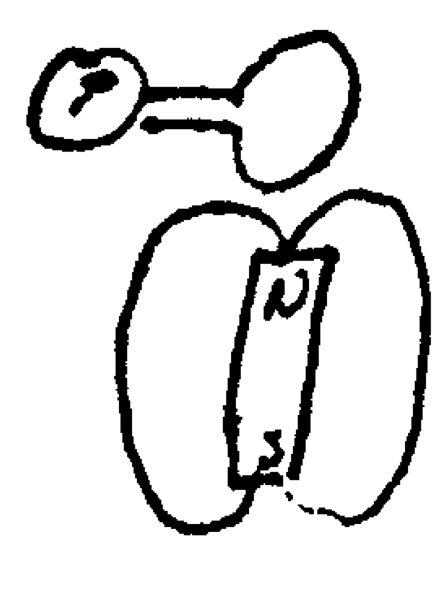
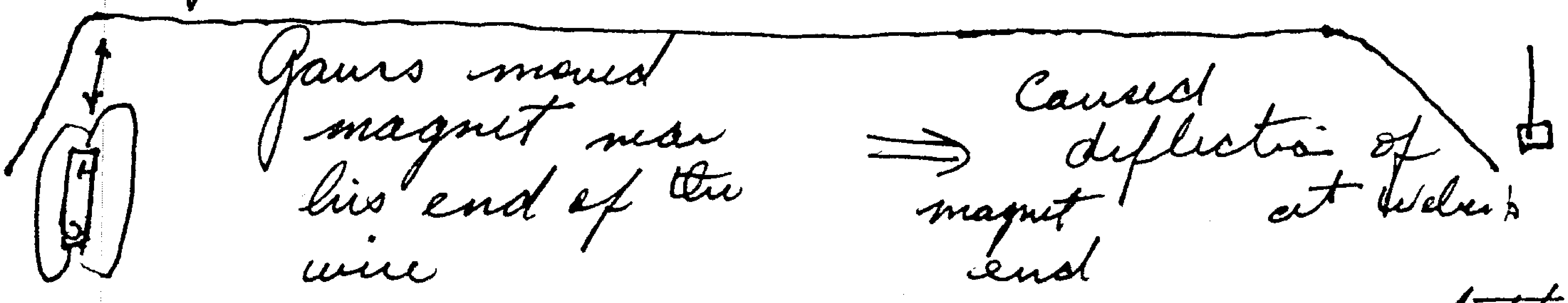
INDUCED CURRENT / EMF

Interest and excitement was generated by discovery that E + Mag related phenomena. From a better understanding of magnetic phenomena it was hoped that one would be able to use magnetic forces to do work. 1820 discovered that currents produce $\vec{B}$ fields and then that current carry wires have forces on them. Real breakthrough in use of magnetic fields to do useful work came w/ discovery by Faraday and independently by Henry that one could use a $\vec{B}$ field to produce a current. Tried various expts to see if could get a current in one wire by passing current through neighboring wire – no go. It was Faraday + Henry who discovered that a <u>changing</u> magnetic produces a current. If you take a galvanometer attached to current loop + pass a bar magnet through it — galvanometer deflects. We have <u>induced</u> a current in the wire. Apparently motion of mag field wrt the wire produces a current.



Gauss + Weber (who developed galvanometer) studied this phenomenon. Strung wire across Göttingen from Gauss' observatory outside town over the North Tower of the church to Weber's laboratory at the edge of the Leine canal. They wanted to see just how far this induced potential could be used to produce current.



Worked out code + could send 8 letters a minute. 1st message "Michelmann kommt" took longer than Michelmann.

Gauss: "vor der die Phantasie fast erschrickt" → "Before which the imagination reels"

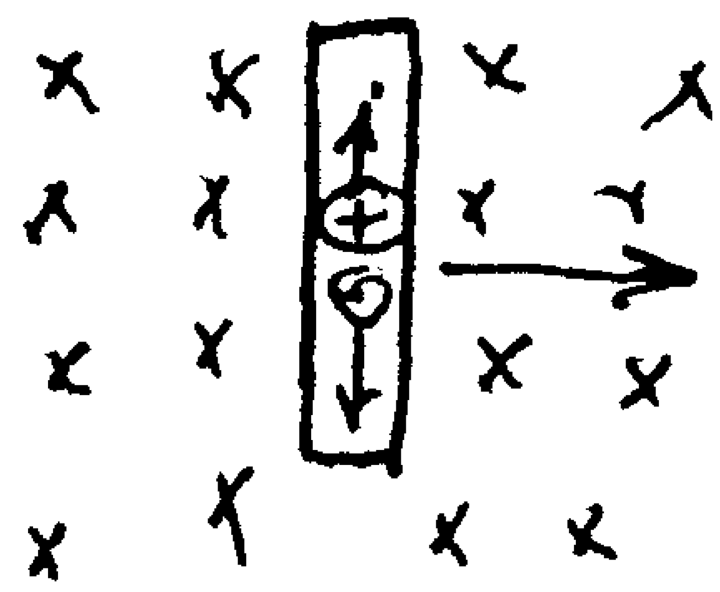
INDUCED EMF

Gauss one of most prolific of scientists

1) Mathematician - worked out geometry which is basis of GR
2) E + M - galvanometer
3) Procedure to determine orbits of minor planets & other astronomical
4) 13 children

