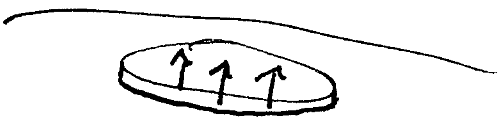
At surface of conductor field lines must be $\perp$ to surface (if there were any $\parallel$ component charges would move.



Surface has charge density σ ($C\ m^{-2}$)
To determine what the field must be we construct an arbitrary Gaussian surface which has walls $\perp$ and $\parallel$ to surface (eg. cylinder) which encloses the volume from just below to just above the surface. Then the charge q within surface $q = \sigma A$

a) Within cylinder $E = 0 \Rightarrow \int E_\perp dA = 0$ for lower surface

b) walls of cyl. $E \parallel$ to walls $\Rightarrow \int E_\perp dA = 0$

c) Upper surface

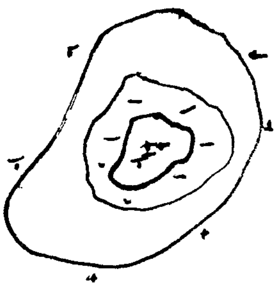
$$\int E_\perp dA = EA = \frac{q}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

just outside surface of conductor

Back to inner part of conductor. Suppose conductor has a cavity which has no charge. Then charge everywhere must still be zero



Place charge +q within cavity. Within conductor

$$E = 0 \Rightarrow \int E_{\perp} dA = 0$$

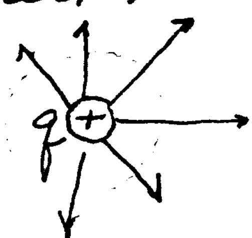
$$\Rightarrow \text{net } q = 0$$

Inside walls of cavity must develop charge $-q$ so that net $q = 0$. If conductor originally neutral then outer surface must develop charge $+q$. If touch chge to inside of cavity wall, chges neutralize leaving outer surface charged w/ $+q$ effectively leaving transferred charge

FARADAY ICE PAIL EXPT.

Coulomb Law

We can see that Gauss's Law is direct result of $\frac{1}{r^2}$ force law



Surface? Symmetry considerations ⇒ sphere

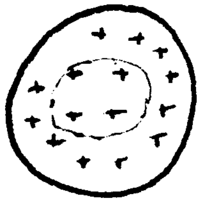
$$\oint E_{\perp} dA = E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi \varepsilon_0 r^2} = \frac{kq}{r^2}$$

NON CONDUCTING

SPHERE

Chg density = ρ ($C\,m^{-3}$)



Spherical symmetry, if chg dist'n is uniform will have only radial $\vec{E}$ component. Gaussian surface?

Sphere will have $E_\perp$ at all points on surface for radial field.

at $r < R$

$$\int E_\perp dA = EA = 4\pi k q \text{ or } \frac{q}{\varepsilon_0}$$

for uniform $\perp$ field

Now $q = \rho . V = \rho . \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

$$E \cdot \cancel{4\pi} \cancel{r^2} = \frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}} \rho / \varepsilon_0$$

$$E = \frac{\rho r}{3\varepsilon_0}$$

at $r > R$

$q = \frac{4}{3}\pi R^3$

$$E \cdot \cancel{4}\pi r^2 = \frac{\cancel{4}}{3}\pi R^3 \rho/\varepsilon_0$$

$$\underline{E = \frac{R^3 \rho}{3 \varepsilon_0 r^2}}$$

$$\varepsilon_0 = \frac{1}{4\pi k} \Rightarrow \frac{4\pi k R^3 \rho}{3 \; r^2} = \frac{k \rho V}{r^2}$$

$$E = \frac{kq}{r^2}$$ Same as for point charge

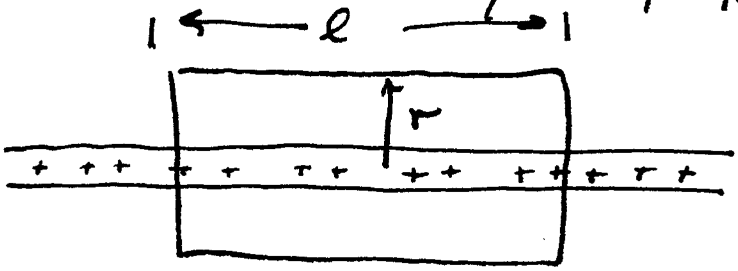
Conductor would have all chg at surface

$$E = \frac{kq}{r^2} \quad r > R$$

but now inside $E = 0$

<u>Long Charged Wire</u>

Chg density λ Cm^{-2}



Symmetry $\Rightarrow$ E radial from wire

Cylindrical surface $E_\perp = E$

$$\int E_\perp \, dA = EA = \quad 4\pi k q$$

$$A = 2\pi r \ell \qquad q = \lambda \ell$$

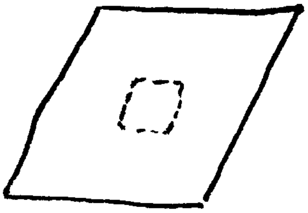
$$EA = E\,\cancel{2}\pi r l = \cancel{4}^{2}\pi k \lambda l$$

$$E = \frac{2k\lambda}{r}$$

Contributions from sides $= 0$ since $E = E_{\parallel}$

INFINITE PLATE

Charge density σ ($C\,m^{-2}$)



Here can use any surface we want as long as its sides are straight + meet plane $\perp$ to surface

Then

$$\int E_{\perp} A = EA = \frac{q}{\varepsilon_0}$$

$$q = \sigma A$$

$$E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

You can see that this is the same as result from integration and ever so much easier