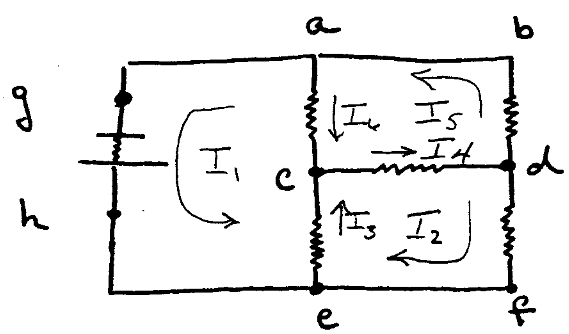
KIRCHOFF'S RULES

Not all networks can be reduced to simple series - parallel combinations e.g.



We use the method described by Kirchoff's rules to solve such networks

Definitions

1) BRANCH POINT - point where 3 or more conductors meet

   Branch points above: a, c, d, e

2) LOOP - any closed conducting path

   Above loops:

   abdea, cdfec, ghecag,

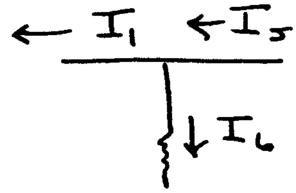
   ghefdbag, abdfeca,

   etc.

2) Use the point rule to reduce the unknown currents. If there are n branch points, apply the point rule to $n-1$ points (last one is not independent

Above ckt:

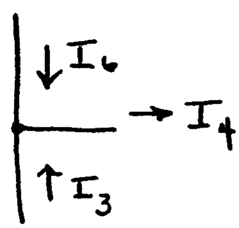
a



$I_5 = I_1 + I_6$

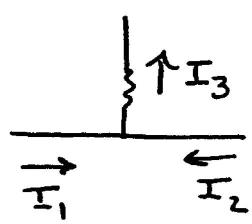
(in) (out)

c



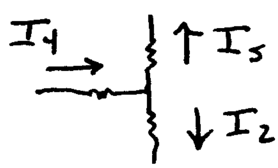
$I_6 + I_3 = I_4$

e



$I_1 + I_2 = I_3$

d



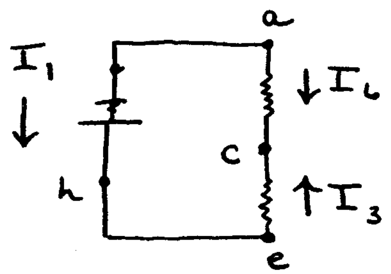
$I_4 = I_5 + I_2$

This is redundant; duplicate info need only 3

3) Separate the circuit into individual closed loops. Choose a closed loop and a direction to proceed around the loop. Apply the loop rule in the chosen direction adding emfs and IR terms as follows:

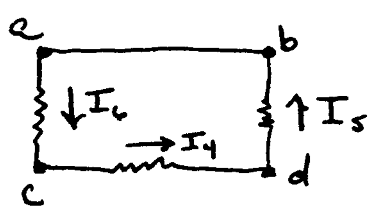
emf is <u>positive</u> if from − to +
<u>negative</u> " " + to −

IR term <u>positive</u> if same direction as current
<u>negative</u> if opposite " "

1) 

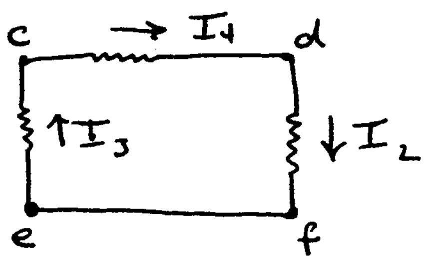
Proceed ccw $\Sigma \mathcal{E} = \Sigma IR$

$$\mathcal{E} = I_1 r + I_3 R - I_6 R$$

2) 

ccw, no $\mathcal{E}$

$$0 = I_6 R + I_4 R + I_5 R$$

3) 

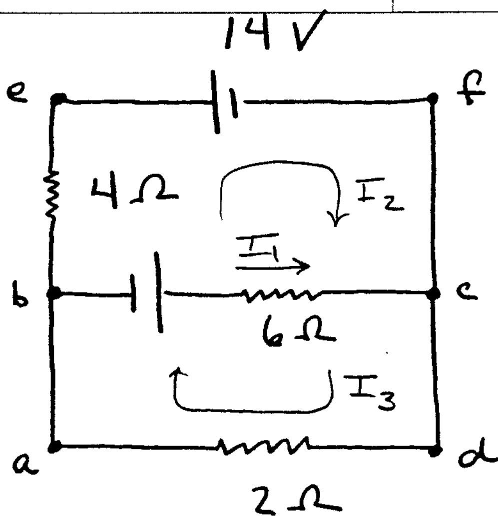
cw

$$0 = I_3 R + I_4 R + I_2 R$$

4) Use as many loops as needed

5) Solve n eqns simultaneously for n unknowns

EXAMPLE



Branch points : b + c

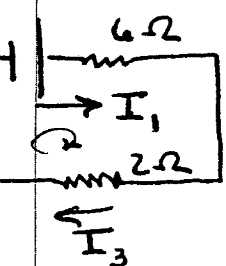
Point rule at c $\quad I_1 + I_2 = I_3 \quad$ ①

