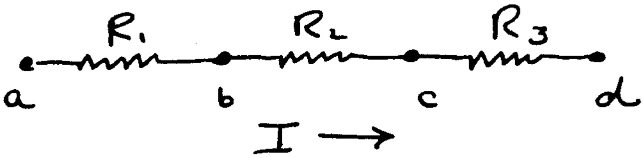
Complex DC Circuits

Resistors in series:



Same current in each resistor

$$V_{ab} = I R_1$$
$$V_{bc} = I R_2$$
$$V_{cd} = I R_3$$

V_{ad} = potential difference across network

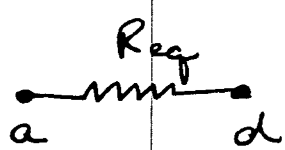
$$= V_{ab} + V_{bc} + V_{cd} = I R_1 + I R_2 + I R_3$$
$$= I (R_1 + R_2 + R_3)$$

What is the "equivalent" resistance such that we can write

$$V_{ad} = I \cdot R_{eq} \quad \text{or} \quad R_{eq} = \frac{V_{ad}}{I}$$

$$V_{ad} = I R_{eq} = I (R_1 + R_2 + R_3)$$

$$R_{eq} = R_1 + R_2 + R_3$$

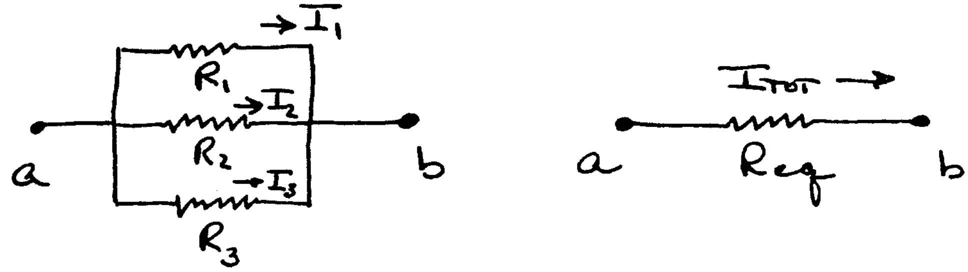


Series resistor

$$\boxed{R_{eq} = \sum R_i}$$

$R_{eq} >$ any R in ckt

Resistors in parallel



Potential difference V_{ab} is the same regardless of path

$$V_{ab} = I_1 R_1 = I_2 R_2 = I_3 R_3$$

Total current a → b

$$I_{TOT} = I_1 + I_2 + I_3$$

$$I_1 = \frac{V_{ab}}{R_1} \quad I_2 = \frac{V_{ab}}{R_2} \quad I_3 = \frac{V_{ab}}{R_3}$$

$$I_{TOT} = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3} = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$I_{TOT} = \frac{V_{ab}}{R_{eq}} \quad \text{Ohm's law}$$

$$\therefore \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Parallel resistors

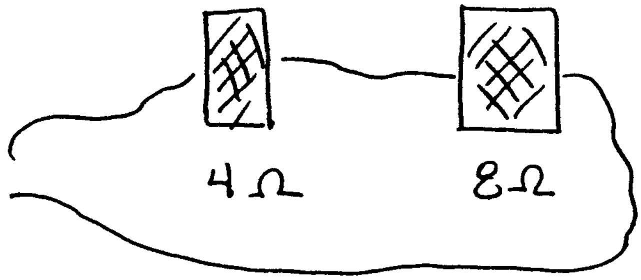
$$\boxed{\frac{1}{R_{eq}} = \sum \frac{1}{R_i}}$$

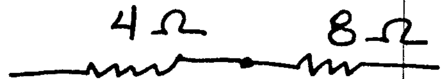
Stereo System

Power Amplifier 150 Watts per channel

Speakers usually $4\ \Omega$ or $8\ \Omega$

In series



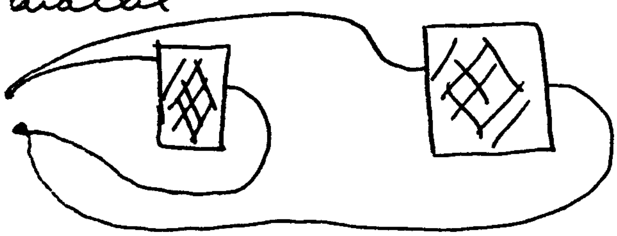


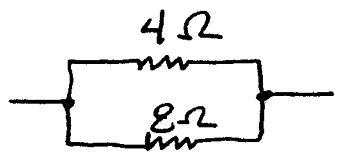
$$R_{eq} = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

$$P = I^2 R \qquad I = \left(\frac{P}{R}\right)^{1/2} = \sqrt{\frac{150\ W}{12\ \Omega}}$$

$$\underline{I = 3.5\ A}$$

Parallel





$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{32\ \Omega^2}{12\ \Omega}$$

$$\underline{R_{eq} = 2.67\ \Omega}$$

note this is lower than either ind. resistance

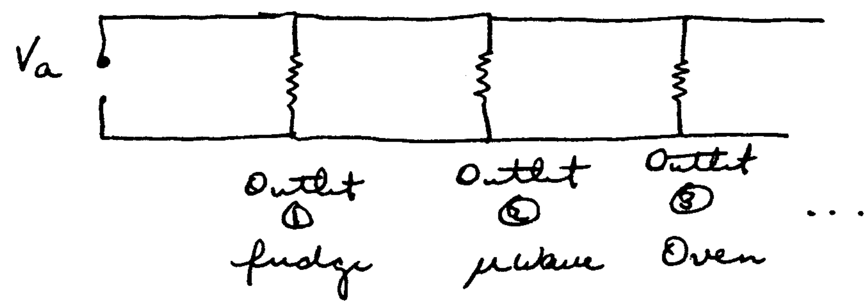
$$P = I^2 R$$

$$I = \sqrt{\frac{150\ W}{2.67\ \Omega}}$$

$$\underline{I = 7.5\ A}$$

Some transistor amps don't like $R < 4\ \Omega$ cause of large currents

Household wiring - lights, appliances is usually parallel

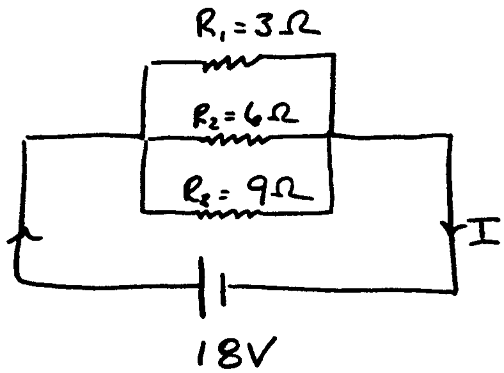


Don't load up too much; as you add add'l parallel resistances the R_{eq} goes down I goes up, trips circuit breaker (e.g. 15 A) or worse ...

ex

What is I

I_1, I_2, I_3



$V = 18V$ across each resistor

$$I_1 = \frac{V}{R_1} = \frac{18V}{3\Omega} = 6\ A$$

$$I_2 = \frac{18V}{6\Omega} = 3A$$

$$I_3 = \frac{18V}{9\Omega} = 2A$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3\Omega} + \frac{1}{6\Omega} + \frac{1}{9\Omega} = \frac{11}{18\Omega}$$

$$R_{eq} = \frac{18}{11}\Omega \quad I = \frac{V}{R_{eq}} = 18/\tfrac{18}{11} = 11\ Amp = I_1 + I_2 + I_3$$