

*Gravitational Instability and  
the Kennicutt-Schmidt Law*

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# *Collaborators*

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- ❖ M. K. Ryan Joungh (Princeton U.)

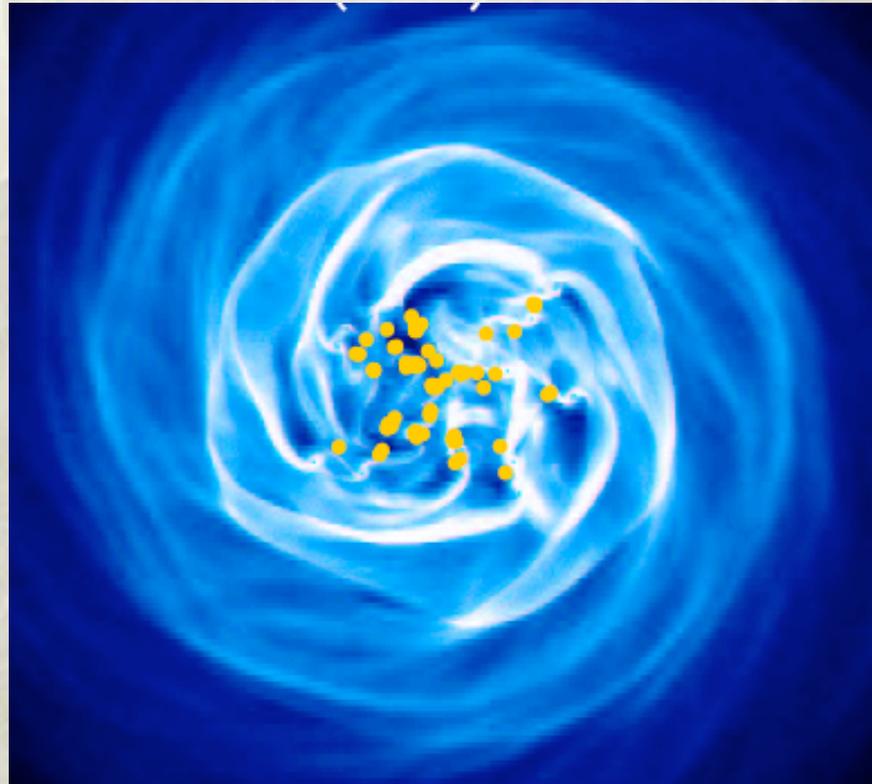


# *Numerical Experiments*

dark matter

stars

gas



Control: initial gravitational instability  
Measure: star formation properties

## *Technical details*

- ❖ **Code:** GADGET v1.1 (Springel, Yoshida & White 2001) + sink particles (Bate et al. 95, Jappsen, Klessen, Li, Mac Low 04) created when  $n > 10^3 \text{ cm}^{-3}$ , interact gravitationally
- ❖ **Galaxy model:** DM halo + disk of stars & isothermal gas (Mo, Mao, White 98, Springel & White 99, Springel 00).
- ❖ **Initial conditions:**
  - Rotational velocities  $50 < V_{\text{rot}} < 220 \text{ km s}^{-1}$
  - Gas fractions 5-90% of disk mass
  - sound speeds  $6 \text{ km s}^{-1}$  (LT) ,  $15 \text{ km s}^{-1}$  (HT)

# *Approximations*

- ❖ Fast molecular cloud formation
- ❖ Isothermal gas
- ❖ Neglect of magnetic fields

Each of these approximations has been addressed with local computational models.

# *Approximations*

## ❖ Fast molecular cloud formation

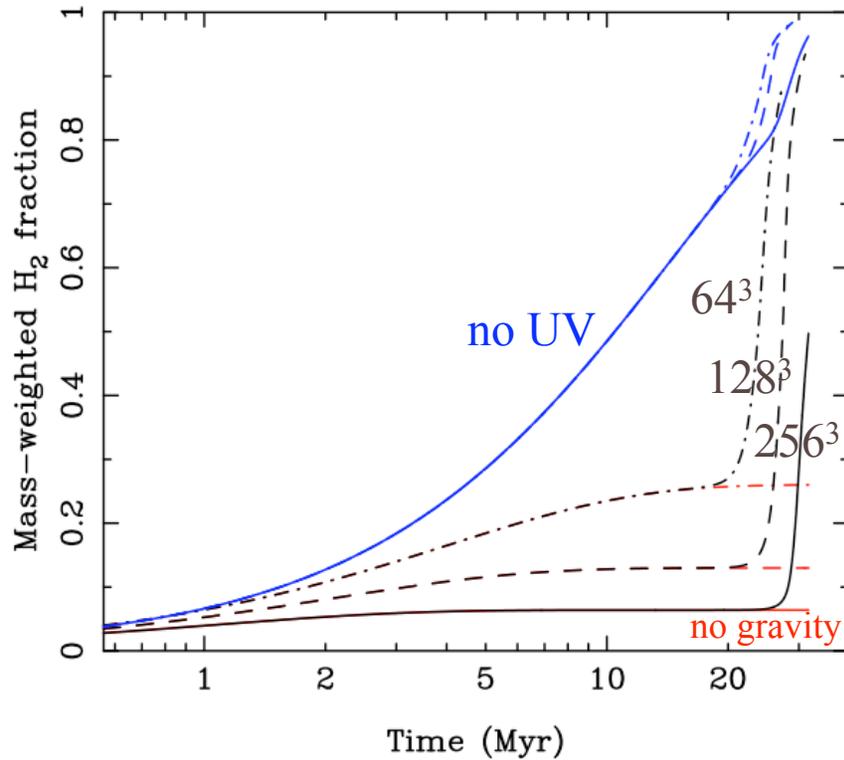
- Follow non-equilib. abundances of  $\text{H}_2$  and  $\text{H}^+$  following Rosen & Smith (2004), fixed abundances of other species ( $\text{CII}$ ,  $\text{OI}$ ,  $\text{SiII}$ )
- Radiative cooling, including  $\text{H}_2$  ro-vib,  $\text{CII}$ ,  $\text{OI}$ ,  $\text{SiII}$  fine structure, dust-gas transfers
- Photoelectric, cosmic ray, photodissoc. heating
- Local approximation for  $\text{H}_2$  self-shielding
- Up to  $512^3$  MHD models with ZEUS-MP (Norman 2000)

## ❖ Isothermal gas

## ❖ Neglect of magnetic fields

## Static collapse

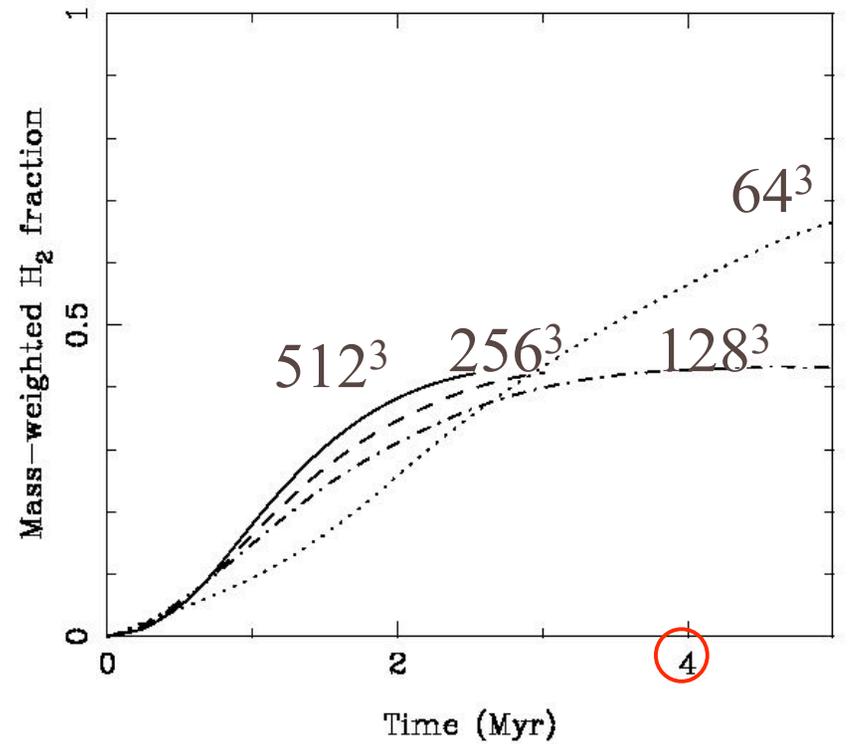
Glover & Mac Low 2006a



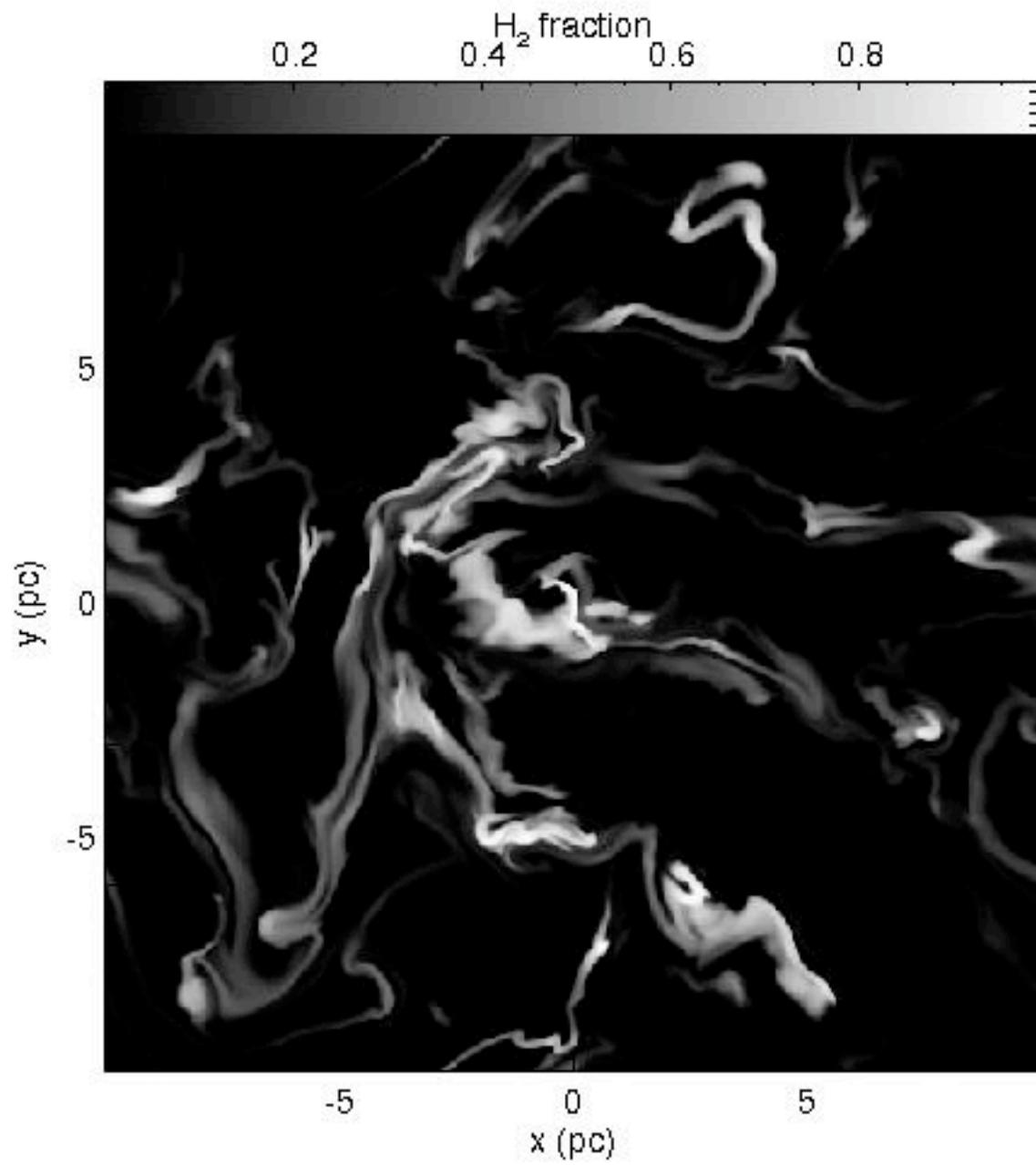
$$L = 40 \text{ pc}, B_0 = 5.85 \mu\text{G}, v_{\text{rms}} = 0.0$$

