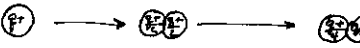
tatic or stationary electrical charges; we will see later that moving charges produce magnetic forces.)

Coulomb in 1784 developed a means of separating charge into known packages. No basic unit of charge, but by bringing identical objects (one charged one not) into contact he reasoned that the charge must be equally distributed between the two objects. He could then divvy up charges into relatively known quantities

e.g. Charge up body w/ same charge "q"

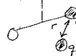


can get $\frac{q}{2}$, $\frac{q}{4}$ ...

Using a torsion balance (similar to that employed by Cavendish to determine grav. force)

Coulomb determined that:

Force depends on product of charges involved



Force depends on the product of the charges involved

$$F \propto q_1 \times q_2$$

Force depends upon inverse square of of separation of charges

$$F \propto \frac{1}{r_{12}^2}$$

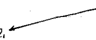
Then there must be some constant of proportionality to make units come out right

$$F = k \frac{q_1 q_2}{r^2}$$

Hopefully this looks a little familiar
Remember

$$F_g = G \frac{m_1 m_2}{r^2}$$

There are some very impt similarities and some very impt differences.



$$F = k \frac{q_1 q_2}{r^2}$$

Properties of Coulomb force

1) Directed along line between q_1 + q_2; Force is a __vector__

2) Repulsive if q_1 + q_2 have the same sign (Force positive)

3) Attractive if q_1 + q_2 have opposite sign (F negative)

In mks system

length = meter (m)
mass = kilogram (kg)
time = second (s)
force = Newton ($N = kg\, m\, s^{-2}$)
charge = Coulomb (C) by definition

then our force constant

$$k = 8.98755\ N m^2 C^{-2} = 10^{-7} c^2$$

we usually write this in a different form

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2}$$

where $\frac{1}{4\pi\varepsilon_0} = k$

$$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$$

turns out this is more natural in some situations

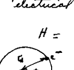
ELECTROSCOPE

Comparison w/ gravitational force

mass ⇒ charge (but in electricity have two forms of charge attraction/repulsion; grav only attraction)

Both have inverse square law

Grav force vs. electrical force; compare grav force of a hydrogen atom vs electrical force



$H =$ proton (carrier of positive charge) $= 1+ = +1.6 \times 10^{-19} C$

mass $= 1 amu: 1.67 \times 10^{-27} kg$

electron (carrier of negative charge) $= 1- = -1.6 \times 10^{-19} C$

mass $= 1/1836 amu = 9.10 \times 10^{-31} kg$

$F_E = k \frac{q_p q_e}{r_{pe}^2}$ $\qquad F_g = G \frac{m_e m_p}{r_{pe}^2}$

In a minute we'll consider the size of the hydrogen atom but for now

$$\frac{F_e}{F_g} = \frac{k q_p q_e}{r_{pe}^2} \Big/ \frac{G m_e m_p}{r_{pe}^2}$$

$$= \frac{k q_p q_e}{G m_e m_p}$$

$$= \frac{9 \times 10^9 \ N m^2 C^{-2} \cdot 1.6 \times 10^{-19} C \cdot 1.6 \times 10^{-19} C}{6.67 \times 10^{-11} N m^2 kg^{-2} \cdot 1.67 \times 10^{-27} kg \cdot 9.1 \times 10^{-31} kg}$$

$$= 2.3 \times 10^{39}$$

Electrical forces are nearly 10^{40} times stronger than gravitational forces. Then why does grav. force _seem_ to dominate daily life. Because charges cancel out → +, − electrical charge but gravitational "charge = mass" has only one sign

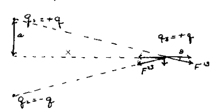
Coulomb's law

$$F = k \frac{q_1 q_2}{r_{12}^2}$$

$k = \frac{1}{4\pi\varepsilon_0}$ "permittivity of free space"

