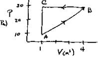
Consider the following cycle



$A \to B$ (A) $P = 10$ Pa (B) $P = 30$ Pa
$V = 1\,m^3$ $V = 4\,m^3$

$$P = P_0 + aV \qquad P_0 = 3.33\ Pa$$
$$W = \int P\,dV \qquad a = 20\ Pa/3m^3 = 6.67\ Pa\,m^{-3}$$
$$= \int (P_0 + aV)\,dV$$
$$= P_0V + \frac{a}{2}V^2 \Big|_{1m^3}^{4m^3}$$
$$= 60\,J$$

$B \to C$ Const pressure $P = 30$ Pa
$$W = \int P\,dV = P\,V \Big|_{4m^3}^{1m^3} = -90\,J$$

$C \to A$ Const Volume
$$W = 0$$

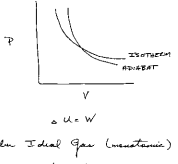
Over the entire cycle $dE_{int} = 0$
$$Q = W = -30\,J$$

ADIABATIC CHANGE

$Q = 0$ - no heat transfer

1) thermal insulation

2) rapid processes tend to be adiabatic because heat flow takes time



$\Delta U = W$

Remember Ideal Gas (monatomic)

$$PV = \frac{2}{3}\langle KE \rangle = \frac{2}{3} U$$

Let's define a constant γ such that

$$PV = (\gamma - 1) U$$

when $\gamma = \frac{5}{3}$ for a monatomic gas. In adiabatic change

$$dW = -dU = P\,dV \quad (\text{from } 1^{st} \text{ Law})$$

Now if $U = PV/(\gamma-1)$ then we can write

$$dU = (P\,dV + V\,dP)/(\gamma-1) = -P\,dV$$

Rearranging

$$\gamma P dV = -V dP$$

or

$$\gamma \frac{dV}{V} + \frac{dP}{P} = 0$$

Integrating

$$\gamma \int \frac{dV}{V} + \int \frac{dP}{P} = 0$$

Remember $\int \frac{dx}{x} = \ln x$ so above becomes

$$\gamma \ln V + \ln P = \text{const}$$

e^( ) xponentiating

$$\boxed{PV^{\gamma} = \text{const}}$$

ADIABATIC EXPANSION/ COMPRESSION

γ turns out to be the ratio of

$$\frac{\text{Specific heat at const Pressure}}{\text{Specific heat at const Volume}} \qquad \gamma = \frac{C_P}{C_V}$$

$\gamma = \frac{5}{3}$ monatomic gas $\qquad \gamma = \frac{7}{5}$ diatomic gas

Car Engine

Piston - fuel is ignited + combustion products expand rapidly + adiabatically

$P_0 = 15\,\text{atm} \quad V_0 = 50\,\text{cm}^3$

$V_1 = 250\,\text{cm}^3$

$\gamma = \frac{7}{5}$ $\quad P_0 V_0^{7/5} = 15 \times 10^5 \, Pa \, (50 \times 10^{-6} m^3)^{1.4}$

$$= 1.43$$

$$P_1 V_1^{7/5} = 1.43 \qquad P = 1.43 \, V^{-1.4}$$

$$dW = P\,dV = \text{const}\, V^{-\gamma} dV$$

$$W = \text{const} \int V^{-\gamma} dV$$

$$= \frac{\text{const}}{(1-\gamma)} V^{(1-\gamma)} \Big|_{V_0}^{V_1}$$

In this case

$$W = \frac{1.43}{0.4} V^{-0.4} \Big|_{250 \times 10^{-6} m^3}^{50 \times 10^{-6} m^3}$$

$$= 89 J \quad \text{Work done by piston}$$

at 4000 RPM, if expansion takes $\frac{1}{2}$ of cycle
$\Delta t = 0.0075 s$

$$P = \frac{\Delta W}{\Delta t} = \frac{89 J}{0.0075 s} = 11,870 W$$

<u>16 hp</u>

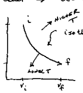
ISOTHERMAL CHANGE

Maintain the system at CONSTANT T e.g. heat bath.

- Unless heat is applied to keep an expanding gas at const T, gas will cool due to work done as gas expands.