## Turbulent flow

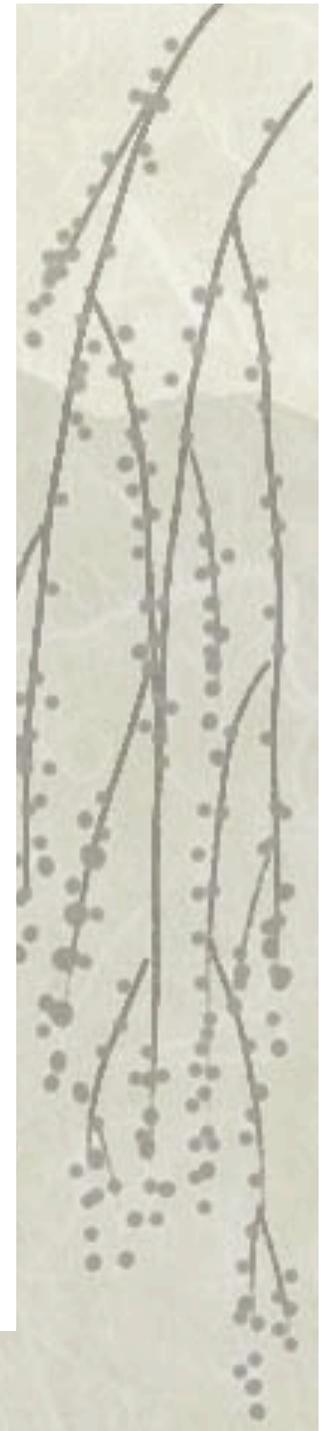
Glover & Mac Low 2006b



$$L = 20 \text{ pc}, B_0 = 5.85 \mu\text{G}, v_{\text{rms}} = 10 \text{ km/s}$$



Glover & Mac Low 2006b



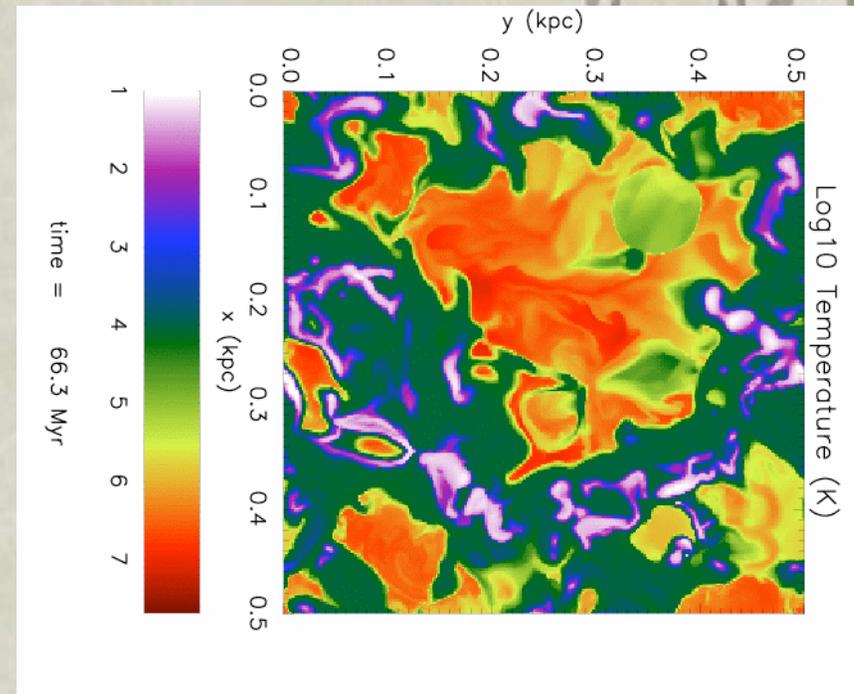
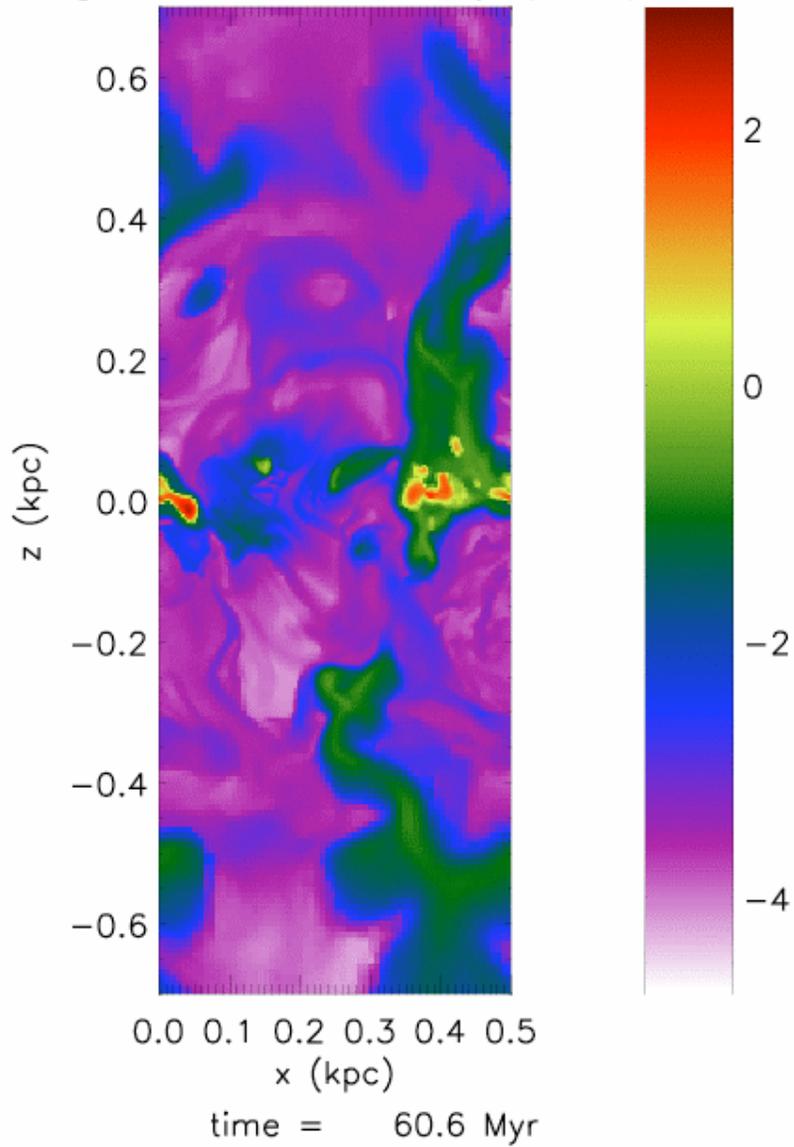
Molecular gas forms quickly in a turbulent medium once densities exceed  $100 \text{ cm}^{-3}$ .

We form sink particles at densities of  $1000 \text{ cm}^{-3}$  in our global disk models. Therefore our assumption that mass in sink particles is primarily molecular appears justified.

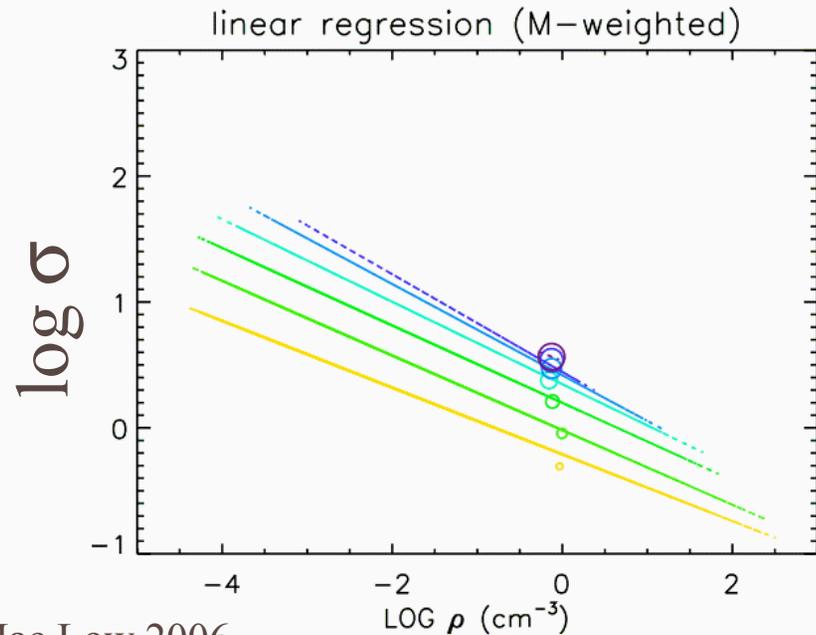
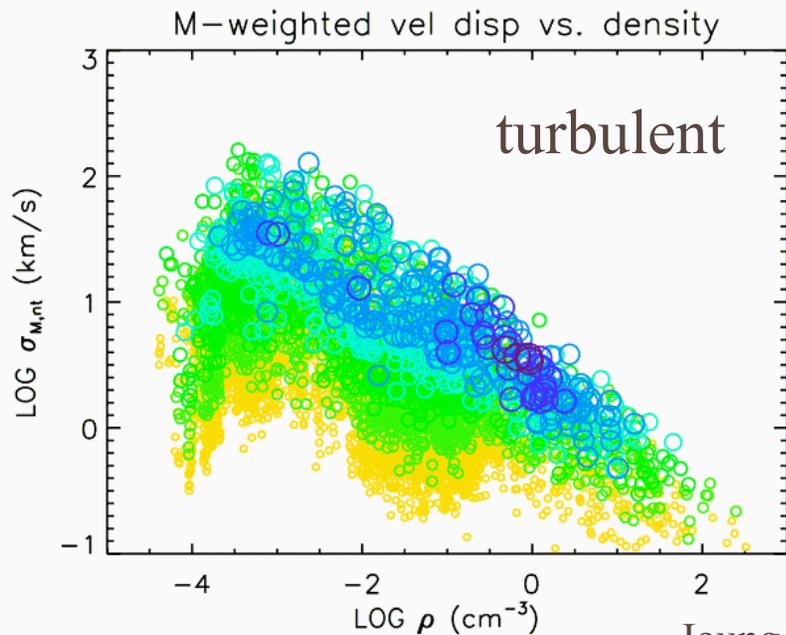
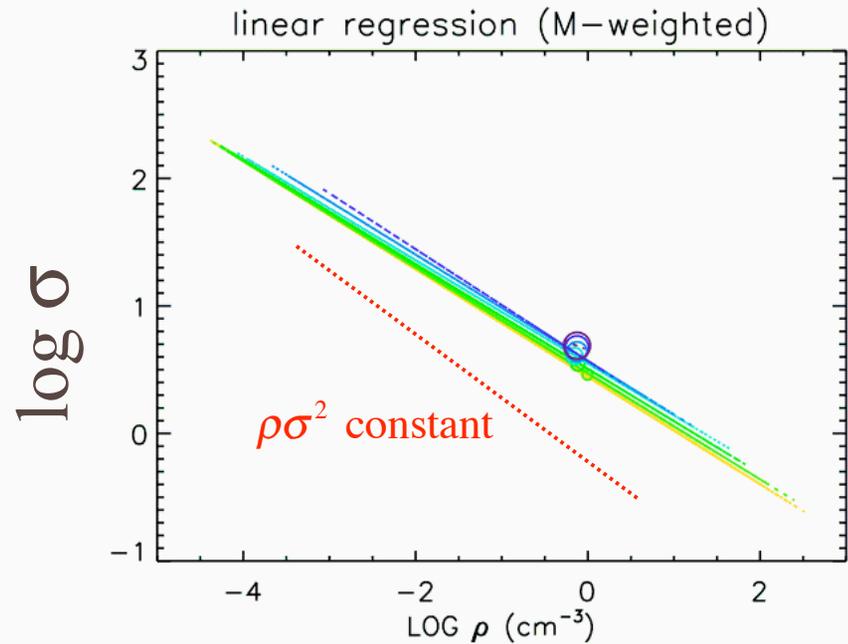
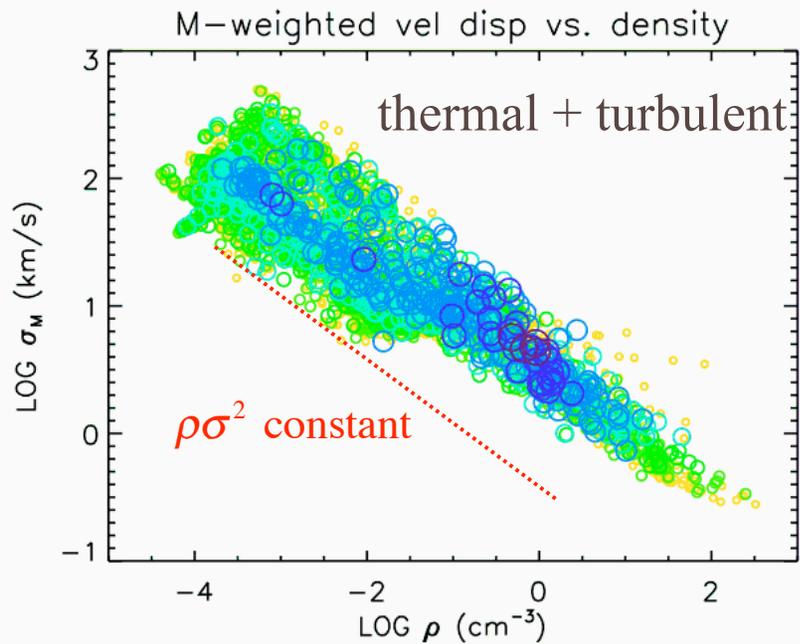
# *Approximations*