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2}$$

Consider 3 charges



Find force on q_3

$$F^{13} = k \frac{q_1 q_3}{r^2} = k \frac{q^2}{a^2+x^2}$$

$$F_x^{13} = k \frac{q^2}{a^2+x^2} \cos\theta \qquad F_y^{13} = -k \frac{q^2}{a^2+x^2} \sin\theta$$

where $\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$

$\sin\theta = \frac{a}{r} = \frac{a}{\sqrt{x^2+a^2}}$

$$F^{23} = \frac{k q_2 q_3}{r^2} = -k \frac{q^2}{a^2+x^2}$$

$$F_x^{23} = -\frac{k q^2}{a^2+x^2} \cos\theta \qquad F_y^{23} = -k \frac{q^2}{a^2+x^2} \sin\theta$$

$$F_x = F_x^{13} + F_x^{23} = 0$$

$$F_y = F_y^{13} + F_y^{23} = -2k \frac{q^2}{a^2+x^2} \frac{a}{\sqrt{a^2+x^2}}$$

$$x >> a \qquad x^2 + a^2 \approx x^2$$

$$F_y = -\frac{2kaq^2}{x^3}$$ Force falls off as $\frac{1}{dist^3}$

Above charge dist'n is called an electric dipole. Because most bodies are electrically neutral have equal # + & - charges, we can often treat electromagnetic interactions of material as interactions of large number of +/- dipoles.

ELECTRIC FIELD

Suppose we place a charged body at point A and a charge q' at point B the charge at B feels a force F. Just as we don't really know "why" matter attracts other matter via gravity, neither do we know the why or the details of the force between charged particles. We have, however the experimental fact that there are forces between charged bodies and that they obey Coulomb's law

$$F = k \frac{q_1 q_2}{r_{12}^2}$$

What if we remove the charge at B. It is useful to think of the space in the vicinity of A as influenced by the presence of the electrical charge. No force exists

·se 19.2 Suppose that the force between the sun and the earth was ...ical origin. Assume that the charges on the sun and the earth are equal magnitude but opposite in sign. Calculate this charge and compare it with ie charge of an electron. [*Ans.*: $q = 2.98 \times 10^{17}$ C = 1.85×10^{36} e.]

19.4. The Concept of Electric Field

Newton's universal gravitation law and Coulomb's law enable us to calculate ie magnitudes as well as the directions of the gravitational and electrical rces, respectively. These laws are limited to describing only interactions of vo point charges (or masses). On the contrary, the field concept is a powerful ol to handle distributed and continuous charges and their interactions. But iere are still no answers to the following more fundamental and basic quesons: (a) what are the origins of these forces? (b) how are the forces transitted from one mass to another or from one charge to another?

The answer to (a) is unknown; the existence of forces is accepted de facto. ut ever since the formulation of the force laws, there have been theories oposed to explain (b). Most scientific thought has been divided along two ies: (1) the action-at-a-distance effect, and (2) the field effect or field theory. the following we limit ourselves to the discussion of Coulomb or electrical rces, but the same remarks apply equally to gravitational forces.

Action-at-a-distance means that the force between two charged bodies is onveyed directly and instantaneously (with no time delay) between the two odies. Accordingly, the force between two charges is considered to be a e-step process. Even though it is an experimental fact that the forces can be nveyed across empty space, the idea of action-at-a-distance has never been a r...ing thought to scientists. There must be some agent or intermediate that transmits force, or change in force, from one point to another.

The concept of a field theory was introduced by Michael Faraday (1791–1867) and is becoming more and more useful and convenient. According to the *field concept*, the interaction between two charges q and q_0 separated by a stance r is explained in the following manner. The charge q produces an ectric field in the space surrounding it. This field exists whether there are her charges present in the space or not. But the presence of this field cannot detected until another charge q_0 is brought into the field. The field of charge interacts with the test charge q_0 to produce an electrical force. Thus, the teraction between q and q_0 is a two-step process: (1) the charge q produces a ld, and (2) the field interacts with charge q_0 to produce a force **F** on q_0. hese two steps are illustrated in Figure 19.9. The two processes are reversible. or example, charge q_0 may produce a field, and if a charge q is brought in the ld of q_0 at a distance of r, it feels a force $-\mathbf{F}$.

