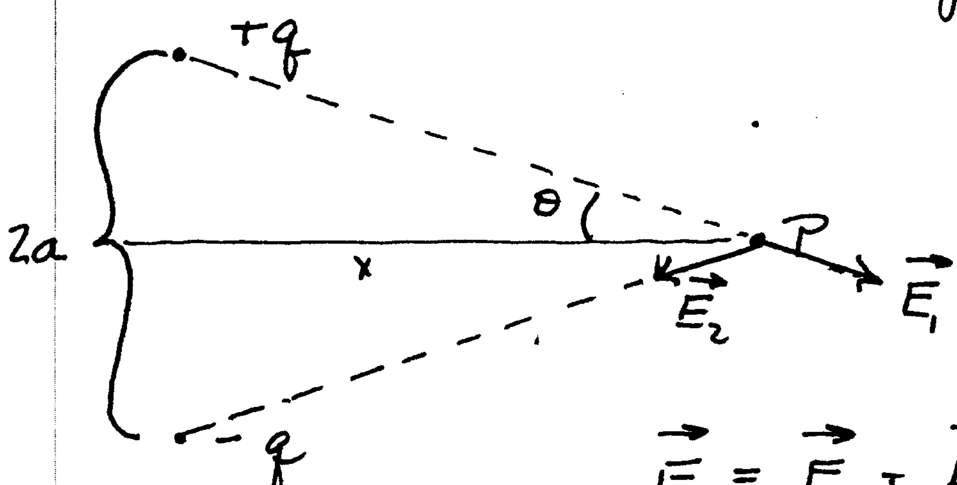
If we have a number of point charges then the electric field at any point is the vector sum of the fields due to each of the individual charges (just as the force would be if we looked at the net $\vec{F}$ on a test charge at that point

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots = k \sum_i \frac{q_i \hat{r}}{r_i^2}$$

Recall our dipole (now w/o third "test charge")



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = (E_{1x}, E_{1y}) \qquad \vec{E}_2 = (E_{2x}, E_{2y})$$

$$\vec{E}_1 = \frac{k\, q\, \hat{r}_1}{r_1^2} = \frac{k\, q\, \hat{r}_1}{a^2 + x^2} \qquad E_2 = -\frac{k\, q\, \hat{r}_2}{a^2 + x^2}$$

$$E_{1x} = E_1 \cos\theta \qquad \text{where } \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$$

$$E_{1y} = E_1 \sin\theta \qquad \sin\theta = \frac{a}{r} = \frac{a}{\sqrt{x^2+a^2}}$$

$$E_{1x} = \frac{kq\,x}{(x^2+a^2)^{3/2}}$$

Similarly

$$E_{2x} = -\frac{kq\,x}{(x^2+a^2)^{3/2}}$$

$$E_{1y} = \frac{kq\,a}{(x^2+a^2)^{3/2}}$$

$$E_{2y} = \frac{kq\,a}{(x^2+a^2)^{3/2}}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (E_x, E_y)$$

$$E_x = E_{1x} + E_{2x} = 0$$

$$E_y = E_{1y} + E_{2y} = \frac{2kq\,a}{(x^2+a^2)^{3/2}}$$

$x \gg a$

$$\vec{E} = \frac{kq\,2a}{(x^2+a^2)^{3/2}}\,\hat{r}_y \approx \frac{kq\,2a}{x^3}\,\hat{r}_y$$

If place chg #3

$q_3 = q$

$$\vec{F} = q_3 \vec{E}$$

$$\vec{F} = \frac{kq^2\,2a}{x^3}\,\hat{r}_y$$

Same as from Coulomb law

The electric field vector is along the radius vector connecting the point charge and the point in question

$\vec{E}$ away from q if q positive

$\vec{E}$ toward q if q negative

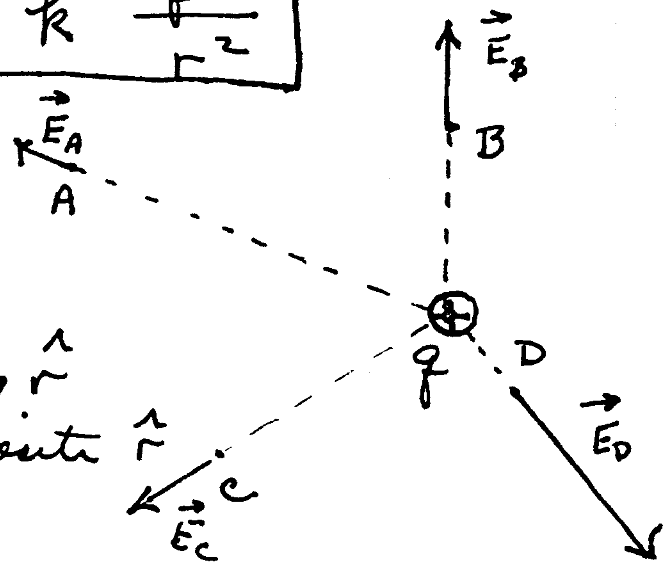
There is convenient notation for specifying this direction

$\hat{r}$ = unit vector (magnitude = 1) along $\vec{r}$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} \Rightarrow |\hat{r}| = \frac{|\vec{r}|}{|\vec{r}|} = 1$$

then

$$\boxed{\vec{E} = k\,\frac{q\,\hat{r}}{r^2}}$$



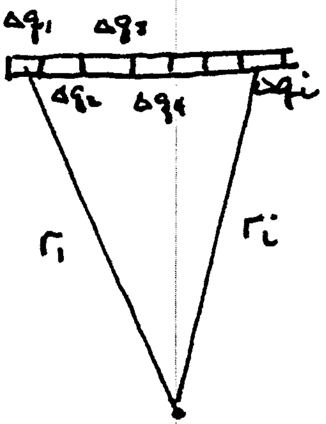
NB

1) $|\vec{E}| = \frac{kq}{r^2}$

2) q positive $\vec{E}$ along $\hat{r}$

3) q negative $\vec{E}$ opposite $\hat{r}$

Very rarely do we see isolated charges, most often we have a continuous distribution of charge. We can approximate by taking chunks of charge Δq so that



$$\vec{E} \approx k \sum \frac{\Delta q \, \hat{r}}{r^2}$$

Clearly the smaller the chunk of charge the better we approximate so that if we could take an infinitesimally small interval $\Delta q \to 0$ we would have an exact relationship

$$\vec{E} = k \lim_{\Delta q \to 0} \sum \frac{\Delta q \, \hat{r}}{r^2}$$

This is just the "VECTOR INTEGRAL"

$$\boxed{\vec{E} = k \int \frac{\hat{r} \, dq}{r^2}}$$

N.B.

1) Must chose limits of integration so that all chg contributing to $\vec{E}$ is included

2) This is vector integral $\Rightarrow E_x = \int dx \quad E_y = \int dy$