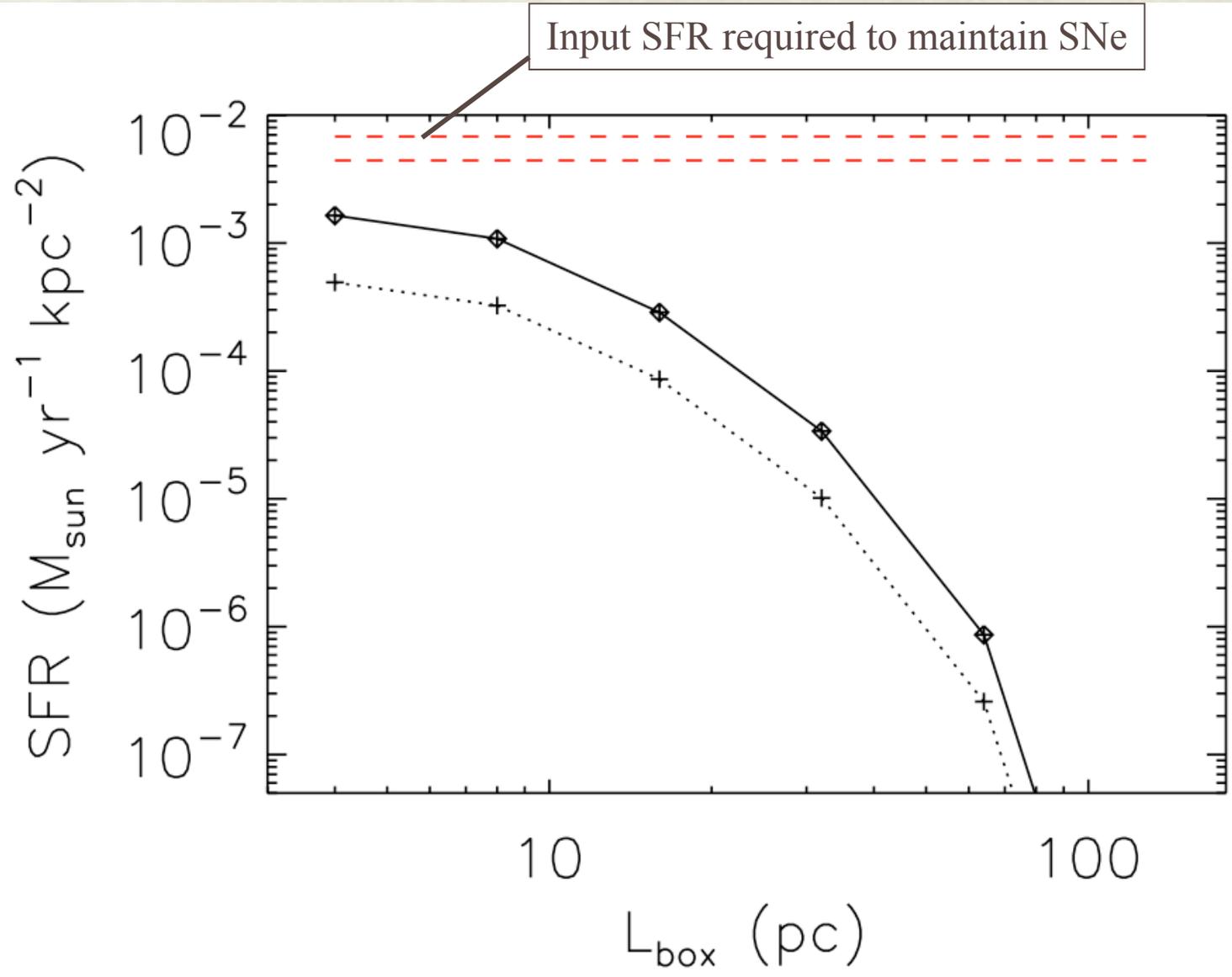
- ❖ Fast molecular cloud formation
- ❖ **Isothermal gas**
  - We used  $T = 10^4$  K to model uniform velocity dispersion in gas in global models
  - Test by comparing with  $(500 \text{ pc})^2 \times 10 \text{ kpc}$  local models of SN driven ISM turbulence (Joung & Mac Low 2006), using Flash AMR hydro code (Fryxell et al 2001).
  - Models include discrete SNe, photoelectric heating and radiative cooling, static gravitational potential
  - $\Delta x_{min} = 1.95 \text{ pc}$  in region  $\pm 200 \text{ pc}$  from plane
- ❖ Neglect of magnetic fields

Log10 Number Density ( $\text{cm}^{-3}$ )



Temperature





Isothermal EOS is a conservative assumption!  
Real ISM is even more compressible.

Barotropic equations of state are not required to prevent unrealistically high SF rates.

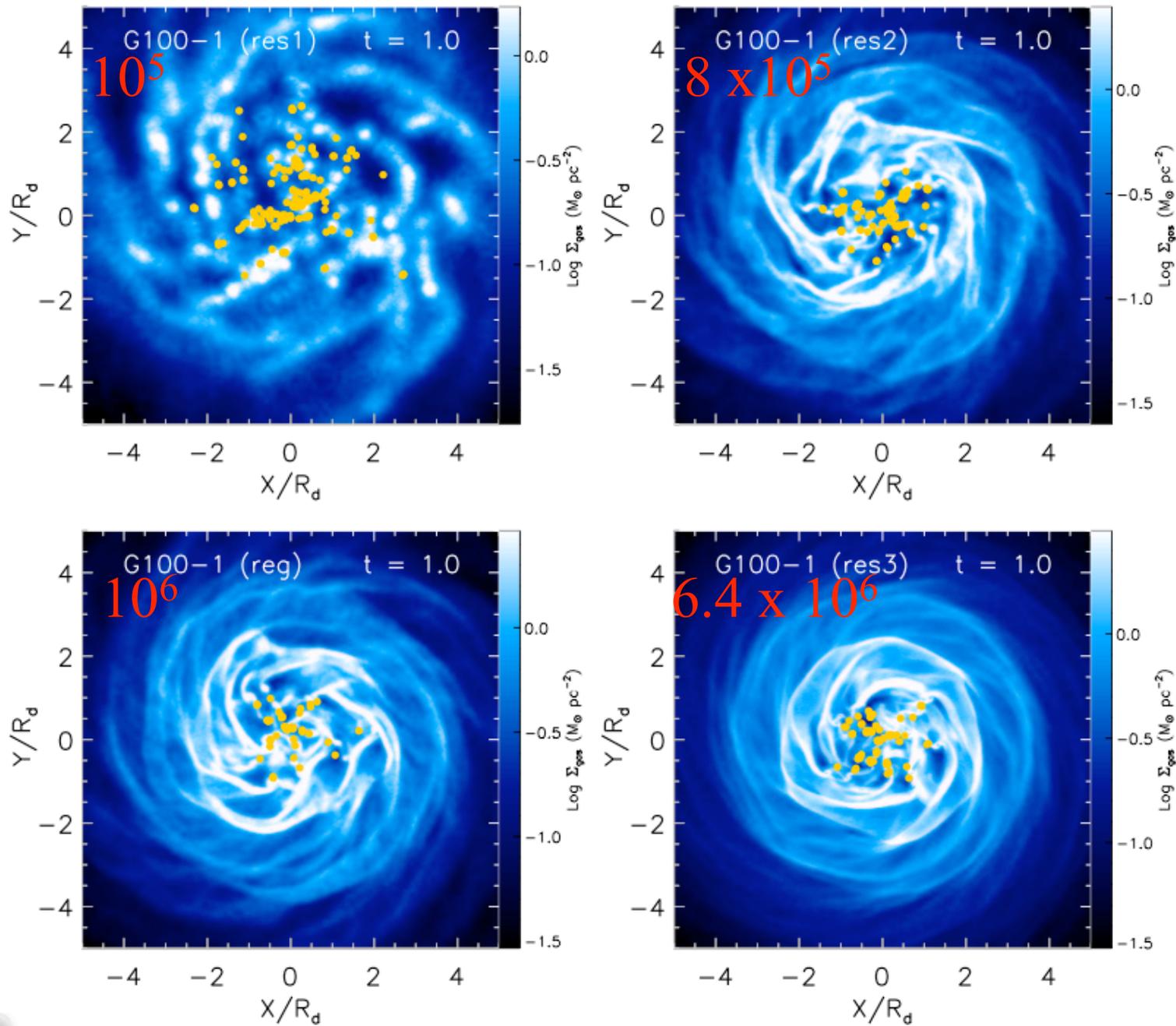
The most important effect this will have on our models is probably to change the mass spectrum (eg Li, Klessen & Mac Low 2003) from log-normal. Perhaps to something more like the observed power law?

# *Approximations*

- ❖ Isothermal gas
- ❖ Fast molecular cloud formation
- ❖ **Neglect of magnetic fields**
  - B fields unlikely to support against collapse at these scales
  - B fields appear not to maintain turbulence.
  - They may slow collapse, make it less efficient.
  - ZEUS models summarized in Mac Low & Klessen (2004), P. Li et al. (2004)

# *Numerical Criteria*

- ❖ *Jeans* for mass resolution (Bate & Burkert 97, Truelove et al. 97)
- ❖ *Gravity-hydro balance* for gravitational softening length (Bate & Burkert 97)
- ❖ *Equipartition* between gas, collisionless particle masses (Steinmetz & White 97).



## Resolution study

Increasing linear res by factor of 2 between each frame.



# Gravitational Instability

Linear analysis of axisymmetric radial gravitational instability:

- collisionless stars (Toomre 64)
- collisional gas (Goldreich & Lynden-Bell 65)

Q instability parameters:

$$\text{Stars: } Q_s = \kappa \sigma_s / (3.36 G \Sigma_s) \quad \text{Gas: } Q_g = \kappa c_g / (\pi G \Sigma_g)$$

Stars & gas (Rafikov 2001):

$$\frac{1}{Q_{sg}} = 2 \frac{[1 - e^{-q^2} I_0(q^2)]}{q Q_s} + \frac{2qR}{Q_g (1 + q^2 R^2)}$$

$\kappa$  -- epicyclic frequency

$I_0$  -- Bessel fcn of order 0,

$q = k \sigma_s / \kappa$ ,  $R = c_g / \sigma_s$ .