Well let's see just what's going on here

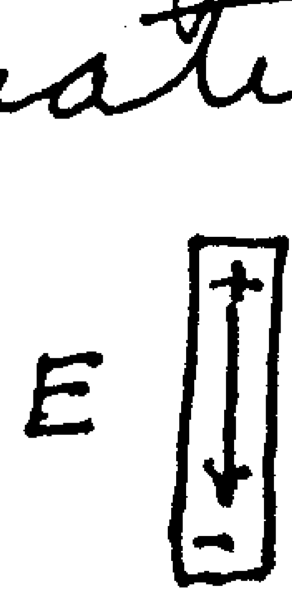
Conductor moving in $\vec{B}$ field



If conductor moves w/ velocity v, then any charge q will feel a force

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

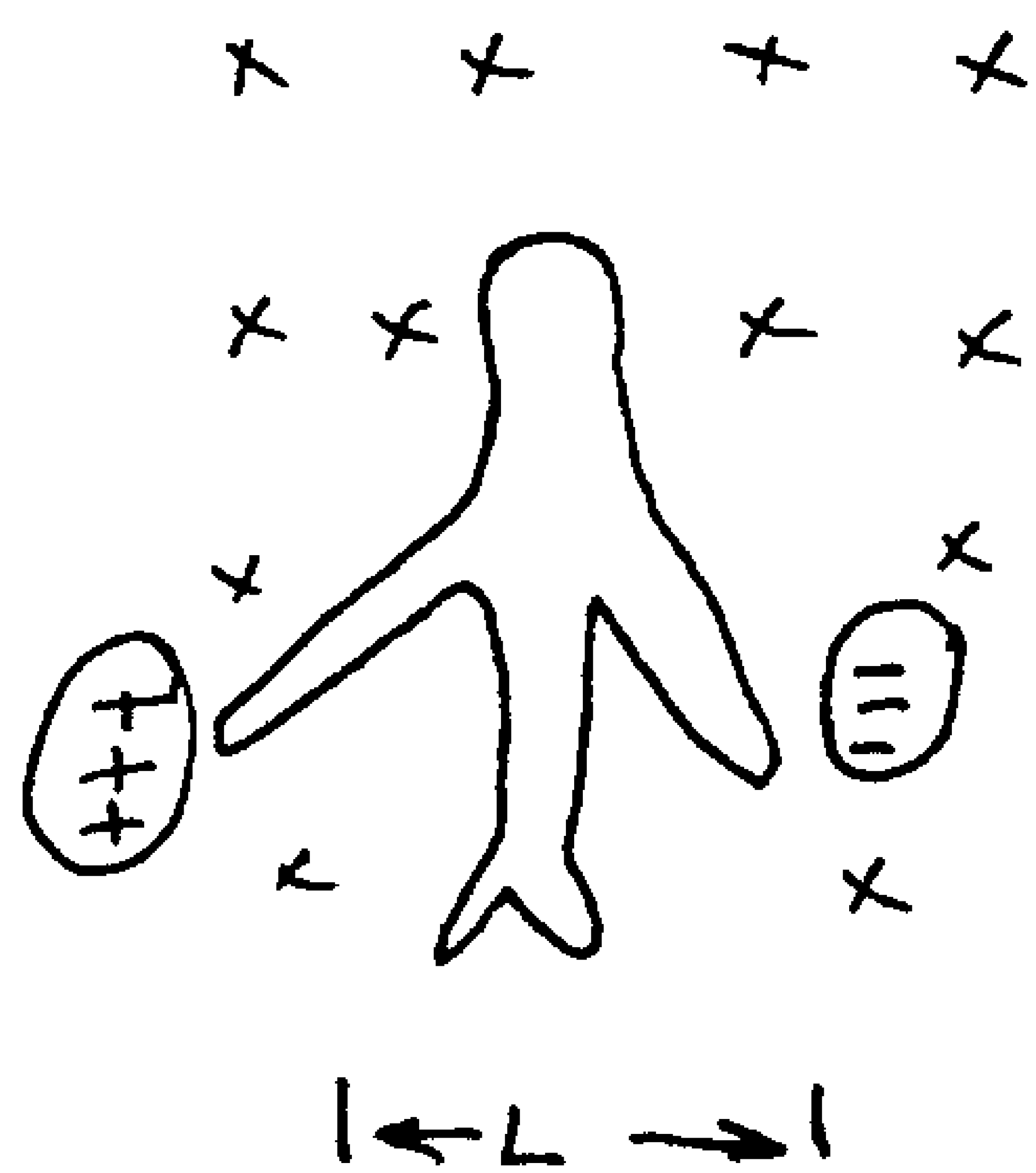
As shown ⊕ charges feel force upward ⊖ charges feel force downward ⇒ separation of charges which Then



creates an electrostatic field E. When suff't charge has been moved

The electrostatic force will balance nonelectrostatic (magnetic force) so that net force is zero

e.g. An 747 aircraft is flying at 500 mph through the earth's magnetic field $B = 1\,G = 10^{-4}\,T$.
What is the motional emf?



