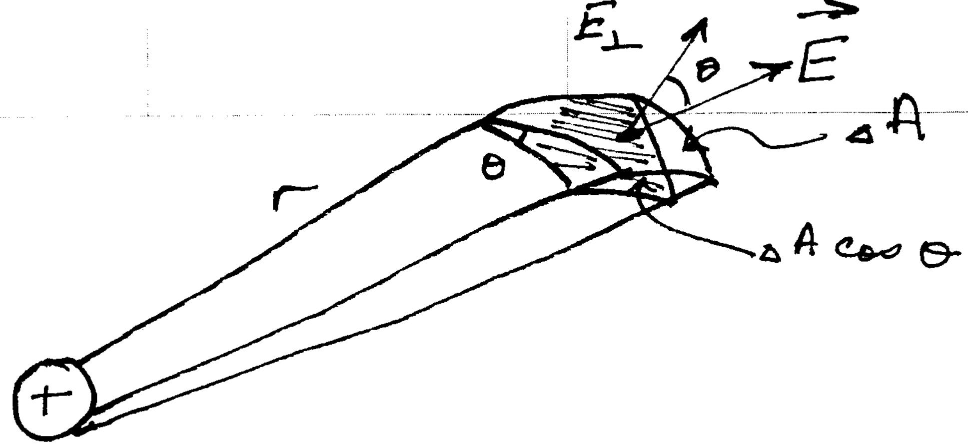


What if we draw a surface that is not a sphere. For a sphere all the field lines are $\perp$ to the surface. For an irregular surface the surface element is larger than a spherical surface element - the projection factor is <u>$\cos\theta$</u>. So if we sum up all the lines $E\Delta A\cos\theta$ we get the same result - the total # of lines is conserved

$$\sum E\Delta A\cos\theta = 4\pi kq$$

<u>provided the surface is closed</u>. Since E varies across surface this is only strictly true if we take infinitesimal surface elements $\Rightarrow$

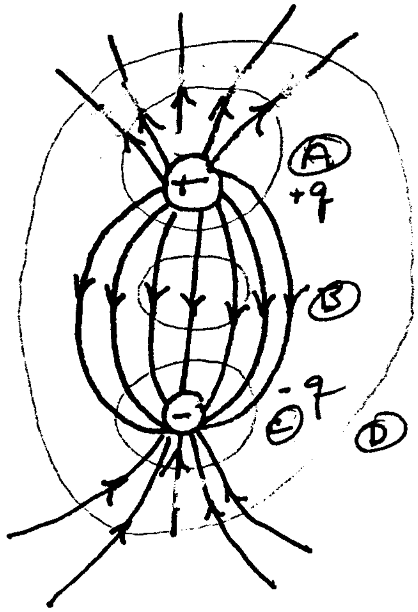
$$\boxed{\oint E\cos\theta\, dA = 4\pi k q \qquad \oint E_\perp dA = 4\pi kq}$$

The quantity $E_\perp dA = E\cos\theta \, dA$ is called the <u>electric flux</u>. If there is more than one charge then

$$Q = \Sigma q \quad \text{and} \quad \varepsilon_0 = \frac{1}{4\pi k}$$

$$\Rightarrow \quad \boxed{\oint E_\perp dA = \frac{Q}{\varepsilon_0}} \qquad \underline{\text{Gauss's Law}}$$

Look at Gauss's Law from point of lines of force. Look at dipole



Lines always away from ⊕ chg toward ⊖ chg

Can place surfaces anywhere we want

1) A encloses + charge all lines of force are outward

$$\int E_\perp dA = \frac{q}{\varepsilon_0}$$

2) B encloses no charge

$$\int E_\perp dA = 0$$

No lines are created within B ⟹ every line that enters B must exit B as

3) C encloses - charge all lines of force are inward

$$\int E_{\perp}\, dA = -\frac{q}{\varepsilon_0}$$

④ D - Net charge within D = 0

$$\int E_{\perp}\, dA = 0$$

For every line entering D there must be a corresponding line exiting D

— o —

We will see that it is often advantageous to break a surface up into segments and look at the contribution to $\int E_{\perp}\, dA$ separately for each segment. The integral is then just the sum of the contributions from each segment. A few simple rules fall out directly

1) If the surface segment has E at right angles everywhere and E has the same magnitude everywhere then $E = E$ $\int E_{\perp}\, dA = EA$

2) If $E \parallel$ to the surface $E_{\perp} = 0$

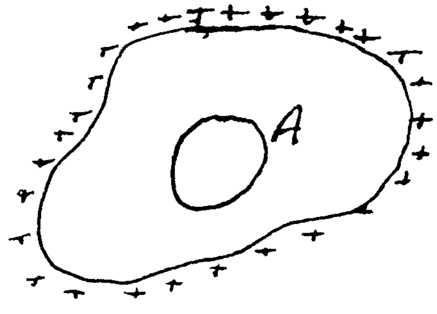
$$\int E_{\perp}\, dA = 0$$

3) If $E = 0$ everywhere $\int E_{\perp}\, dA = 0$

How do we apply Gauss's Law. There are a variety of useful applications but they often require a bit of creativity in setting them up

1) <u>Conductors</u> - we have <u>already</u> discussed that conductors are materials in which the electrons move freely. We can see quite simply that the $\vec{E}$ field intensity inside a conductor must be zero unless there is a current. If $\vec{E}$ is not zero then any charge q will feel a force $q\vec{E}$. Being free to move a current will be created (charges will be moving) No current $\Rightarrow$ no electric field.

Now suppose we charge up a conductor; what can we say about where charge will be located. Any surface within the conductor must have



$E = 0 \Rightarrow \int E_\perp dA = 0 = 4\pi q$

There can be no net charge anywhere within a conductor $\Rightarrow$ charge (