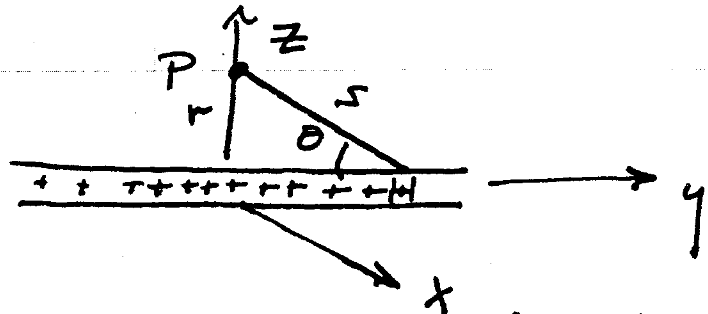


Long line of charge (essentially infinite)

Consider point in y z plane on z axis

If charge density on line of charge is λ (Coulombs per meter) then

$dq = \lambda\, dy$

$$\vec{E} = k \int \frac{\hat{s}\, dq}{s^2} = k\lambda \int \frac{\hat{s}\, dy}{s^2}$$

Since $\hat{s}$ lies in y z plane $\hat{s}_x = 0$

$$E_x = 0$$

$$E_y = k\lambda \int \frac{\hat{s}_y\, dy}{s^2}$$

$$\hat{s}_y = |\hat{s}| \cos\theta = \cos\theta$$

or (since $\cos\theta = \frac{y}{s}$)

$$E_y = k\lambda \int \frac{y\, dy}{s^3}$$

but $s = \sqrt{r^2 + y^2}$

$$E_y = k\lambda \int_{-\infty}^{+\infty} \frac{y\, dy}{(y^2+r^2)^{3/2}}$$

$$= k\lambda \left(\frac{-1}{(y^2+r^2)^{1/2}} \right) \Bigg|_{-\infty}^{+\infty}$$

$$= 0$$

$$E_z = k\lambda \int \frac{\hat{s}_z \, dy}{s^2}$$

$$\hat{s}_z = |\hat{s}| \sin\theta = \sin\theta$$

$$\sin\theta = \frac{r}{s}$$

$$E_z = k\lambda \int_{-\infty}^{+\infty} \frac{r \, dy}{(y^2+r^2)^{3/2}}$$

$$= k\lambda r \left. \frac{y}{r^2(y^2+r^2)^{1/2}} \right|_{-\infty}^{+\infty}$$

As $y \longrightarrow \infty \quad (y^2+r^2)^{1/2} \longrightarrow (y^2)^{1/2} \rightarrow |y|$

$$E_z = k\lambda \cancel{r} \left. \frac{y}{r^{\cancel{2}}|y|} \right|_{-\infty}^{+\infty}$$

$$= 2\frac{k\lambda}{r}$$

If we had taken a point on x axis

$$E_z = 0$$

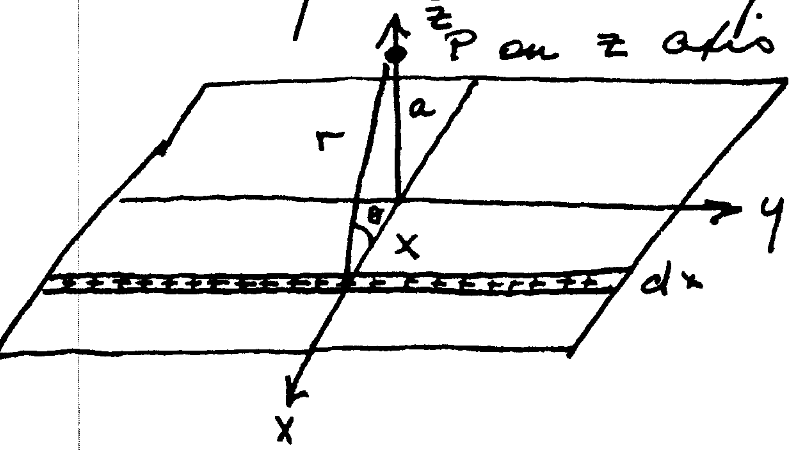
$$E_y = 0$$

$$E_x = \frac{2k\lambda}{r}$$

Clearly anywhere surrounding line of charge $\vec{E} = 2\frac{k\lambda}{r}\hat{r}$

use 2 types of arguments
1) Break up into tractable (familiar pieces)
2) Argue from commonsense

Infinite sheet of charge.



Treat plane as sum of large number of lines of charge

If there is charge density σ per unit surface area

then the charge density per unit length

$$d\lambda = \sigma\, dx$$

then from previous result

$$dE = 2k\frac{d\lambda}{r} = 2k\frac{\sigma\, dx}{r}$$

$$E_z = \int 2k\sigma\frac{\hat{s}_z\, dx}{r}$$

$$\hat{s}_z = \sin\theta = \frac{a}{r} = \frac{a}{(x^2+a^2)^{1/2}}$$

$$E_z = 2k\sigma a\int_{-\infty}^{+\infty}\frac{dx}{x^2+a^2}$$

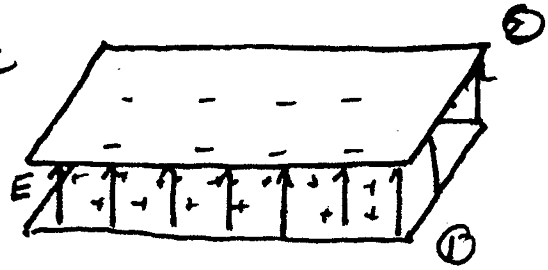
$$= 2k\sigma \cancel{a}\left[\frac{1}{\cancel{a}}\tan^{-1}\frac{x}{a}\right|_{-\infty}^{+\infty}$$

$$= 2k\sigma\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right] = 2\pi k\sigma$$

$$\underline{\vec{E} = 2\pi k\sigma\,\hat{z}}$$

UNIFORM FIELD

Can use superposition to find other fields



$$\vec{E} = \vec{E_1} + \vec{E_2}$$

Between plates $E_1 = 2\pi k \sigma \uparrow$

$E_2 = 2\pi k \sigma \uparrow$

$$E = 4\pi k \sigma \qquad \varepsilon_0 = \frac{1}{4\pi k}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Above − plate

$E_1 = 2\pi k \sigma \uparrow$

$E_2 = 2\pi k \sigma \downarrow$

$$E = 0$$

Below + plate $E_1 = 2\pi k \sigma \downarrow$

$E_2 = 2\pi k \sigma \uparrow$

$$E = 0$$