WINGSPAN = 64m

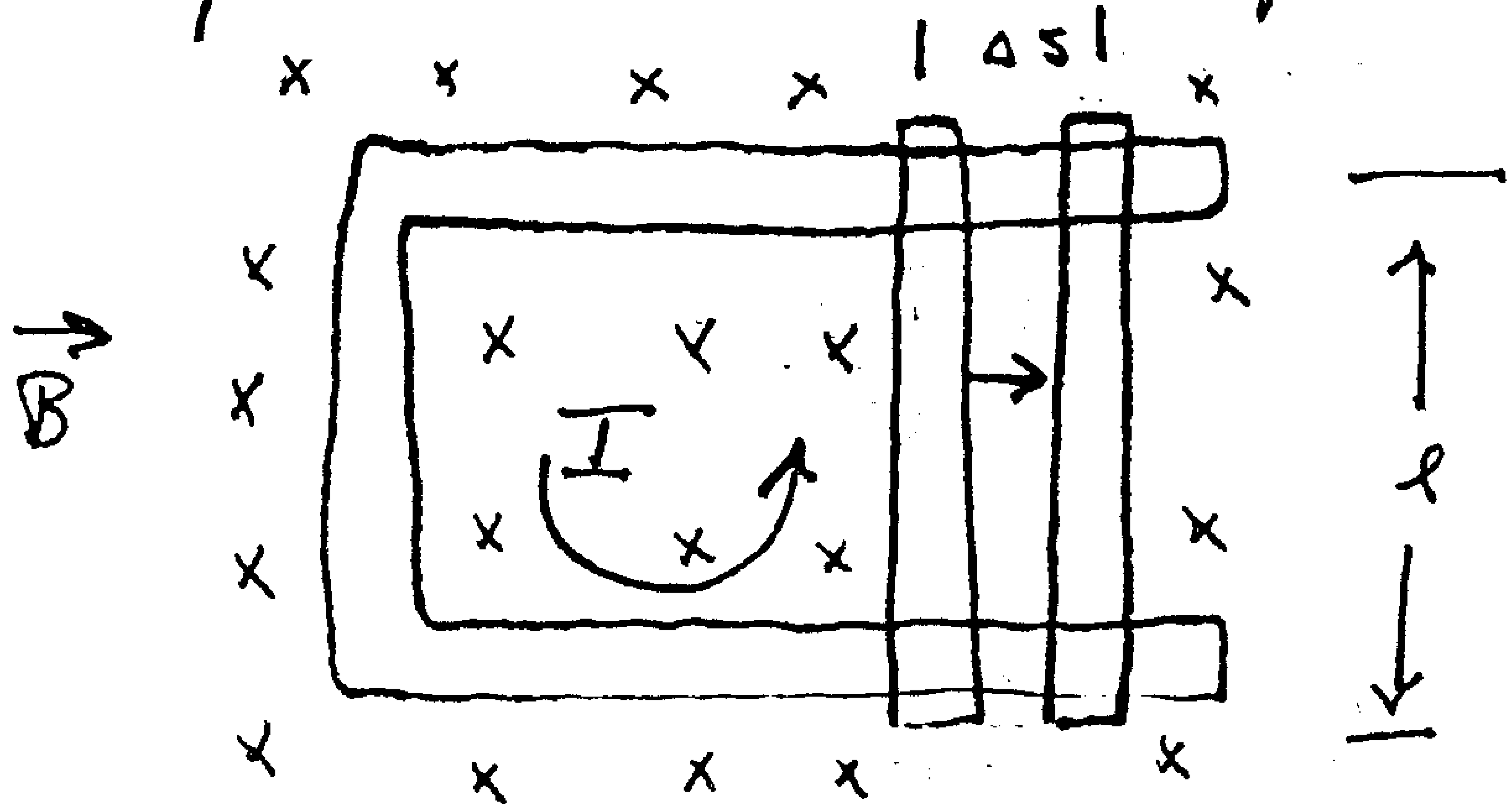
$$\mathcal{E} = vBL$$
$$= 225\,m\,s^{-1} \cdot 10^{-4}\,T \cdot 64\,m$$
$$= 1.4\,V$$

Which wing is at higher potential?

LEFT WING is + ; RIGHT is −

$$E = v \times B$$

If we have a complete circuit



Motion of bar causes charge separation in bar, remainder of conductor

feels a "push" $\Rightarrow$ current will flow around loop

Moving conductor - "seat of induced emf"

From our previous def'n of emf

$$\mathcal{E} = \int_b^a \vec{E}_n \cdot d\vec{s} = E_n \ell = \frac{F}{q} \ell$$

$$= v B \ell$$

If R is the resistance of the loop

1) Current through loop

$$I = \frac{\mathcal{E}}{R} = \frac{vB\ell}{R}$$

2) Since there is a current there is a force $\quad F = I \ell B$

3) Since the force opposes motion, must do work to move the bar

$$W = \int F \cdot ds = F \Delta s$$
$$= I \ell B \Delta s$$

or power
$$\Delta W / \Delta t = I \ell B \frac{\Delta s}{\Delta t} = I \ell B v$$

Define a new quantity $\Phi = \int \vec{B} \cdot d\vec{A}$

or $d\Phi = \vec{B} \cdot dA$ "magnetic flux"

$$= \vec{B} \, \ell \Delta s$$

look at $d\Phi / dt$

$$\frac{d\Phi}{dt} = B \ell \frac{\Delta s}{\Delta t}$$

$$= v B \ell$$

3) Power input to move conductor

$$P = Fv = I\ell B \cdot v$$

$$I = \frac{vB\ell}{R}$$

$$P = \frac{vB\ell}{R} \cdot \ell B v = \frac{(vB\ell)^2}{R}$$

4) Rate of energy conversion

$$P = \mathcal{E} I = (vB\ell)\left(\frac{vB\ell}{R}\right) = \frac{(vB\ell)^2}{R}$$

Translating mechanical power (moving the conductor) to electrical power

FARADAY'S LAW

Can consider induced emf another way. For our loop as the conductor moves we have

$$dA = \ell\, ds$$

If area is changing then flux through circuit is changing

$$d\Phi = \vec{B} \cdot d\vec{A} = B\,dA = B\ell\, ds$$

$$\frac{d\Phi}{dt} = B\ell \frac{ds}{dt} = vB\ell$$

But $vBl = \mathcal{E}$ (induced emf)