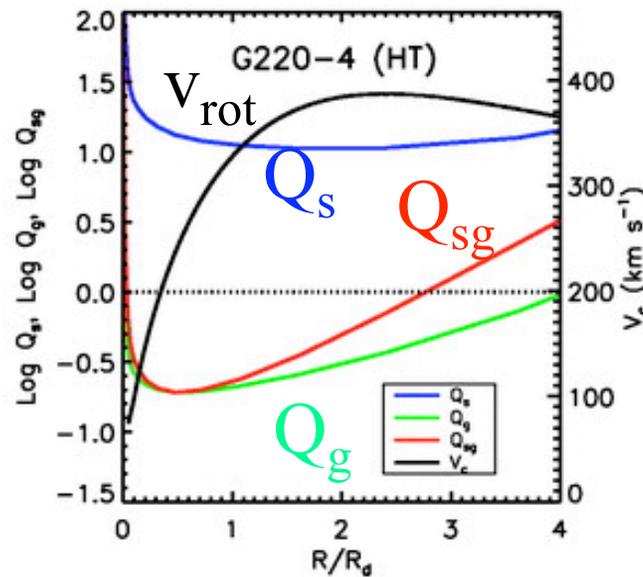
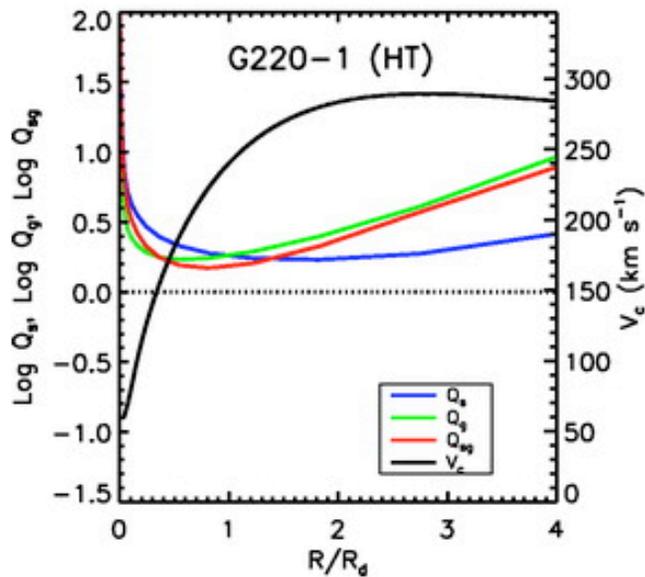
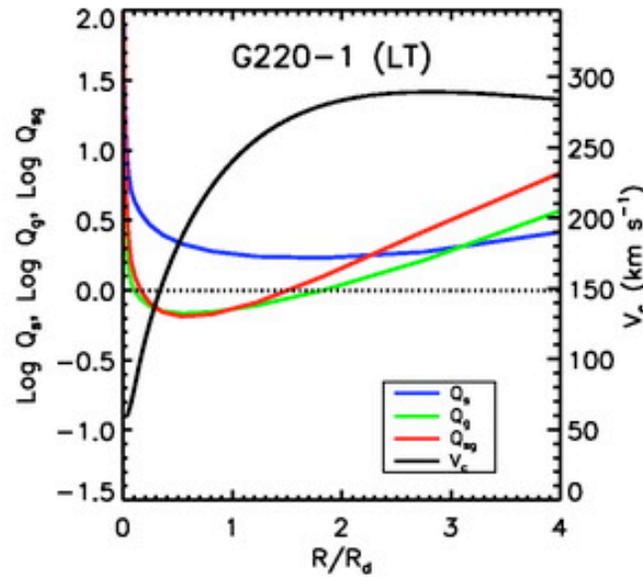
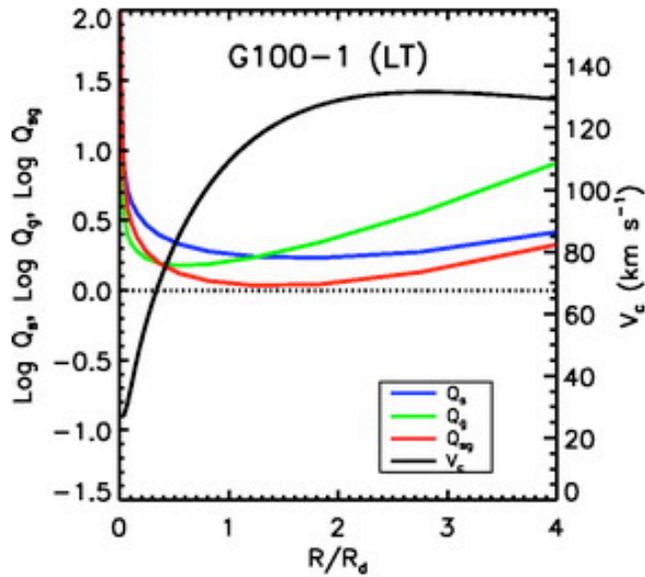
$\Sigma_s, \Sigma_g$  -- stellar, gas surf. den

$\sigma_s$  -- radial stellar vel disp

$c_g$  -- isotherm gas sound spd

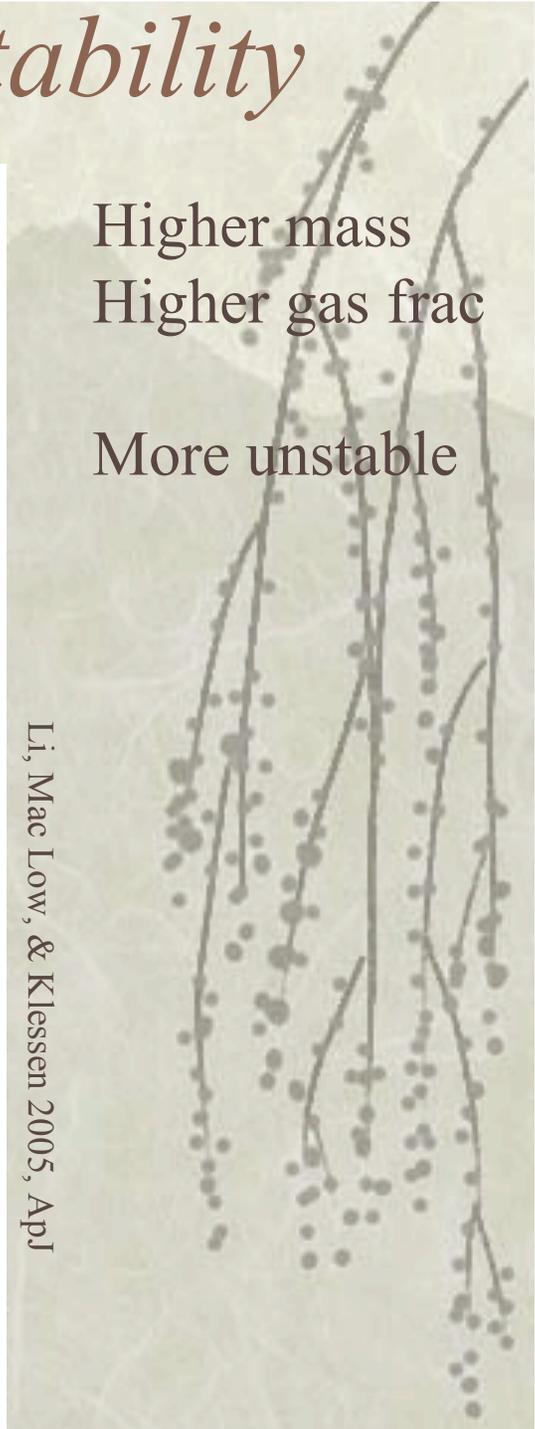
Instability when  $Q_{sg} < 1$ .

