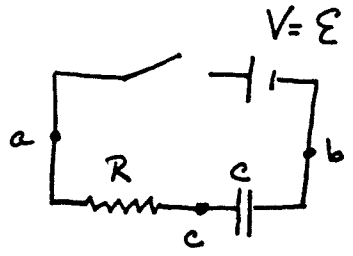
R-C Circuit



When switch is thrown current begins to flow through ckt, capacitor charges up creating p.d V_{cb} decreasing p.d. V_{ac} $[V = V_{ac} + V_{cb}]$ thus decreasing the current $V_{ac} = iR$. At any instant

$$V_{ac} = iR \qquad V_{cb} = \frac{q}{C}$$

$$V_{ab} = V = iR + \frac{q}{C}$$ Solving for the current $$i = \frac{V}{R} - \frac{q}{RC}$$

at $t=0$ $q=0$ $i = I_0 = \frac{V}{R}$ as ① charge increases current decreases until finally drops to zero $q \to Q$ $Q = CV$ ②

To find out what happens between ① + ② must do some math. Recall def'n of current

$$i = \frac{dq}{dt} = \frac{V}{R} - \frac{q}{RC}$$

which we rearrange to get all charge on one side of eqn

$$\frac{dq}{dt} = \frac{VC - q}{RC}$$

$$\frac{dq}{VC - q} = \frac{dt}{RC}$$

$$\int \frac{dq}{VC-q} = \frac{1}{RC}\int dt$$

Integrating

$$\int \frac{1}{q} = \ln q \qquad \int \frac{dq}{c-q} = -\ln(c-q)$$

$$-\ln(VC-q) = \frac{t}{RC} + \text{const}$$

$t=0$ $q=0$ Thus

$$\text{const} = -\ln(VC)$$

$$\ln(VC-q) - \ln VC = -\frac{t}{RC}$$

$$\ln\left(\frac{VC-q}{VC}\right) = -t/Rc$$

Exponentiating

$$e^{\left(\ln \frac{VC-q}{Vc}\right)} = \frac{VC-q}{VC} = 1-\frac{q}{VC}$$

$$1-\frac{q}{VC} = e^{-t/RC}$$

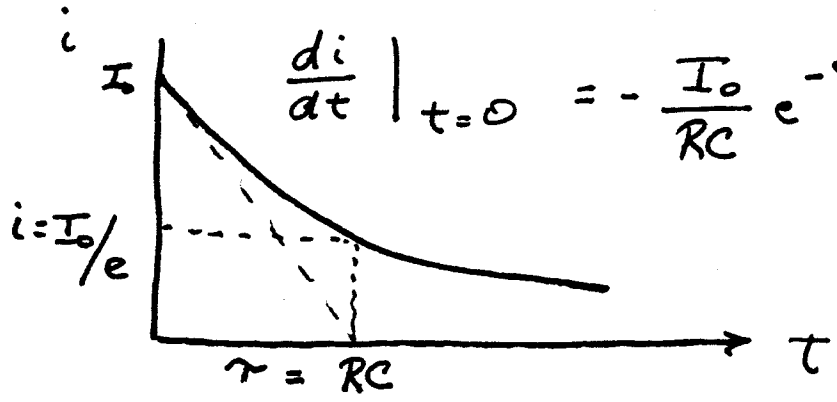
Rearranging $\boxed{q = VC\,(1-e^{-t/RC}) = Q\,(1-e^{-t/RC})}$

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(\cancel{VC}^{0} - VCe^{-t/RC}\right)$$

Differentiating

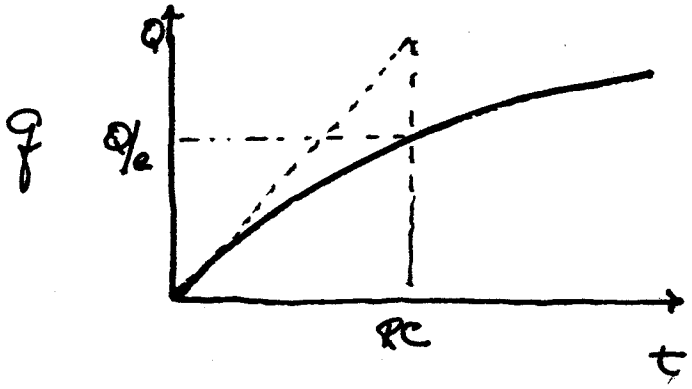
$$\frac{d}{dt}\left(e^{-t/RC}\right) = -\frac{e^{-t/RC}}{RC}$$

$$\boxed{i = \frac{V}{R}e^{-t/RC} = I_0 e^{-t/RC}}$$



$$\left.\frac{di}{dt}\right|_{t=0} = -\frac{I_0}{RC} e^{-0/RC} = \frac{I_0}{RC} = \frac{I_0}{\tau}$$

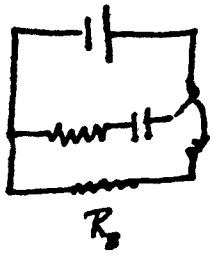
Initially current decreasing as $\frac{I_0}{\tau}$. If continued to decrease at that rate. $I = 0$ at $t = \tau = RC$. In real life when $t = \tau$ $i = I_0/e = \frac{I_0}{2.718281828}$



$v_{cb} = \frac{q}{C}$
So capacitor voltage looks like this too

$\tau = RC$ is "time constant" or "relaxation time" of circuit. This is how automobile turn signals work. Filament of bulb = R_3, place capacitance in series

Also clustmas tree lights, etc



When v_{bc} reaches some value (det'd by time constant of circuit) switch is activated discharging through lamp — light flashes.

Discharging capacitor, now $V_{ab} = 0$

$$0 = v_{ac} + v_{cb}$$

$$i = \frac{q}{RC}$$

$$t = 0 \quad q = Q \qquad I_0 = \frac{V_0}{R} = \frac{Q_0}{RC}$$

$$i = I_0 e^{-t/RC}$$

$$q = Q_0 e^{-t/RC}$$

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How do we measure current, voltage resistance? Not necessarily a simple matter, esp. if we want to make measurement w/ high accuracy because placing instrument in circuit introduces its own characteristics alters circuit