Conventions

$\mathcal{E}$ - positive if results in current in clockwise direction

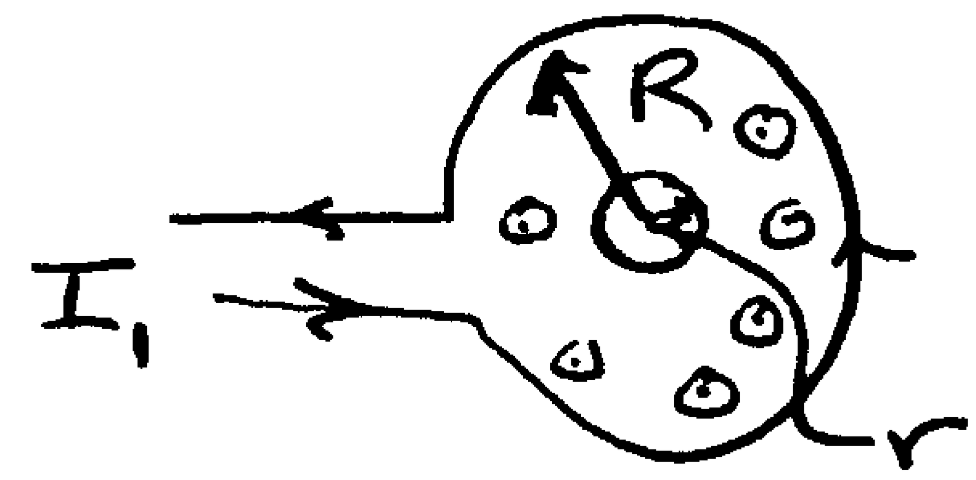
$\frac{d\Phi}{dt}$ - positive if increase in flux directed away from observer

$\mathcal{E}$ and $\frac{d\Phi}{dt}$ always have opposite sign

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$ Faraday's Law

This expands our principle; says that any change in Φ results in induced emf

Ex:



Wire loop with current I_1

$I_1 = 10\,A$ $\quad R = 10\,cm$

$$B = \frac{2\pi k' I}{R} = \frac{2 \cdot 3.14 \cdot 10^{-7} \cdot 10A}{0.1m}$$

$$B = 6.28\times 10^{-5}\,T$$ out of page

Small loop w/ center at $R=0$, $r = 1cm$
let resistance around small loop

$$\Phi = \int \vec{B}\cdot d\vec{A} = BA$$
$$= 6.28\times10^{-5}\,T \cdot \pi r^2$$
$$= 2\times 10^{-8}\,Wb$$

Now suppose the current varies so that

$$\frac{di}{dt} = -0.1\,As^{-1}$$

$$\frac{dB}{dt} = \frac{2\pi k'}{R}\frac{di}{dt} = 6.28\times10^{-6}\ (0.1\,As^{-1})$$
$$= -6.28\times10^{-7}\,Ts^{-1}$$

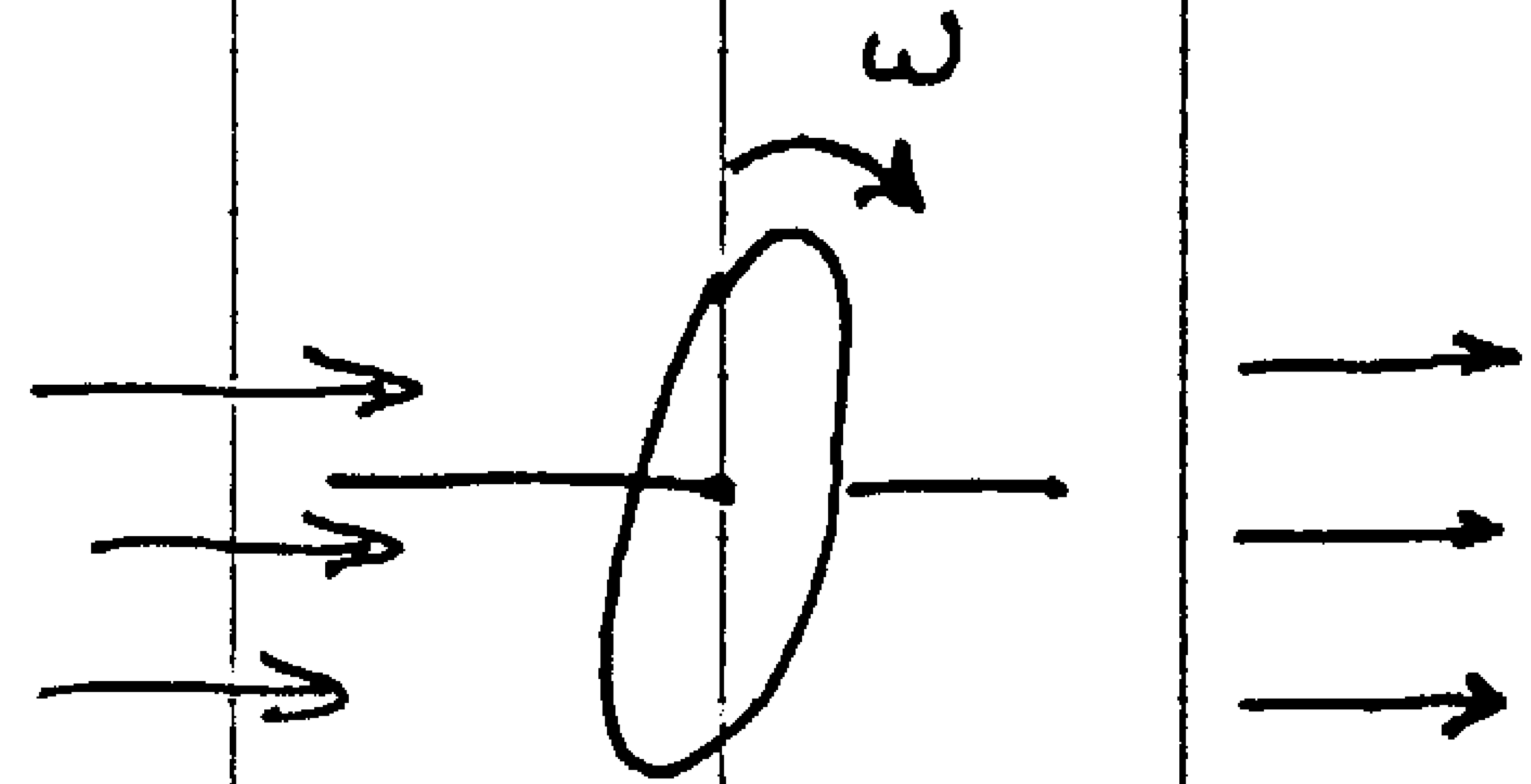
n.b. $B_{loop} = 2\pi k' I_2 / r$ $= 1.2\times10^{-14}\,T$ = much smaller than change in B