$T = \text{const} \Rightarrow U = \text{const}; \quad \Delta U = 0$

but in general
$W \neq 0$
$Q \neq 0$



$$P = \frac{nRT}{V} = \frac{NkT}{V} = \frac{\text{const}}{V}$$

$$W = \int_{V_i}^{V_f} P(V)\, dV = \int_{V_i}^{V_f} \frac{nRT}{V}\, dV$$

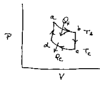
$$= nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\boxed{W = nRT \ln \frac{V_f}{V_i}}$$

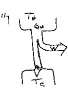
ISOTHERMAL PROCESS

How can we use Heat to do Work

A system that does work via some sort of cycle is an ENGINE or more precisely a HEAT ENGINE



Schematically



Because the engine is eg starting + returning to "a", the change in internal energy $\Delta U = 0$ so by FIRST LAW

$$W = Q \quad \text{around cycle.}$$

Let's build an engine:

e.g. Boil some liquid at T_H; heat Q_H is taken from the boiler to do work W (eg run a turbine) and heat Q_c is delivered to a condenser

$$\Delta U = 0 \quad \text{over cycle}$$

$$W = Q_H - Q_c$$

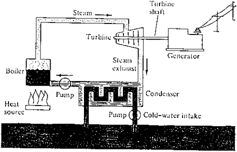


Figure 11.10. The major components of a steam electric power-generating plant. The heat source may be coal or oil or a uranium-fueled reactor.

EVAPORATOR COILS

Refrigeration box

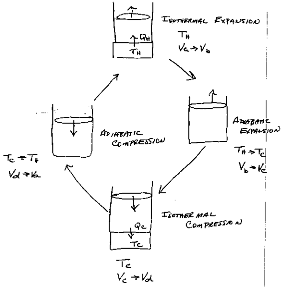
Low-pressure

High-pressure

Condenser

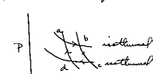
FIGURE 12-11
Typical refrigeration system.

CARNOT CYCLE



These are reversible processes — This is then an IDEAL ENGINE - best that can be done

Efficiency of an ideal engine (CARNOT CYCLE)



Isothermal expansion

$$W_{ab} = \int p\,dV = \int_a^b nRT_H \frac{dV}{V} \qquad (1)$$

$$Q_H = W_{ab} = nRT_H \ln \frac{V_b}{V_a} \qquad (2)$$

Compression

$$Q_C = -nRT_C \ln \frac{V_c}{V_b} \qquad (3)$$

Now to relate $V_{b,c}$ $V_{a,d}$ we use the adiabatic relation

$b \to c$

$$P_b V_b^{\gamma} = nRT_H \frac{V_b^{\gamma}}{V_b} = P_C V_c^{\gamma} = nRT_C \frac{V_c^{\gamma}}{V_c}$$

$$T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1} \qquad (4)$$

similarly

$$T_H V_a^{\gamma-1} = T_C V_d^{\gamma-1} \qquad (5)$$

(4) ÷ (5)

$$\frac{V_b}{V_a} = \frac{V_c}{V_d}$$

(2)+(3)

$$\frac{Q_H}{T_H} = -\frac{Q_c}{T_c} \quad \text{or} \quad \frac{|Q_H|}{T_H} = \frac{|Q_c|}{T_c}$$

$$\mathcal{E}_{CARNOT} = \frac{W}{Q_H} = \frac{T_H - T_c}{T_H}$$

Let's look at this quantity $\frac{Q}{T}$; around the cycle

$$\frac{Q_H}{T_H} + \frac{Q_c}{T_c} = 0$$

We can generalize this relationship for any closed cycle (reversible processes)

$$\Sigma \frac{Q}{T} = 0 \quad \text{or for}$$

differential

$$\oint \frac{dQ}{T} = 0$$

This relationship suggests that we can

The Efficiency

$$\varepsilon = \frac{\text{Work out}}{\text{Heat (Energy) in}} = \frac{W}{Q_H}$$

Subst.

$$\varepsilon = \frac{Q_H - Q_C}{Q_H}$$

and we can see that

$$\boxed{\varepsilon = \frac{T_H - T_C}{T_H}}$$

eg. $Q = C m \Delta T$

This is the theoretical maximum efficiency for a heat engine (CARNOT)

Note: Higher $T_H - T_C \Rightarrow$ higher efficiency
You can raise T_H - superheated steam
or lower T_C

Steam Turbine $100°C \rightarrow 0°C$
nb T is always in Kelvin for efficiency

$$\varepsilon = \frac{T_H - T_C}{T_H} = \frac{373 - 273}{373\,K}$$

$$\varepsilon = 27\%$$

The only way to get 100% efficiency is to get $T_c \to 0K$.

