_3} \varepsilon = 4 \, \text{V}
\]

\[
\varepsilon_3 = \frac{Q_{2+3}}{C_3} = \frac{C_2}{C_2 + C_3} \varepsilon = 8 \, \text{V}
\]
After the capacitor is fully charged, no current flows through the capacitor, so the voltage across the resistor is zero. Therefore,

\[ \varepsilon_1 = \varepsilon = 12 \text{ V} \]

(b) When the switch is open, the circuit becomes a \( RC \) circuit with \( C_1, C_2 \) and \( C_3 \) in series, and the capacitance equivalence is given by

\[ C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3} = 1.6 \mu F \]

Current at any time \( t \) is given by

\[ I(t) = I_0 \exp\left(-\frac{t}{RC}\right) \]

When the current decreases to half of its maximum value,

\[ I(t) = \frac{I_0}{2} = I_0 \exp\left(-\frac{t}{RC}\right) \]

\[ \Rightarrow \frac{1}{2} = \exp\left(-\frac{t}{RC}\right) \]

\[ \Rightarrow \ln 2 = \frac{t}{RC} \]

\[ \Rightarrow t = RC \ln 2 = 1.6 \times 10^{-5} \text{ s} \]