$$\mathcal{E} = -\frac{d\Phi}{dt} = -A\frac{dB}{dt} = 3.14\times10^{-4}\,m^2(-6.28\times10^{-7}\,Ts^{-1})$$
$$= 2\times10^{-10}\,Wbs^{-1} = 2\times10^{-10}\,V$$

$$I_2 = \frac{\mathcal{E}}{R} = 2\times10^{-10}\,V/1\,\Omega = 2\times10^{-10}\,A$$

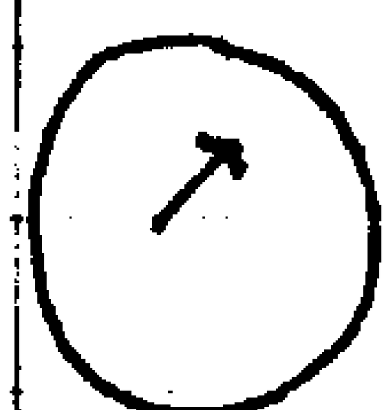
I will be ~~out~~ ccw ($B(I_2)$ out of page opposing change in flux)

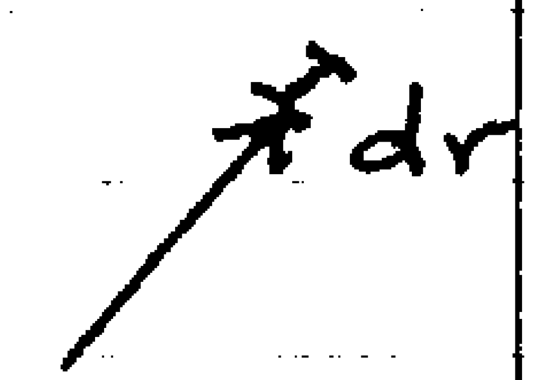
FARADAY DYNAMO



at r

small ring of width dr





$$v = \omega r$$

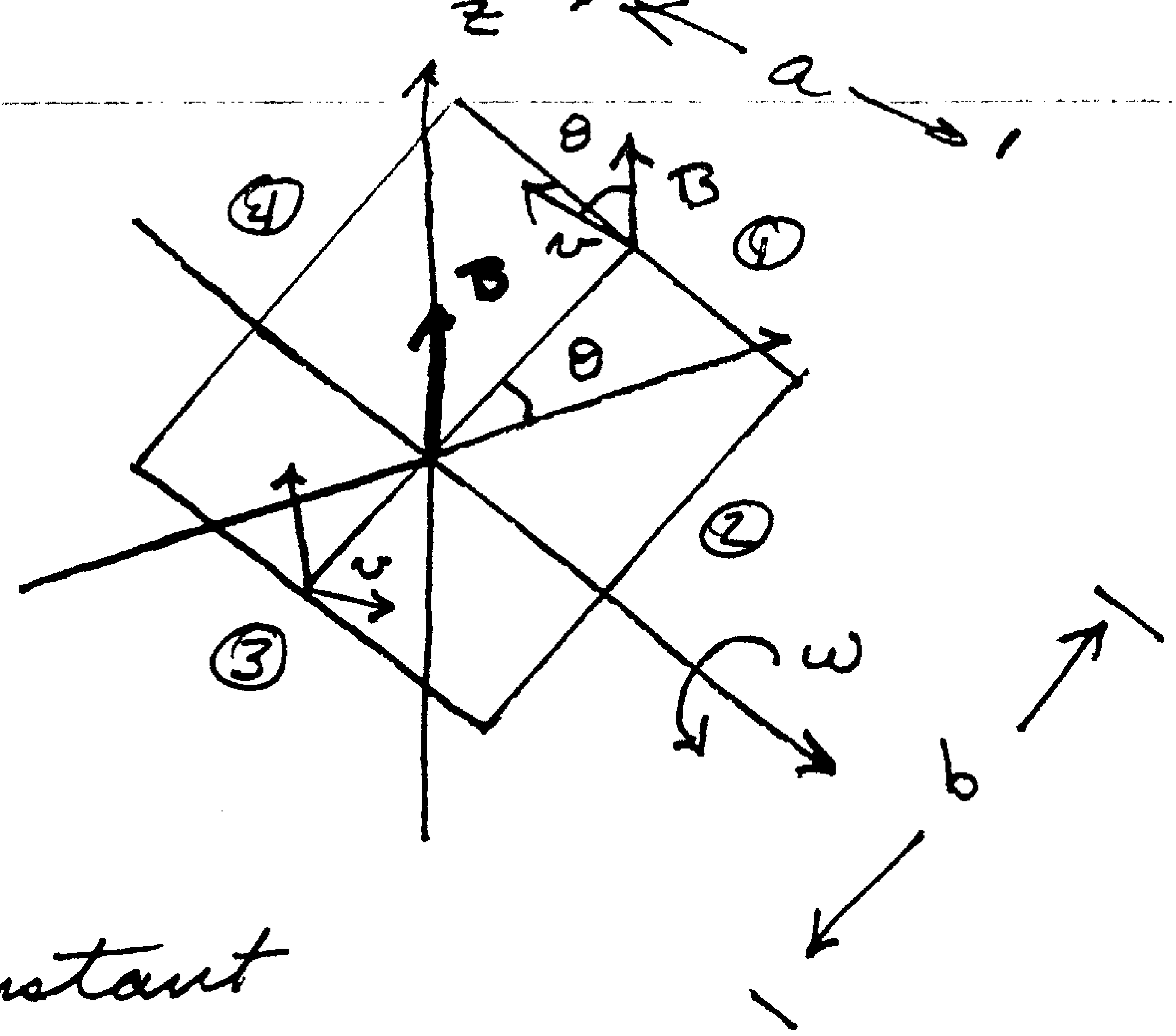
$$d\mathcal{E} = v B l \quad \text{between } r \quad r + dr$$