Increase T_H - run steam at higher pressure
$\Rightarrow T_H$ higher

$W = \int P dV$ work is higher

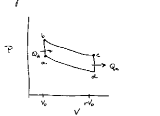
Superheated steam at 800K

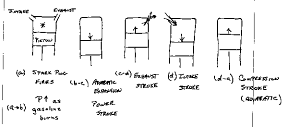
$$\varepsilon = \frac{800 - 273}{800} = 66\%$$

actually hard to get $T_c \to 0$

THE AUTO (OTTO) CYCLE

Idealized version of the internal combustion engine





COMPRESSION RATIO $r = \frac{V_{max}}{V_{min}}$

$$Q_H = mC\,(T_b - T_a)$$

$$Q_C = mC\,(T_d - T_c) \qquad \text{n.b. } Q_C \text{ is negative use } |Q_C|$$

$$\varepsilon = \frac{Q_H - |Q_C|}{Q_H}$$

$$\varepsilon = \frac{T_b - T_a - T_c + T_d}{T_b - T_a}$$

Adiabatic processes:

$$PV^{\gamma} = \text{const}$$

$$P = \frac{nRT}{V} \Rightarrow TV^{(\gamma-1)} = \text{const}$$

$$T_b V_0^{(\gamma-1)} = T_c (rV_0)^{(\gamma-1)}$$
$$T_a V_0^{(\gamma-1)} = T_d (rV_0)^{(\gamma-1)}$$

Substitute efficiency for T_b, T_a

$$\varepsilon = \frac{T_c r^{\gamma-1} - T_d r^{\gamma-1} - T_c + T_d}{T_c r^{\gamma-1} - T_d r^{\gamma-1}}$$

$$= \frac{(T_c - T_d)(r^{\gamma-1} - 1)}{(T_c - T_d) r^{\gamma-1}}$$

$$\varepsilon = 1 - \frac{1}{r^{\gamma-1}}$$

A typical compression ratio for today's cars is $r = 8$. Using $\gamma = \frac{7}{5} = 1.4$ appropriate for an ideal gas of diatomic molecules

$$\varepsilon = 56\%$$

with $T_H \sim 3300\,K$ and $T_c = 1400\,K$. So, if you want to increase efficiency just increase "r". That's why when folks wanted more fuel efficient cars the auto makers went to "high-compression

engines. But increasing r increases T_a at the end of the compression stroke and can cause the air-gas mixture to preignite $\Rightarrow$ "KNOCKING"
Need "HIGH OCTANE" gas to go to $r = 10$

Ex - An automobile engine produces 60 hp at freeway speed of 60 mph with a "fuel efficiency" of 20 mpg. What is its true efficiency

$$60 \text{ mph} / 20 \text{ mpg} = 3 \text{ gallons/hr} \cdot 3788 \text{ cm}^3/\text{gal}$$

$$= 11{,}364 \text{ cm}^3/\text{hr}$$

$\rho_{gas} = 0.7 \text{ g/cm}^3$ so the car consumes

$$16{,}234 \text{ g/hr} \div 3600 \text{ s/hr}$$

$$4.51 \text{ g s}^{-1}$$

$L_c = 5 \times 10^4 \text{ J/g}$ heat of combustion

$$P_{\text{burn}} = 4.51 \text{ g s}^{-1} \cdot 5 \times 10^4 \text{ J/g}$$

$$= 225{,}500 \text{ W} \div 756 \text{ W/hp}$$

$$= 300 \text{ hp}$$

$$\varepsilon = \frac{60 \text{ hp}}{300 \text{ hp}} = 20\%$$

4-Stroke Cycle Process