GAS LAWS

BOYLE'S LAW $\quad P \propto V^{-1}$

$$PV = \text{constant}_1 \qquad \text{const } T, m$$

CHARLES' LAW $\quad V \propto T$

$$\frac{V}{T} = \text{constant}_2 \qquad \text{const } P, m$$

GAY-LUSSAC'S LAW $\quad P \propto T$

$$\frac{P}{T} = \text{const}_3 \qquad \text{const } V, m$$

$$PV \cdot \frac{V}{T} \cdot \frac{P}{T} = \text{const}_1 \cdot \text{const}_2 \cdot \text{const}_3$$

$$\frac{P^2 V^2}{T^2} = \text{const}$$

$$\frac{PV}{T} = \text{const}$$

IDEAL GAS LAW

$$\boxed{PV = nRT}$$

n = # moles of gas

R = Universal Gas Const

$= 8.31441 \quad J/\text{mole}\cdot K$

Recall $N_A = 6.022 \times 10^{23}$ molecules/mole

AVOGADRO'S #

Define

$$\boxed{k_B = \frac{R}{N_A} = 1.3806 \times 10^{-23}\ J/K}$$

BOLTZMANN'S CONSTANT (one of the FUNDAMENTAL CONSTANTS)

and rewrite

$$\boxed{PV = N k_B T}$$

This is the Physicist's version of the IDEAL GAS LAW. This law has an amazing range of validity

e.g. Center of sun (mostly hydrogen)

$$\rho = 150\ \text{grams}/cm^3 \quad (150 \times \rho_{H_2O}$$
$$= 1.5 \times 10^5\ kg/m^3 \quad 10 \times \rho_{lead})$$

$$T = 15\ \text{million}\ K$$

Atomic hydrogen has a mass of 1 gram/mole

$V = 1\,m^3$ has 1.5×10^5 moles of H

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$= \frac{1.5\times10^5_{mol} \cdot 8.314_{J/mol \cdot K} \cdot 1.5\times10^7 K}{1\,m^3}$$

$$= 1.87 \times 10^{13} Pa$$

$$= 1.84 \times 10^8\ atm$$

The previous calculation is not quite right because hydrogen is ionized. 1 mole of ionized hydrogen has

$2 \cdot N_A\ (p^+ + e^-)$ particles

both of which contribute to P

$$PV = N k_B T$$

$$P = 1.5\times10^5_{mol} \cdot 2 \cdot 6.022\times10^{23} \cdot 1.38\times10^{-23} J/K \cdot 1.5\times10^7 K / 1\,m^3$$

See fig 14.17

$P = 3.74 \times 10^{13}\,\text{Pa} = 3.68 \times 10^{8}$ atmospheres

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$\langle\ \rangle$ = avg (mean) $\langle v^2 \rangle$ = mean square velocity

Your book derives the relationships between P, E, v, T. We'll look at it a little differently. Ludwig Boltzmann (1844-1906) developed the theory of atomic motions - KINETIC THEORY - at a time when the existence of atoms was still controversial.

Boltzmann. In a gas with N particles

$$N(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T}$$

or in terms of energy $\frac{8\pi}{m}\left(\frac{m}{2\pi kT}\right)^{3/2} KE \cdot e^{-KE/k_B T}$

From this it is easily shown that

$$P = \frac{1}{3}\rho \langle v^2 \rangle \qquad \rho = \frac{Nm}{V}$$

so that

$$PV = \frac{1}{3} Nm \langle v^2 \rangle \qquad \langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$PV = \frac{2}{3} N \langle KE \rangle$$

From the IDEAL GAS LAW

$$PV = N k_B T = \frac{2}{3} N \langle KE \rangle$$

$$\boxed{\langle KE \rangle = \frac{3}{2} k_B T}$$

The importance of this relationship not sufficiently emphasized in the Book. It is FUNDAMENTAL

TEMPERATURE IS A MEASURE OF ENERGY

Then we have

$$\langle v^2 \rangle = \frac{3k_B T}{m}$$

$$v_{\text{root mean square}} = \sqrt{\langle v^2 \rangle} = \left(\frac{3k_B T}{m} \right)^{1/2}$$

At room temperature, the speed of the molecules in this room

$$v_{rms} = \left(\frac{3 \cdot 1.38 \times 10^{-23}\ \text{J/K} \cdot 300\text{K}}{\begin{matrix} 28\,(N_2) \\ 32\,(O_2) \end{matrix} \cdot 1.67 \times 10^{-27}\ \text{kg}} \right)$$

$$\simeq 500\ \text{m s}^{-1} \quad (\sim 1000\ \text{mph})$$

See fig 14.21