$$= \omega r B \, dr$$

$$\mathcal{E} = \int_0^R d\mathcal{E} = \int_0^R \omega B r \, dr$$

$$= \frac{1}{2} \omega B r^2$$

GENERATOR



At any instant

$$v = \omega r = \omega \frac{b}{2}$$

The "motional field

$$E_m = \vec{v} \times \vec{B} = vB\sin\theta = \omega \frac{b}{2} B \sin\theta$$

$$\mathcal{E} = \oint \vec{E} \cdot \vec{ds}$$

From ① $\int E \cdot ds = Ea = \omega \frac{b}{2} a B \sin\theta$

② $E \perp ds \Rightarrow \int E \cdot ds = 0$

③ $\int E \cdot ds = Ea = \omega \frac{b}{2} a B \sin\theta$

④ $\int E\, ds = 0$

Thus around the entire circuit

$$\mathcal{E} = \oint \vec{E} \cdot \vec{ds} = \omega b a B \sin\theta$$

$$ba = A$$

If at $t=0$ $\theta=0$ then we have $\theta=\omega t$

$$\boxed{\mathcal{E}=\omega AB\sin\omega t}$$

Can get the same result from looking at the change in flux. For uniform B

$$\Phi=\vec{B}\cdot\vec{A}$$

at $t=0$ $\theta=0$ $\vec{A}\parallel\vec{B}$

$$\Phi=BA\cos\omega t$$

then

$$\mathcal{E}=-\frac{d\Phi}{dt}=-\left(\frac{d}{dt}\left[BA\cos\omega t\right]\right)$$

$$=-BA\frac{d}{dt}(\cos\omega t)$$

$$=\omega BA\sin\omega t$$

This is the basis for an alternating current generator or alternator. If current $\mathcal{E}=IR$

$$I=\frac{\omega BA}{R}\sin\omega t$$

alternating current

Remember $\mathcal{E}$ is potential energy per unit charge. Have taken energy from where. Must do work

$$F = I\,\vec{l} \times \vec{B}$$

Take e.g. rectangular loop tilted wrt plane (yz) $\perp$ to $\vec{B} = B_z$

Lower Side a $\quad I \perp B \Rightarrow F = IaB$ (+x direction)

Upper " $\quad$ " $\quad F = IaB$ (−x direction)

Right Side b $\quad \vec{l} \times \vec{B} = b \sin(90-\alpha)$ $\quad F = IbB \sin(90-\alpha)$ (+y direction)

Left " $\quad F = IbB \sin(90-\alpha)$ (−y direction)

