Solution of Final Exam
September 8, 2000

1. D.
Drift velocity is given by \( v_d = \frac{I}{\eta A e} \) where \( A = \pi r^2 = 3.14 \times 10^{-4} \text{ m}^2 \), \( \eta = 1 \times 10^{28} \text{ m}^{-3} \), \( e = 1.602 \times 10^{-19} \text{ C} \) and \( I = 100 \text{ A} \), so \( v_d = 2 \times 10^{-4} \text{ m/s} \).

2. C.
Electric field is always zero inside a perfect conductor, so for \( r < R \), \( E = 0 \). Outside the conductor, Gauss' Law gives \( E \propto 1/r^2 \) for \( r > R \).

3. C.
Electric field vanishes implies electric potential is constant inside a conductor, \( V = \text{constant} \) for \( r < R \). Outside the conductor, \( V \propto -\int E \, dr \propto 1/r \) for \( r > R \).

4. B.
Imagine the wire of current, which flows in +z direction, and the test charge both lie on the yz-plane, such that the magnetic field on the test charge points in \(-x\) direction. To produce a force pointing in the direction of current (i.e. +z direction), the test charge must move in +y direction according to the law of magnetic force, \( F \cdot \mathbf{\hat{y}} = qv \times B(- \mathbf{\hat{x}}) \)

5. B.
Thermal energy of the ideal gas particle equals its kinetic energy, \( \frac{3}{2} k_B T = \frac{1}{2} mv_{rms}^2 \), therefore, \( v_{rms} = \sqrt{\frac{3k_B T}{m}} \).

6. E.
Left side and right side force on charge \(-2q\) are equal in magnitude but opposite in direction, so the resultant force is zero.
7. E.
In an adiabatic process, \( P_f V_f^2 = P_i V_i^2 \), so if \( V_f = 2V_i \), then \( \frac{P_f}{P_i} = \left( \frac{V_i}{V_f} \right)^2 = \left( \frac{1}{2} \right)^2 = 0.31. \)

8. C.
In a Carnot engine, \( Q_1/T_1 = Q_2/T_2 \), so the entropy change of the system is \( \Delta S = Q_2/T_2 - Q_1/T_1 = 0. \)

9. E.
In a discharging circuit, \( Q = Q_0 \exp\left(-\frac{t}{RC}\right) \). Since energy \( E = \frac{Q^2}{2C} \), therefore \( E = \frac{Q_0^2}{2C} \exp\left(-\frac{2t}{RC}\right) = E_0 \exp\left(-\frac{2t}{RC}\right) \). Consequently, \( E = E_0/2 \) when \( t = \frac{RC}{2} \ln(2) \).

10. E.
At first, the charged particle moves along the electric field in straight line. Since the magnetic field is parallel to the velocity of the particle, it has no effect on the particle, so the particle remains travelling in straight line.

11. A.
Use a circle of radius \( r \) which concentric with the cable as the Ampérian loop. The current enclosed in this loop is zero because the currents on the inner and outer conductors are equal in magnitude and opposite in direction. Thus, Ampére’s Law gives \( \oint B \cdot ds = \mu_0 I_{end} = 0 \) and the magnetic field at point \( P \) is zero.

12. E.
Imagine the wire of current and the loop both lie on the \( yz \) plane. If the current flows in \( +z \) direction, then the magnetic field on the loop points in \( -x \) direction. Since, the magnetic field of the wire of current decreases as the loop moves away from wire \( (B \propto 1/r) \), the magnetic
flux of the loop decreases. Therefore, the induced current will produce a magnetic field that points to the same direction \((-x)\) as the magnetic field of wire according to Lenz’s Law. Using right-hand rule, we expect the induced current flows in clockwise direction. Once we find out the direction of induced current, we can deduce the direction of force on either side of the loop by the law of magnetic force, \(\vec{F} = LI \times \vec{B}\).

13. (a) Heat loss by copper equals heat gain by lead. Let the final temperature be \(T_f\).

\[
m_{Cu}C_{Cu}(400K - T_f) = m_{pb}C_{pb}(T_f - 200K)
\]

\[\Rightarrow T_f = \frac{200m_{pb}C_{pb} + 400m_{Cu}C_{Cu}}{m_{pb}C_{pb} + m_{Cu}C_{Cu}} = 320K\]

(b) Change of entropy is given by:

\[
\Delta S = \int_{400K}^{T_f} m_{Cu}C_{Cu} \frac{dT}{T} + \int_{200K}^{T_f} m_{pb}C_{pb} \frac{dT}{T}
\]

\[= m_{Cu}C_{Cu} \ln\left(\frac{T_f}{400K}\right) + m_{pb}C_{pb} \ln\left(\frac{T_f}{200K}\right)\]

\[= 1.75 J/K\]

14. (a) Process \(CA\) does not contribute to the work done because it is a
constant volume process.

\[ W_{AB} = \int P \, dV = \int_{V_1}^{V_2} \frac{P_1 V_1}{V} \, dV \]

\[ = P_1 V_1 \ln \left( \frac{V_2}{V_1} \right) = 3296 \, J \]

\[ W_{BC} = \int_{V_2}^{V_1} P_2 \, dV \]

\[ = P_2 (V_1 - V_2) = -2000 \, J \]

\[ \Rightarrow \text{Total work} \quad W = W_{AB} + W_{BC} = 1296 \, J \]

Since this process does positive work on its surroundings, it is a heat engine.