We know from Coulomb's law that the magnitude of the force **F** depends pon the separation distance r between the two charges. In terms of the field ncept, we say that the strength of the field is very high near charge q and creases as $1/r^2$. We may define the *electric field strength* or *electric field tensity* **E** due to a charge q at a distance r from it to be

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \tag{19.7}$$

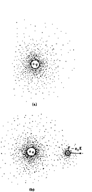


FIGURE 19.9. (a) Dots surrounding the positive charge indicate the presence of the electric field. The density of the dots is proportional to the strength of the electric field at different points. (b) Interaction of the field with the charge q_0.

at B if there is no charge present; nevertheless we say that there is an <u>ELECTRIC FIELD</u> present due to the charges at A. If a "test charge" is placed at B it will feel a force; we may describe this force as due to the electric field at B. The electric field is defined as the force felt by the test charge divided by the amount of the test charge

$$\boxed{\vec{E} = \frac{\vec{F}}{q}}$$

Units $N\,C^{-1}$

In this view the electric field is a property of space in the neighbourhood of A. Any charge placed in the field will feel a force $\vec{F} = q'\vec{E}$

(N.B. If "+" charge $\vec{F}$ same direction as $\vec{E}$
If − " $\vec{F}$ opposite direction

Now what we want in discussing the electric field are the characteristics of the space purely due to the charge at A. Obviously if we place significant charge at B we will alter the ~~field~~ distribution of charge on body A and hence the electric field, thus what we really want is

$$\vec{E} = \lim_{q' \to 0} \frac{\vec{F}}{q'}$$

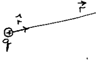
$\vec{E}$ FIELD OF A POINT CHARGE

If we have a point charge q + test charge q', by Coulomb's law

$$F = k \frac{q q'}{r^2}$$ at the point of the test charge. Then the electric field at that point

$$E = \frac{F}{q'} = k \frac{q}{r^2}$$

The electric field vector is along the radius vector connecting the point charge and the point in question



$\vec{E}$ away from q if q positive

$\vec{E}$ toward q if q negative

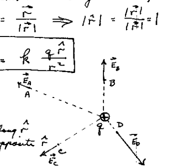
There is convenient notation for specifying this direction

$\hat{r}$ = unit vector (magnitude = 1) along $\vec{r}$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} \Rightarrow |\hat{r}| = \frac{|\vec{r}|}{|\vec{r}|} = 1$$

then

$$\boxed{\vec{E} = k \frac{q\,\hat{r}}{r^2}}$$



NB

1) $|\vec{E}| = \frac{kq}{r^2}$

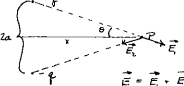
2) q positive $\vec{E}$ along $\hat{r}$

3) q negative $\vec{E}$ opposite $\hat{r}$

If we have a number of point charges then the electric field at any point is the vector sum of the fields due to each of the individual charges (Just as the force would be if we looked at the net $\vec{F}$ on a test charge at that point

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots = k \sum_i \frac{q_i \hat{r}}{r_i^2}$$

Recall our dipole (now w/o third "test charge")



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = (E_{1x}, E_{1y}) \qquad \vec{E}_2 = (E_{2x}, E_{2y})$$

$$\vec{E}_1 = \frac{k\, q\, \hat{r}_1}{r_1^2} = \frac{k q\, \hat{r}_1}{a^2 + x^2} \qquad E_2 = -\frac{k q\, \hat{r}_2}{a^2 + x^2}$$

$$E_{1x} = E_1 \cos\theta \qquad \text{where } \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$$

$$E_{1y} = E_1 \sin\theta \qquad \sin\theta = \frac{a}{r} = \frac{a}{\sqrt{x^2+a^2}}$$

$$E_{1x} = \frac{kq\,x}{(x^2+a^2)^{3/2}} \qquad \text{Similarly } E_{2x} = -\frac{kq\,x}{(x^2+a^2)^{3/2}}$$

$$E_{1y} = \frac{kq\,a}{(x^2+a^2)^{3/2}} \qquad E_{2y} = \frac{kq\,a}{(x^2+a^2)^{3/2}}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (E_x, E_y)$$

$$E_x = E_{1x} + E_{2x} = 0$$

$$E_y = E_{1y} + E_{2y} = \frac{2kq\,a}{(x^2+a^2)^{3/2}}$$

$x \gg a$

$$\vec{E} = \frac{kq\,2a}{(x^2+a^2)^{3/2}}\,\hat{r}_1 \approx \frac{kq\,2a}{x^3}\,\hat{r}_1$$

If place chg #3 $q_3 = q$

$$\vec{F} = q_3\vec{E}$$

$$\vec{F} = \frac{kq^2\,2a}{x^3}\,\hat{r}_1$$

Same as from Coulomb law