# Initial Gravitational Instability

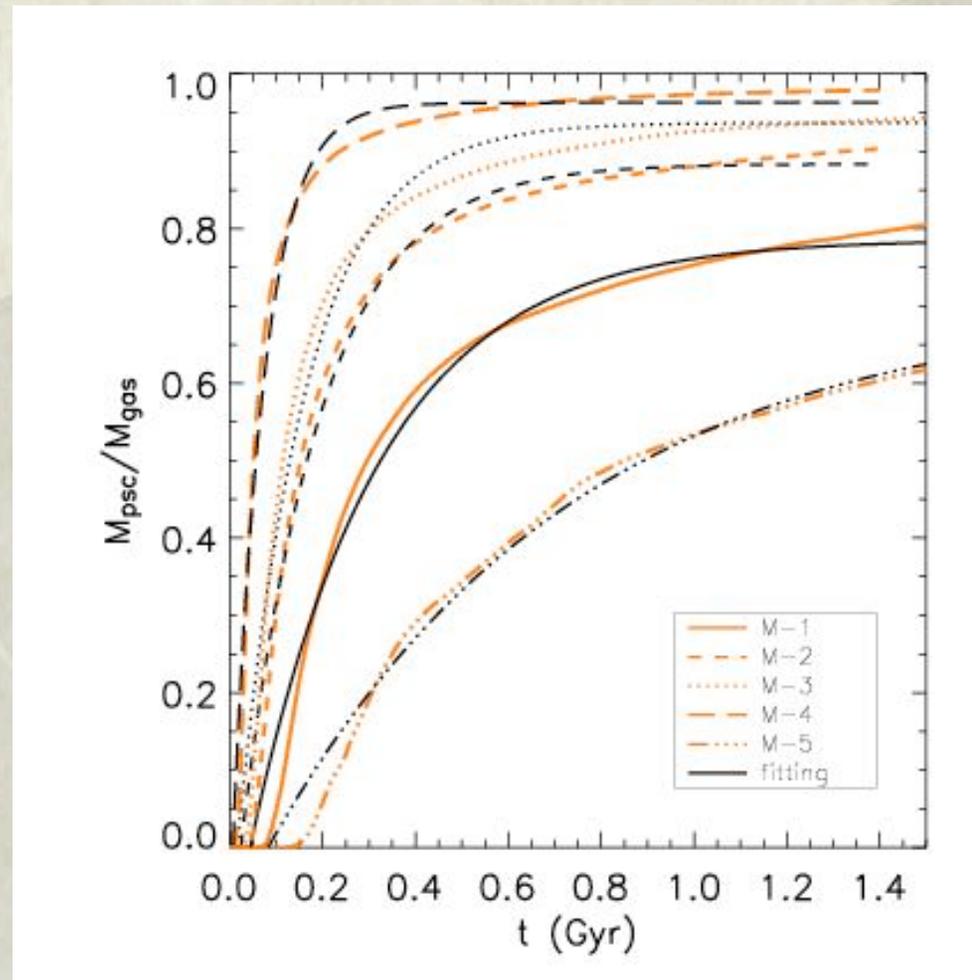


Higher mass  
Higher gas frac  
More unstable

Li, Mac Low, & Klessen 2005, ApJ



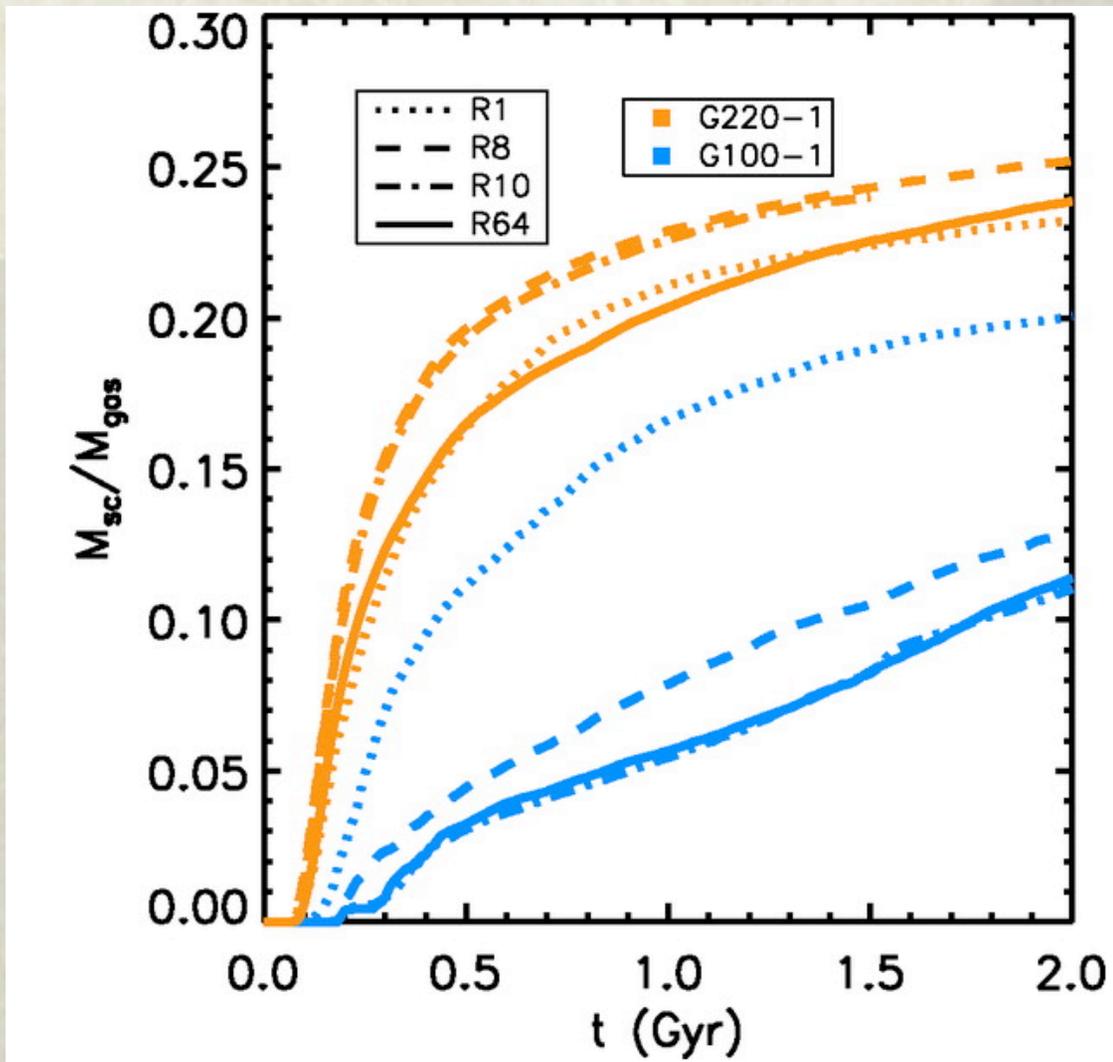
# Collapse Timescale



Li, Mac Low, & Klessen 2005, ApJ

$$M_* = M_0 \left( 1 - \exp \left[ -t / \tau_{sf} \right] \right)$$

# Resolution Study

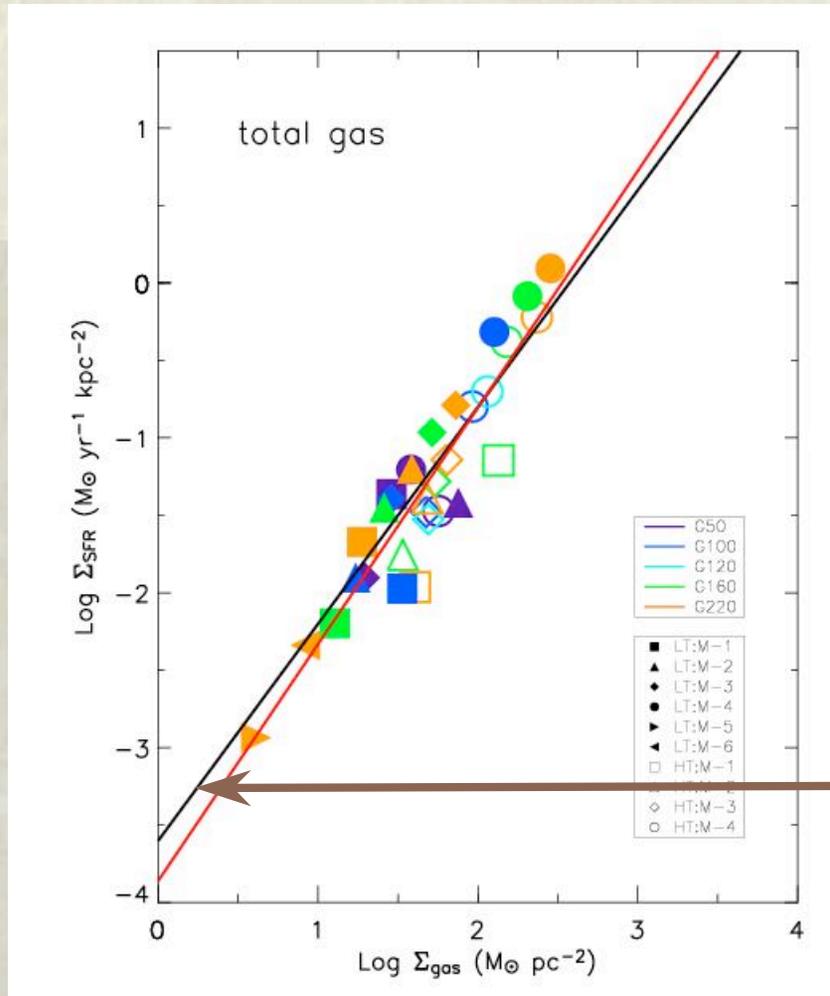


Li, Mac Low, & Klessen 2005, ApJ



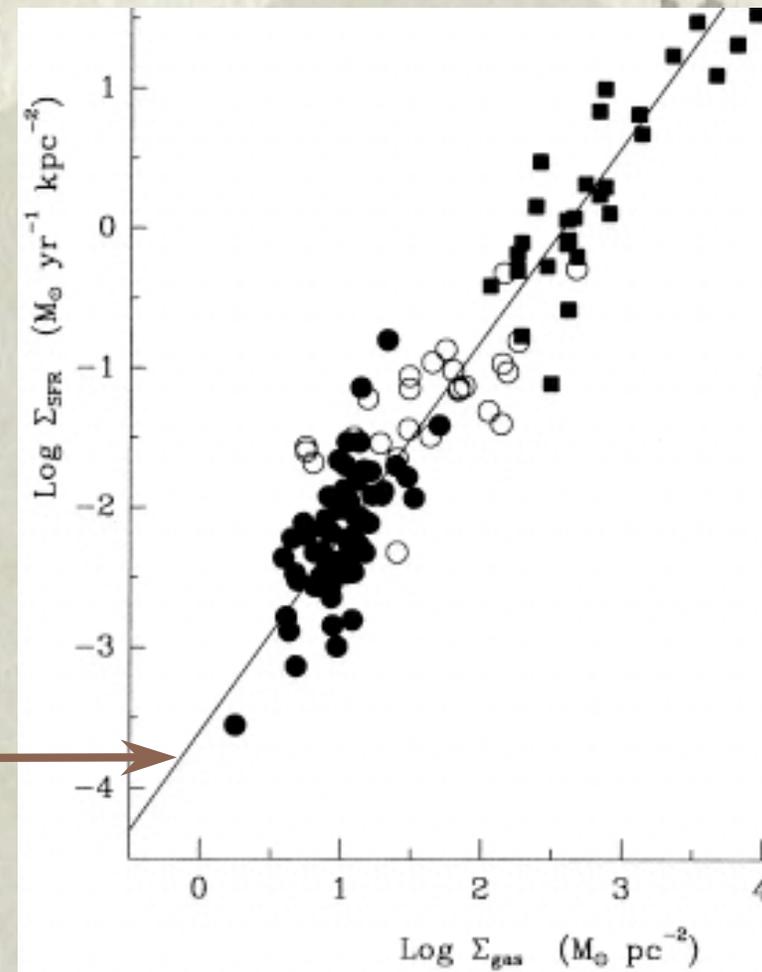
# Global Schmidt Law

models



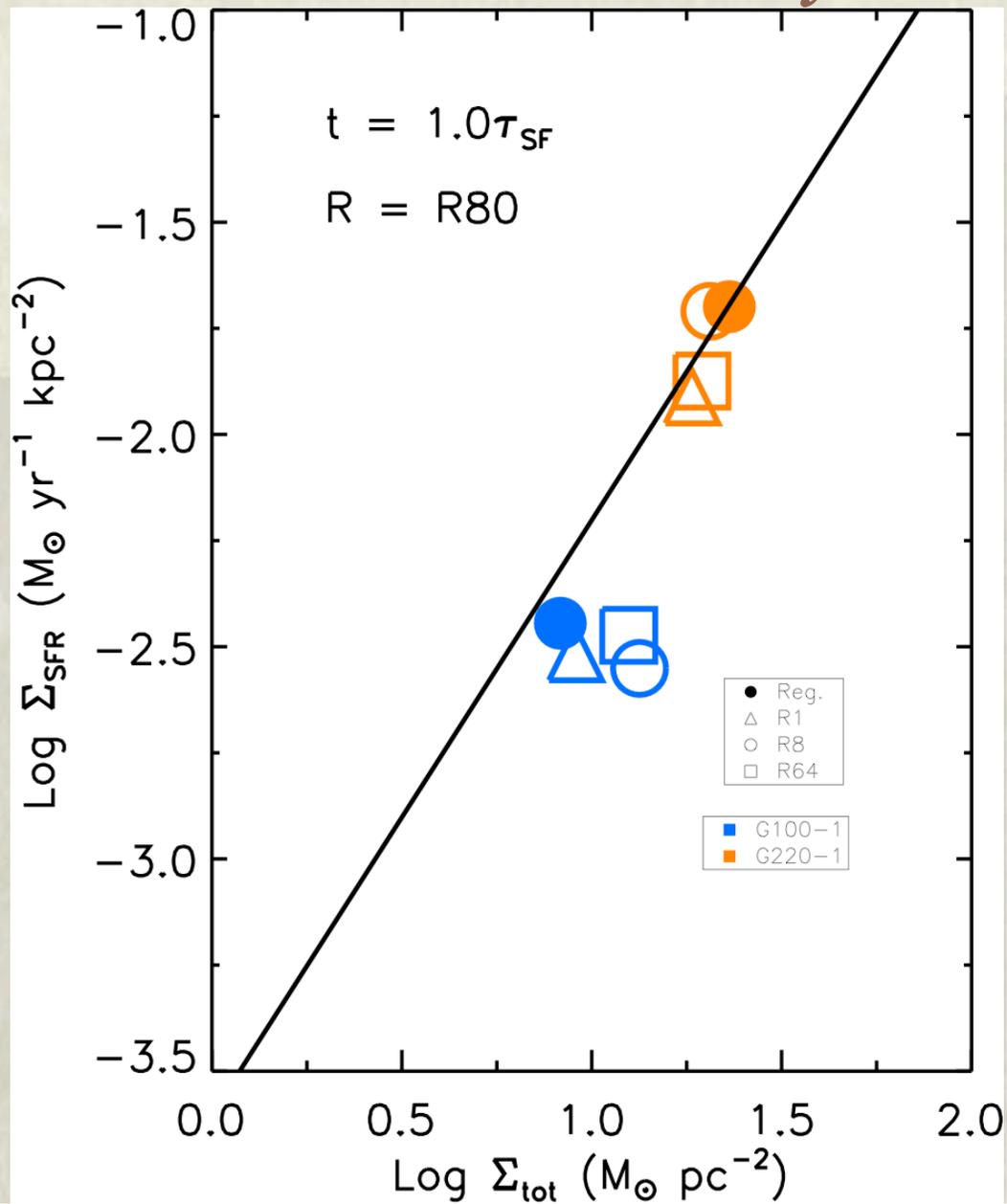
Li, Mac Low, & Klessen 2005, ApJLett, 2006, ApJ