or

$I_1 + I_2 - I_3 = 0$

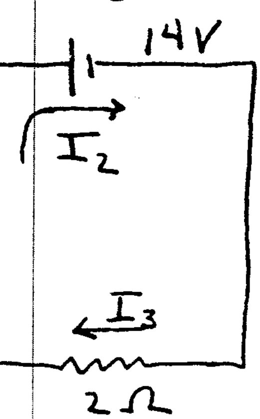
(note applying point rule at b yields $I_3 = I_1 + I_2$ which is equivalent; 2 branch points ⇒ 1 eqn)

Loops abcda, abefcda, befcb



Loop rule abcda proceeding cw

$10\,V = I_1\, 6\,\Omega + I_3\, 2\,\Omega \quad$ ②



Loop rule abefcda proceeding cw

$-14\,V = I_2\, 4\,\Omega + I_3\, 2\,\Omega \quad$ ③

3 eqns 3 unknowns Subst ① in ②

$10\,V = I_1\, 6\,\Omega + (I_1 + I_2)\, 2\,\Omega$

reduces to $\quad I_2 = 5 - 4 I_1 \quad$ Subst in ③

$-14\,V = (5 - 4I_1)\, 4\,\Omega + 2\,\Omega\,(I_1 + 5 - 4 I_1)$

$$-14 = 20 - 16 I_1 + 2I_1 + 10 - 8I_1$$

Collecting terms

$$22 I_1 = 44$$

$$I_1 = 2 \text{ amps}$$

$$I_2 = 5 - 4 I_1 = 5 - 8$$

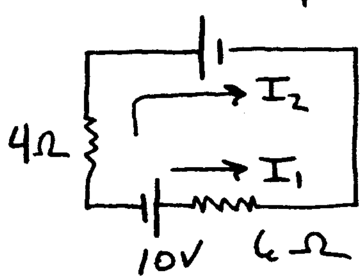
$$= -3 \text{ amps}$$

$$I_3 = I_1 + I_2 = 2 - 3$$

$$= -1 \text{ amp}$$

Could equally well have used loop befcb

Proceed cw



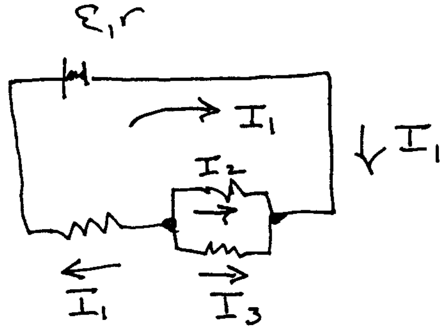
$$-14\,V - 10\,V = -I_1\, 6\,\Omega + I_2\, 4\,\Omega$$

$$-24 = -6 I_1 + 4(5 - 4I_1)$$

$$22 I_1 = 44$$

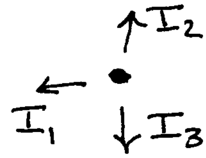
$$I_1 = 2 \text{ amps} \ldots$$

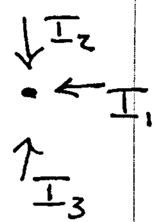
Yesterday's PROBLEM USING KIRCHOFF'S RULES



NODES

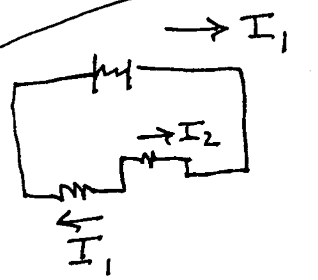
a, b are nodes





$$0 = -I_1 - I_2 - I_3$$

LOOPS



$$\sum \varepsilon = \sum IR$$

CW $\quad 0 = I_2 R_2 - I_3 R_3$

CCW $\quad \varepsilon = -I_1 r - I_1 R_1 + I_2 R_2$

$$18V = -I_1\, 2\Omega - I_1\, 2\Omega + I_2\, 6\Omega$$

$$= -I_1\, 4\Omega + I_2\, 6\Omega$$

$$I_2 = I_3 \frac{R_3}{R_2} \quad \text{or} \quad I_3 = I_2\, R_2/R_3$$

$$I_1 = -I_2 - I_2\, R_2/R_3$$

$$= -I_2\,(1 + R_2/R_3)$$

$$I_2 = \frac{-I_1}{1 + R_2/R_3} = -\frac{I_1}{1 + 6\,\Omega/3\,\Omega}$$

Subst

$$18\,V = -I_1 \cdot 4\,\Omega + 6\,\Omega\left(\frac{-I_1}{3}\right)$$

$$= -I_1 \cdot 6\,\Omega$$

$$I_1 = -3\,A$$ oops I_1 goes other way

$$I_2 = -\frac{I_1}{3} = 1\,A$$

$$I_3 = I_2 \cdot 2 = 2\,A$$