Resultant $F = 0$

But forces on side a are not colinear

$$T = r_\perp F$$

$$= \frac{b}{2} \sin\alpha\, IaB + \frac{b}{2} \sin\alpha\, IaB$$

(Both forces act to rotate loop in same direction

$$T = IB\,ab \sin\alpha$$

ab = area of loop

$$\boxed{T = IBA \sin\alpha}$$

or if we take $\vec{A}$ = vector of area A $\perp$ to plane

$$\boxed{T = I(\vec{A} \times \vec{B})}$$

We can show that this is true of a plane loop of *any* shape

$$W = \int T \cdot d\theta$$

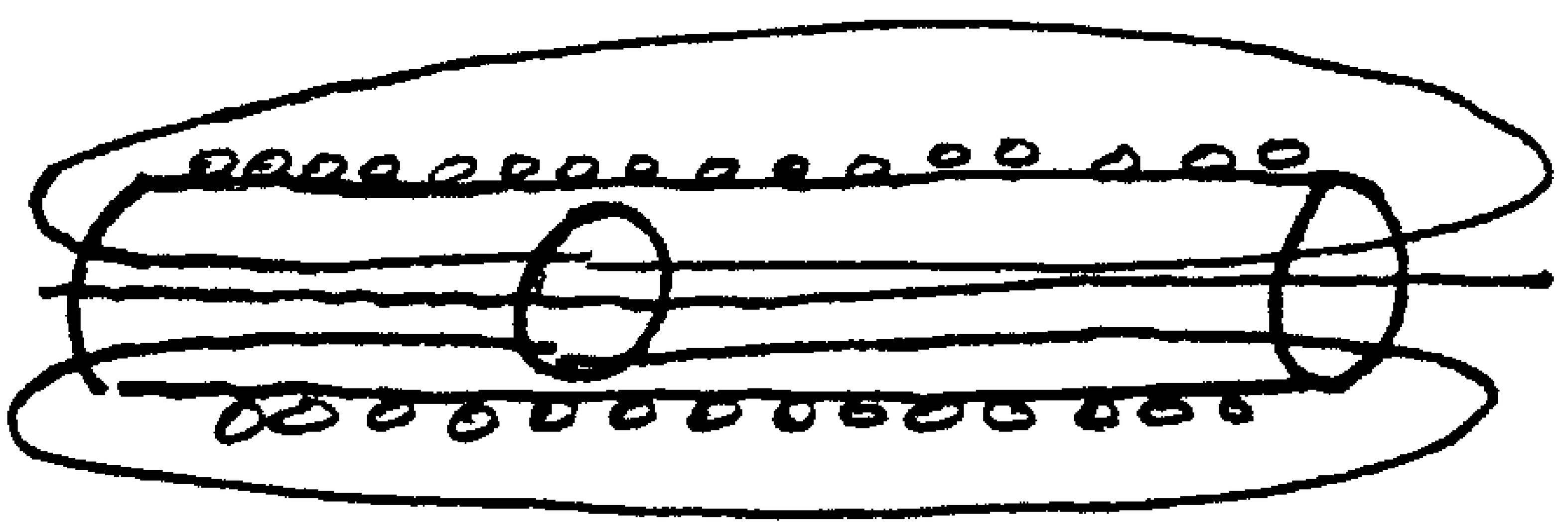
$$= \int I B A \sin\theta \, d\theta$$

$$\theta = \omega t \qquad d\theta = \omega\, dt$$

Subst $\quad W = \dfrac{\omega^2 B^2 A^2}{R} \int \sin \omega t \, dt$

Compare this w/ rate of power in the circuit. This comes from falling water, steam, automobile engine, etc!

<u>Induced E field</u>



Inside solenoid which has current changing at $\dfrac{dI}{dt}$

Φ through circle of radius r

uniform

$$\Phi = BA = \mu_0 n I \cdot \pi r^2$$

$$\boxed{\mathcal{E} = \int \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt}}$$

$$\frac{d\Phi}{dt} = \mu_0 n \frac{dI}{dt} \cdot \pi r^2$$

Line integral around ds

$$E \cdot 2\cancel{\pi}\cancel{r} = \mu_0 n \cancel{\pi} r^{\cancel{2}} \frac{dI}{dt}$$

$$E = \frac{\mu_0 n r}{2} \frac{dI}{dt}$$

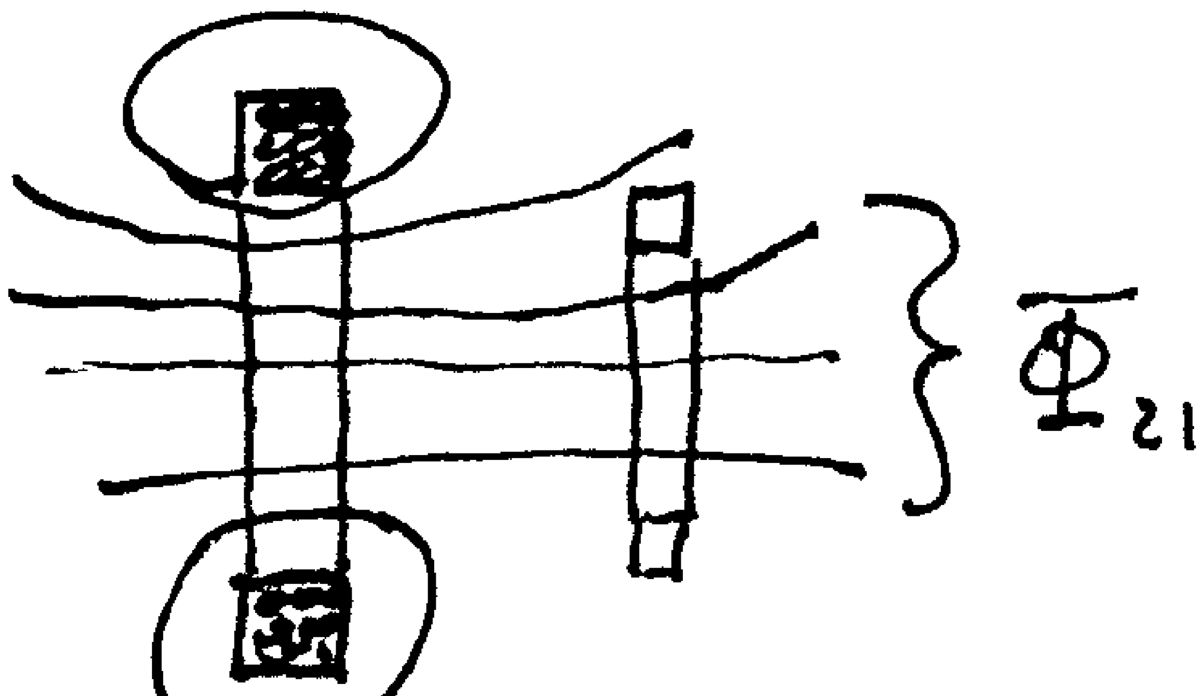
Changing $\vec{B}$ field due to changing I generates electric field. But N.B.

$$\oint \vec{E} \cdot d\vec{s} \neq 0$$ not a conservative field

$$\int \vec{E} \cdot d\vec{A} = 0$$ (not produced by charge but by $\vec{B}$ field)

INDUCTANCE

Let's look at this phenomenon a little more closely. When we want to talk about induction in a circuit it is more convenient to express things in terms of current rather than B, Φ. Consider pair of coils



1 2

Φ_{21} = flux through coil 2 due to i_1 in coil 1

N.B. Decrease in $i_1 \Rightarrow i_2$ will increase to oppose decrease in Φ + vice versa

$M_{21} \equiv$ mutual inductance

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$$

where $N_2 \Phi_{21}$ = # of flux linkages

AC Power/Circuits

How do we generate and transmit electrical power? We've already remarked that this was a longstanding controversy between Edison & Tesla!