observations

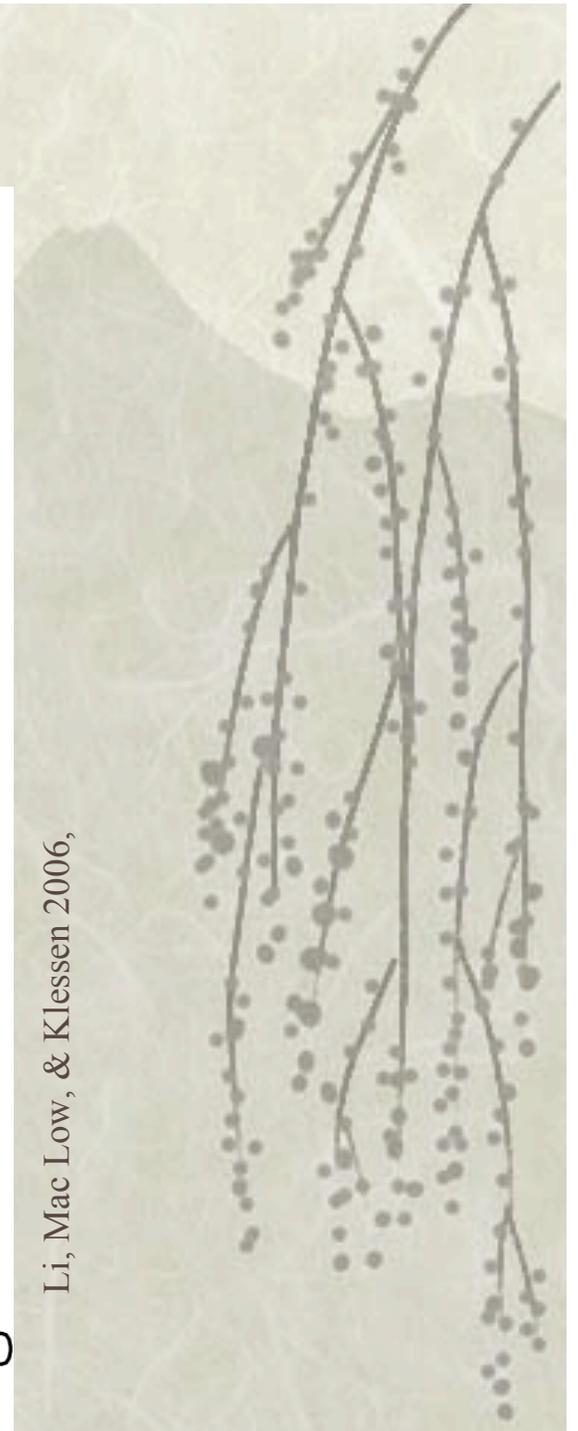


Kennicutt 1998

# Resolution Study



Li, Mac Low, & Klessen 2006,



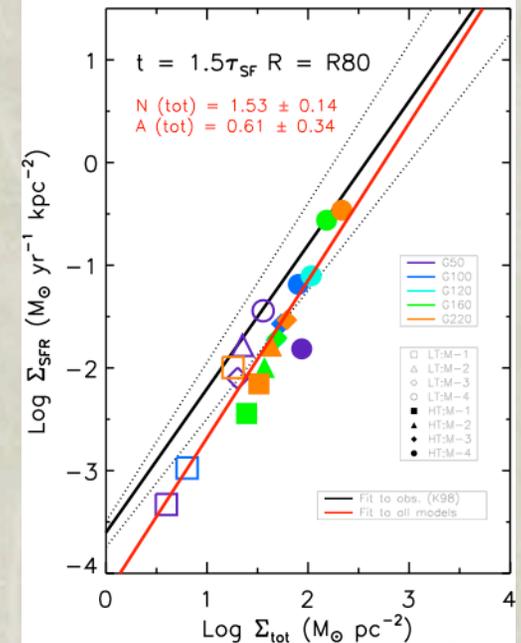
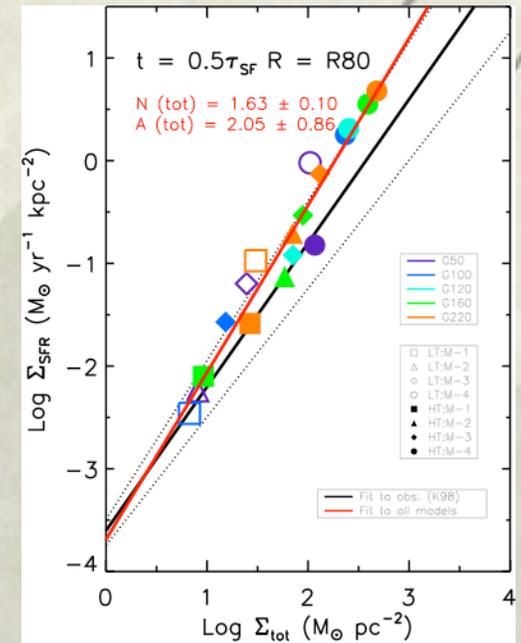
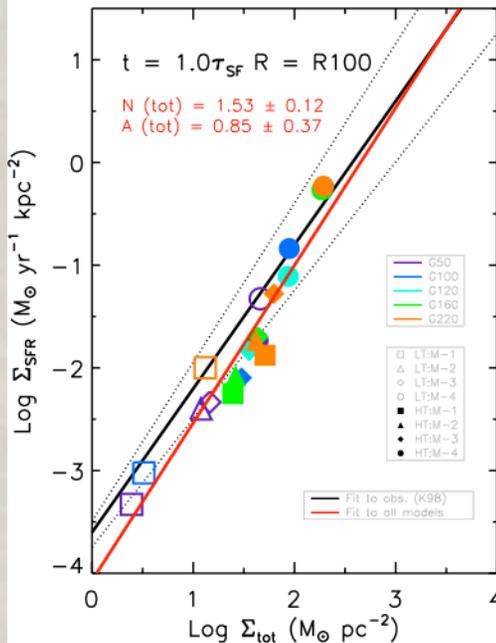
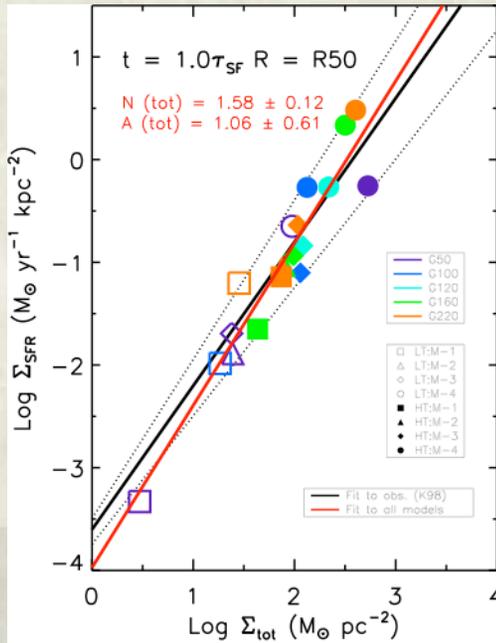
# Variations

Observed

$$\Sigma_{SFR} = (2.5 \pm 0.7) \times 10^{-4} \left( \frac{\Sigma_{gas}}{1 M_{\odot} pc^{-2}} \right)^{1.4 \pm 0.15}$$

$$M_{\odot} year^{-1} kpc^{-2},$$

That is,  $A = 2.5 \pm 0.7$   
 $N = 1.4 \pm 0.15$



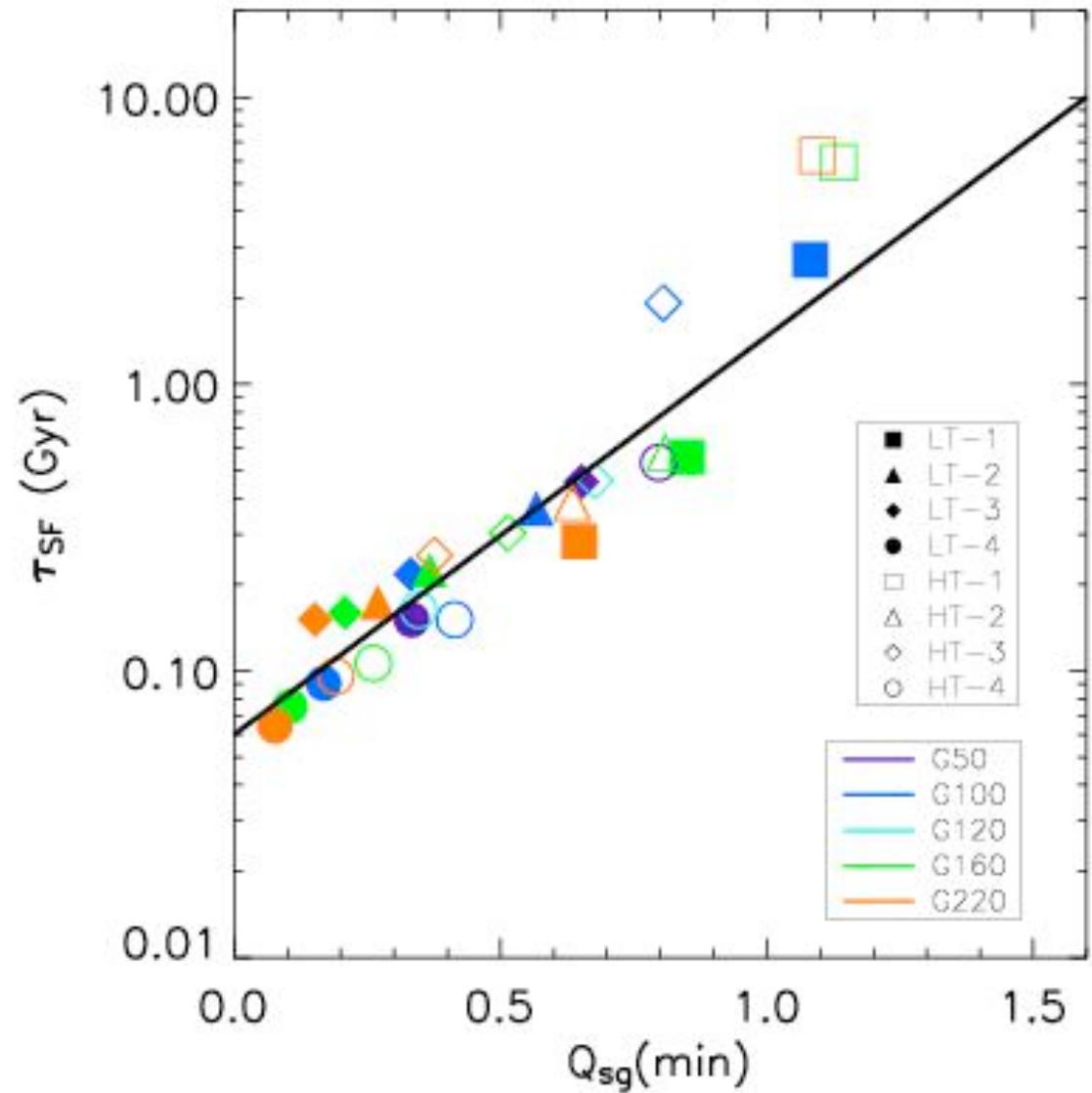
Varying SFE:

- $\epsilon = 0.1 \Rightarrow A = 0.7 \times 10^{-4}$
- $\epsilon = 0.9 \Rightarrow A = 3.2 \times 10^{-4}$

# *Instability drives SF*

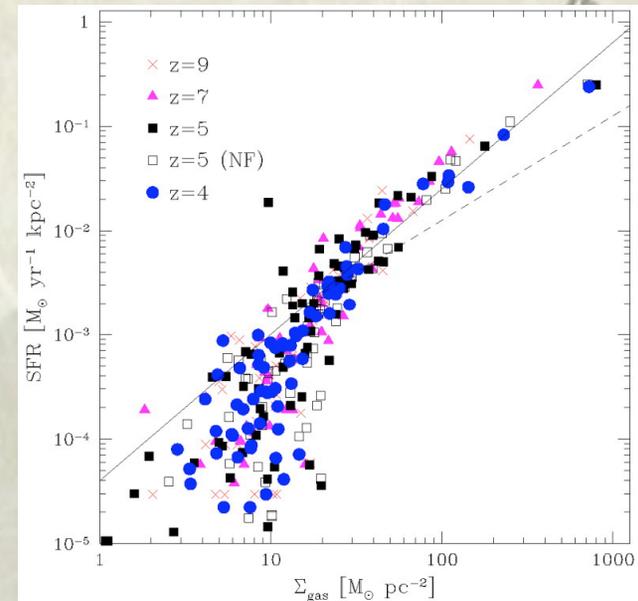
$$\tau_{SF} \propto e^{\alpha Q_{sg}}$$

$$\alpha \approx 3$$



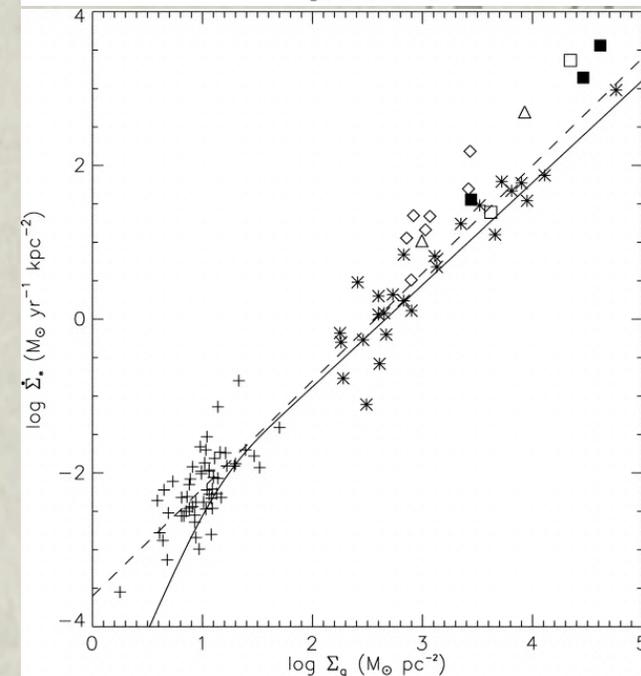
# Complementary Derivations

- Kravtsov (2003):
  - Cosmological ICs
  - Star formation law  $\dot{\rho}_* \propto \rho_g$
  - Measured SF in many galaxies in one model