(b) In this heat engine,

\[ Q_{in} = W + Q_{out} = 8296 \, J \]

\[ \Rightarrow \text{Efficiency} \quad \varepsilon = \frac{W}{Q_{in}} = 15.6\% \]

15. (a) After being accelerated by a potential \( \varepsilon \), kinetic energy of the electron equals the electrical potential energy,

\[ \frac{1}{2} m v^2 = e \varepsilon \]

\[ \Rightarrow v = \sqrt{\frac{2e \varepsilon}{m}} = 1.88 \times 10^7 \, m/s \]

(b) For the electron to pass through undeflected, the electrical force \( (eE) \) on the electron must equal the magnetic force \( (evB) \),

\[ eE = evB \Rightarrow E = vB = 2.29 \times 10^7 \, V/m \]
16. (a) The Gaussian surface in this problem is a spherical surface, with radius \( r \) where \( b > r > a \) and concentric with the metal shells. The electric field is constant on this surface since it depends only on distance \( r \). Thus Gauss’ Law gives

\[
\epsilon_0 \int \vec{E} \cdot d\vec{A} = q \Rightarrow \epsilon_0 E \int dA = q \Rightarrow E = \frac{q}{4\pi \epsilon_0 r^2}
\]

and the electric field points from positive charge to negative charge (i.e. outward direction).

(b) Potential difference \( V \) is given by

\[
V = -\int_b^a \vec{E} \cdot d\vec{r} = -\int_b^a \frac{q}{4\pi \epsilon_0 r^2} \, dr = \frac{q}{4\pi \epsilon_0} \left( \frac{1}{a} - \frac{1}{b} \right) = \frac{q}{4\pi \epsilon_0} \left( \frac{b-a}{ab} \right)
\]

(c) Capacitance is given by

\[
C = \frac{q}{V} = \frac{q}{4\pi \epsilon_0} \left( \frac{ab}{b-a} \right)
\]

(d) As \( a \to b \to R \) and \( b - a \to d \), capacitance becomes

\[
C = 4\pi \epsilon_0 \left( \frac{ab}{b-a} \right) \to 4\pi \epsilon_0 \frac{R^2}{d} = \epsilon_0 \frac{A}{d}
\]

which is the capacitance of a parallel plates capacitor \((A = 4\pi R^2\) is the surface area of a sphere of radius \( R \))

17. Resistors \( R_3, R_4 \) and \( R_5 \) are in series, so their resistance equivalence is given by

\[
R_{3+4+5} = R_3 + R_4 + R_5 = 50 \, \Omega
\]
Resistors \( R_1, R_2 \) and \( R_{3+4+5} \) are parallel to each other, so the total equivalent resistance \( R \) is given by

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{3+4+5}}
\]

\[
\Rightarrow R = \frac{R_1 R_2 R_{3+4+5}}{R_1 R_2 + R_1 R_{3+4+5} + R_2 R_{3+4+5}}
\]

\[
= \frac{R_1 R_2 (R_3 + R_4 + R_5)}{R_1 R_2 + R_1 (R_3 + R_4 + R_5) + R_2 (R_3 + R_4 + R_5)}
\]

\[
= 4.55 \Omega
\]

18. (a) Constant current \( I \) flows in the counter-clockwise direction in a uniform magnetic field \( B \) points into the page. From the magnetic force law, \( \vec{F} = LI \times \vec{B} \), the magnetic force on the rod points to the right. However, as the rod starts to move towards its right, the magnetic flux \( \Phi \) enclosed by the rod and rails decreases and that will create an induced current. According to Lenz’s Law, the induced current \( i \) will flow in the direction that opposes the decreasing of magnetic flux. This means the induced current flows in a clockwise direction such that its magnetic field points in the same direction as the background uniform field \( B \). The induced current \( i \) also experiences a magnetic force by the background uniform field \( B \) and that force points towards the left side of the rod. In other words, magnetic forces on constant current \( I \) and induced current \( i \) point in opposite direction.

Assume the rod moves in velocity \( v \), then the induced emf \( \varepsilon \) is given by

\[
| \varepsilon | = \frac{d\Phi}{dt} = BLv
\]

so that the induced current \( i \) is given by

\[
i = \frac{| \varepsilon |}{R} = \frac{BLv}{R}
\]
(b) The magnetic force due to the background field $B$ on the induced current $i$ is given by

$$F_i = iLB = \frac{B^2L^2}{R}v$$

And the force due to the background field $B$ on the constant current $I$ is given by

$$F_I = ILB$$

If we define vector pointing towards right as positive and vector pointing towards left as negative, then the net force is given by

$$F_{net} = F_I - F_i = ILB - \frac{B^2L^2}{R}v$$

c) Using Newton's second law, net force $F = ma$, equation of motion of the rod is then given by

$$F_{net} = m\frac{dv}{dt} = ILB - \frac{B^2L^2}{R}v$$

The rod reaches its terminal velocity when its acceleration drops to zero, i.e. $\frac{dv}{dt} = 0$. In that case,

$$m\frac{dv}{dt} = 0 = ILB - \frac{B^2L^2}{R}v_{terminal}$$

$$\Rightarrow \text{Terminal Velocity } v_{terminal} = \frac{IR}{BL}$$