DC Power   AC Power

Most of what we've discussed up to now has been specifically applied to DC setups but much also applies to AC

DC uses battery, DC generator by mech. processes creates electrostatic fields $\Rightarrow V$ which can do work.

AC as we shall see uses induced EMF, relationship between $i, B, \mathcal{E}, \Phi$.

A few simple considerations

P loss to resistance (e.g. resistivity of power line

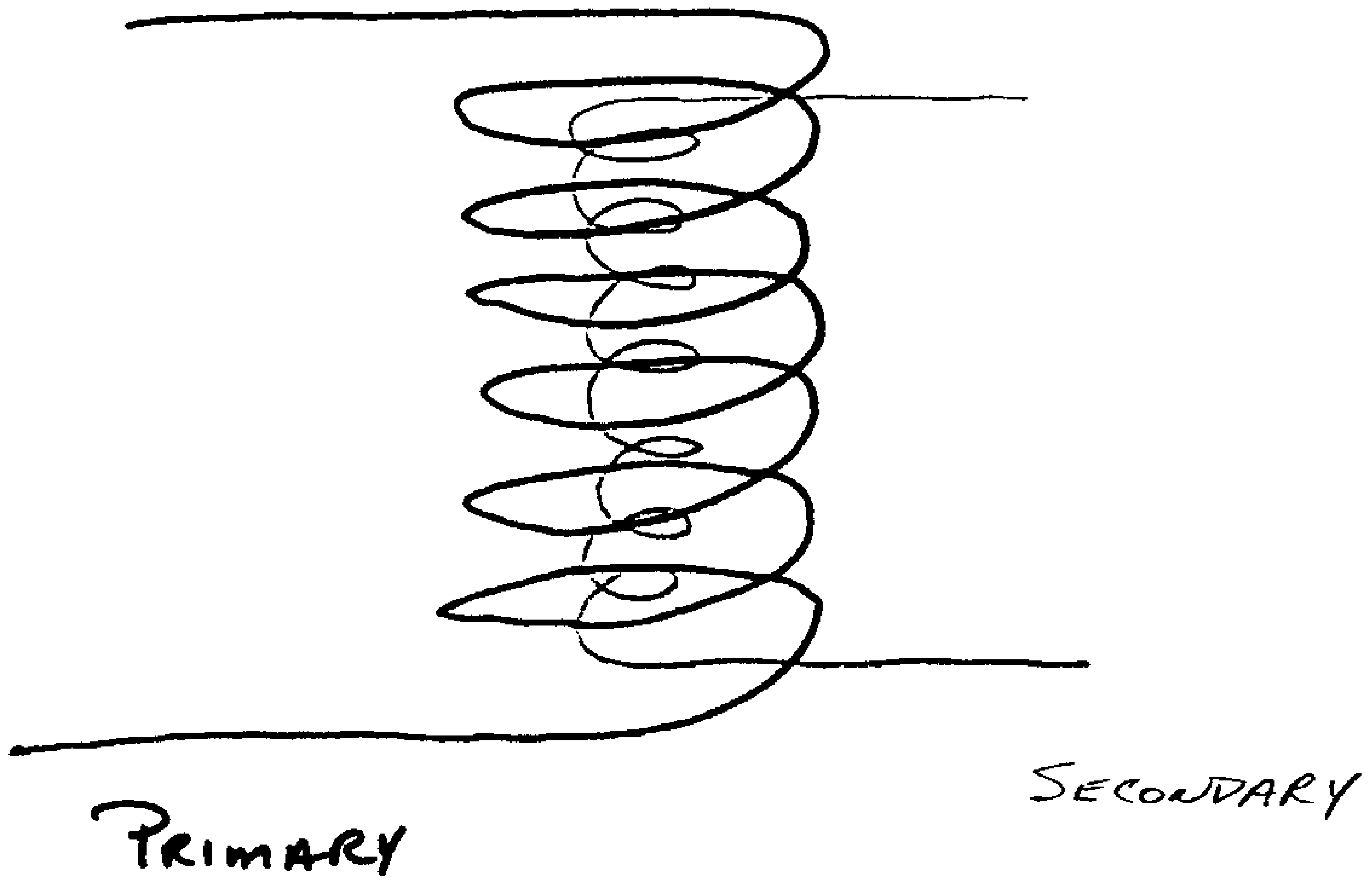
$$P = I^2 R \Rightarrow \text{want } I \text{ small}$$

Power supplied at given Voltage

$$P = IV \Rightarrow \text{want } V \text{ high so } I \text{ can be small}$$

But Want V inside homes small for safety considerations

TRANSFORMER



$$\mathcal{E}_1 = \mathcal{E}_2 = -\frac{d\Phi}{dt} \quad \text{in one coil}$$

but suppose Primary has N_1 turns, secondary N_2 turns, then

$$\mathcal{E}_1 = -N_1 \frac{d\Phi}{dt} \qquad \mathcal{E}_2 = -N_2 \frac{d\Phi}{dt}$$

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Voltage "transformer" HV lines carry power at high voltage to substations where power is "stepped down to 120V a-c

V_1 is fixed

When secondary ckt is closed (say through load R

$$V_1 I_1 = V_2 I_2 \quad \text{no losses}$$

$$I_1 = \frac{V_1}{(N_1/N_2)^2 R}$$