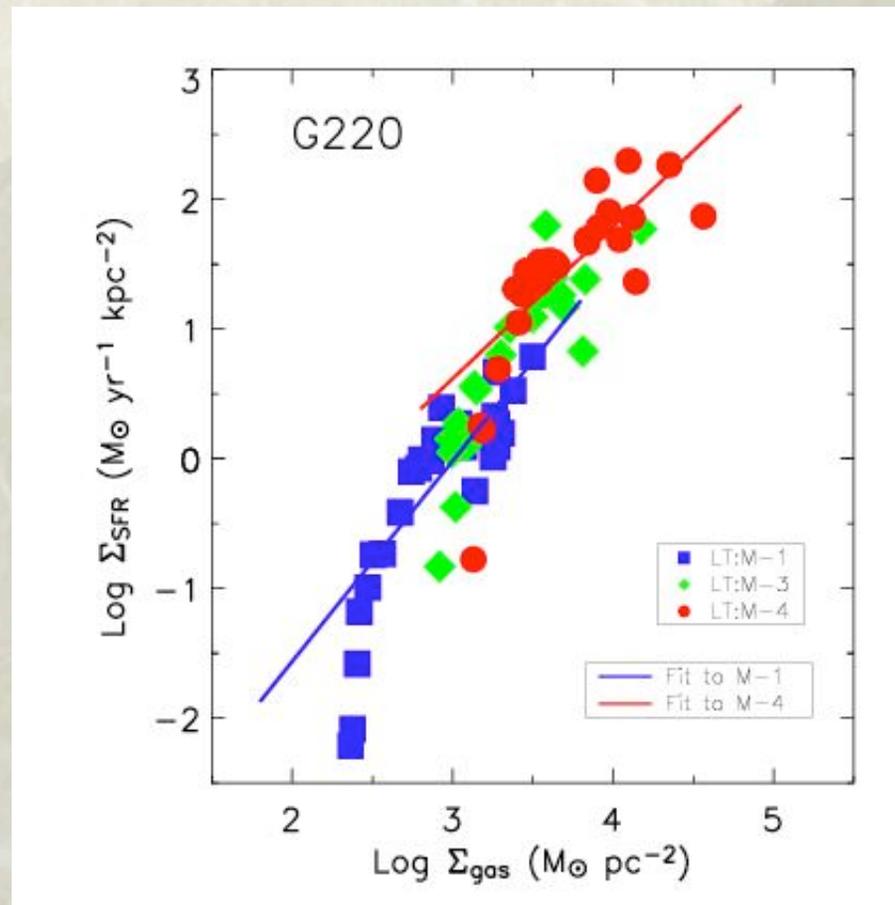
Kravtsov 2003

- Krumholz & McKee (2005):
  - Take observed distribution of GMCs, H II regions as *input*
  - GMCs in virial equilibrium
  - Derive Schmidt law (*solid*) from density PDF of supersonic turbulence (see Padoan & Nordlund)

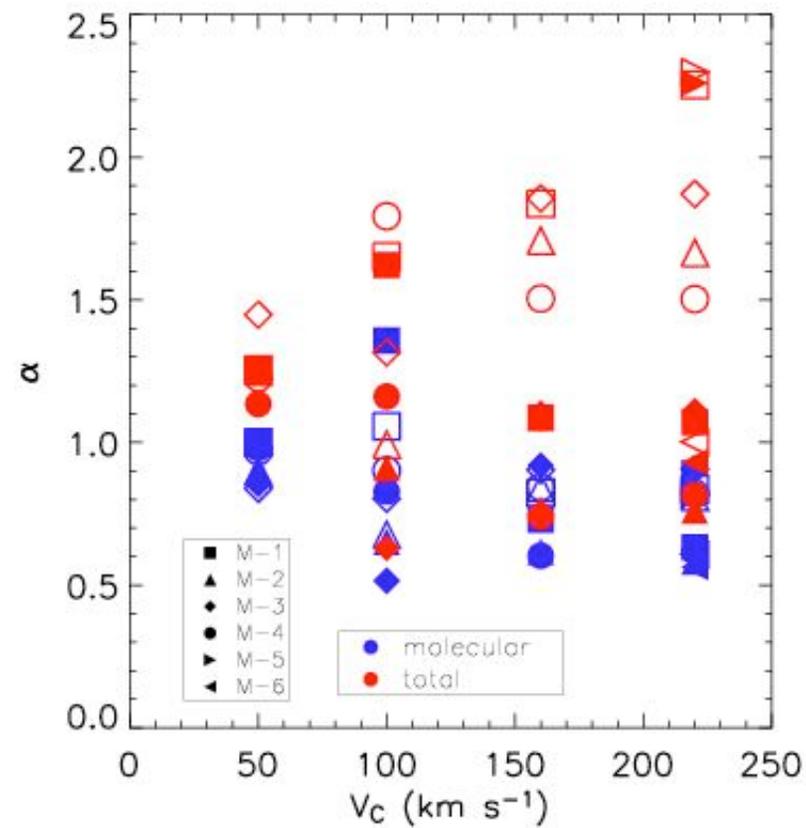


• Krumholz & McKee 2006

# *Local Schmidt Laws*

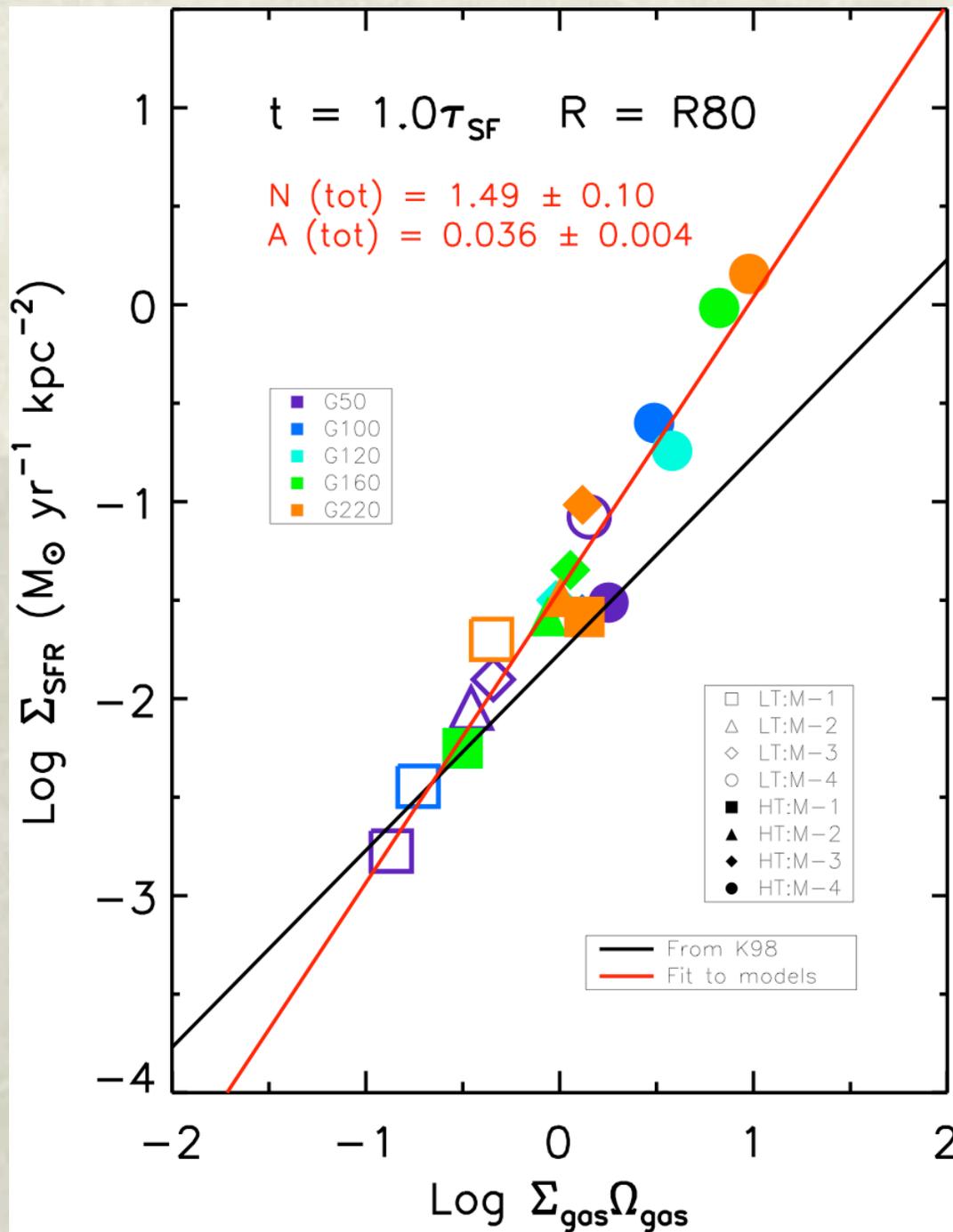


# *Local Schmidt Law Indices*



Wyse-Silk  
Law disagrees?

Primarily  
for very  
gas-rich  
objects  
foreign to  
the local  
universe,  
though.



# LMC Stability

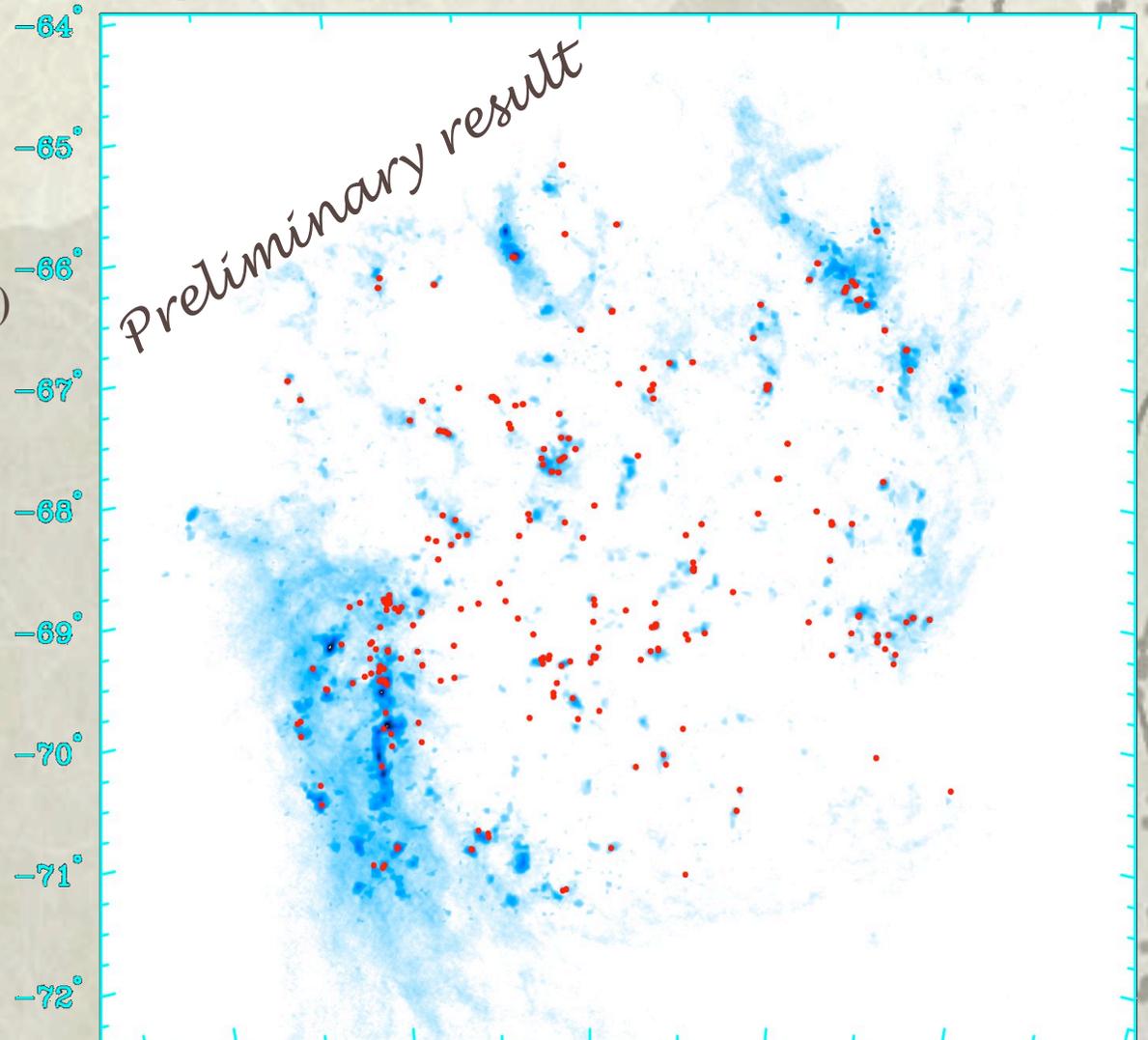
$$Q_g = \frac{KC_g}{\pi G \Sigma_{g_g}} < 1 \quad (\text{gas})$$

Atomic gas: HI (Kim et al)  
Molecules: CO (Fukui et al)

YSOs: Spitzer 8  $\mu\text{m}$

$Q_g$ : blue scale

YSOs: red dots



From Y.-H. Chu, 2006, IAU Symp 237

# LMC Stability

Stars: Spitzer 3  $\mu\text{m}$

Atomic gas: HI (Kim et al)

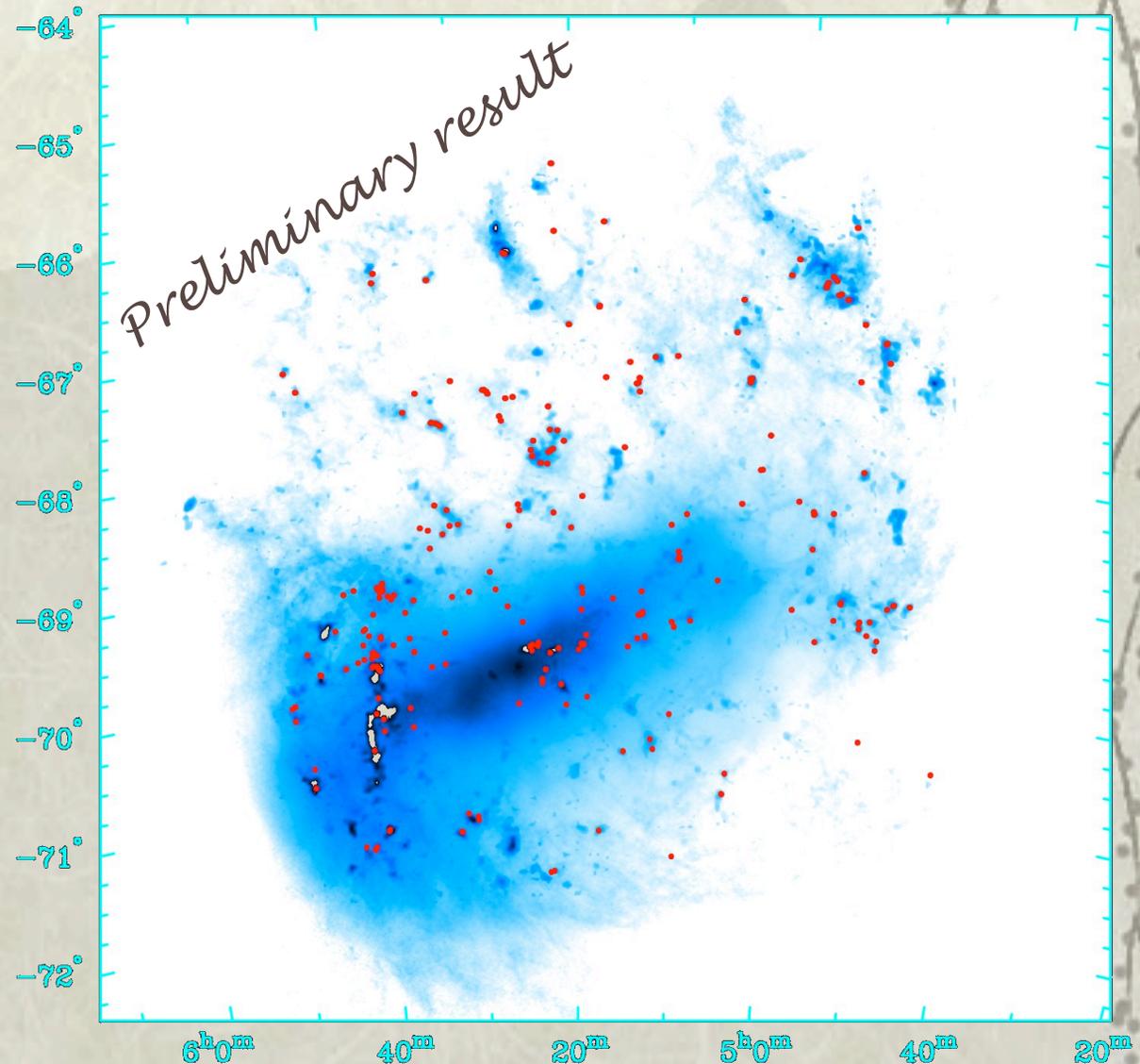
Molecules: CO (Fukui et al)

YSOs: Spitzer 8  $\mu\text{m}$

$Q_{\text{sg}}$ : blue scale

YSOs: red dots

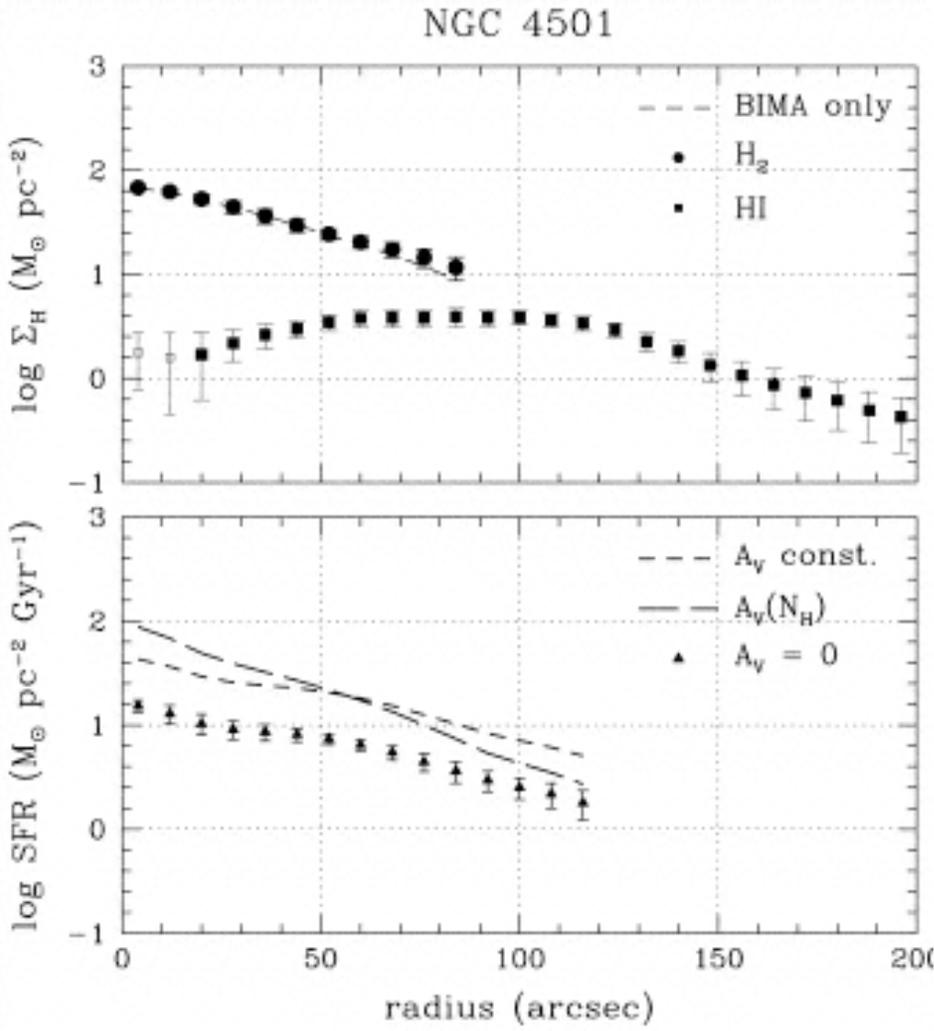
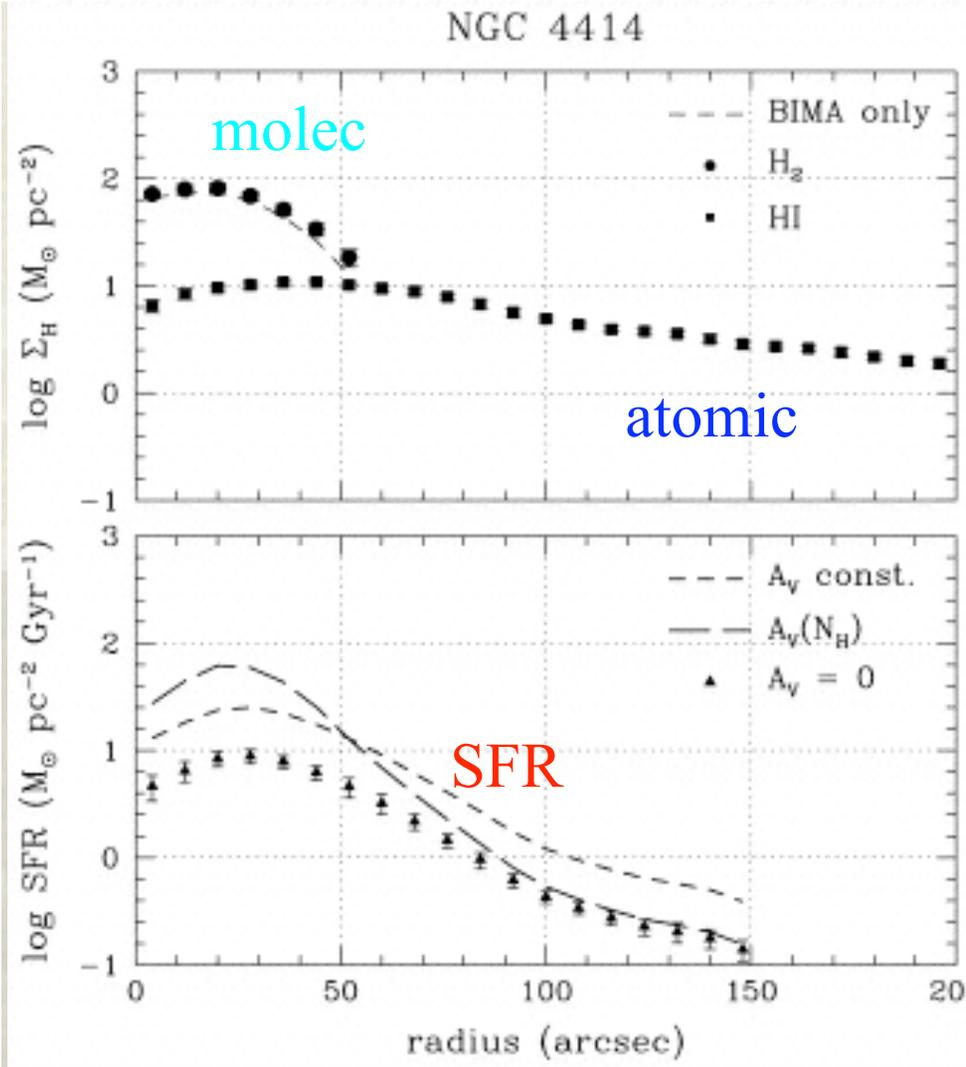
**Stars & gas**



From Y.-H. Chu, 2006, IAU Symp 237



# Observed radial distributions of gas



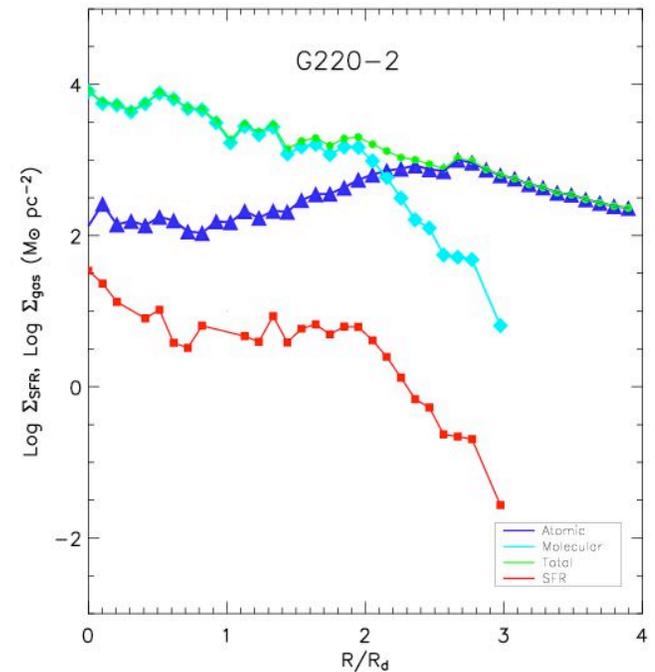
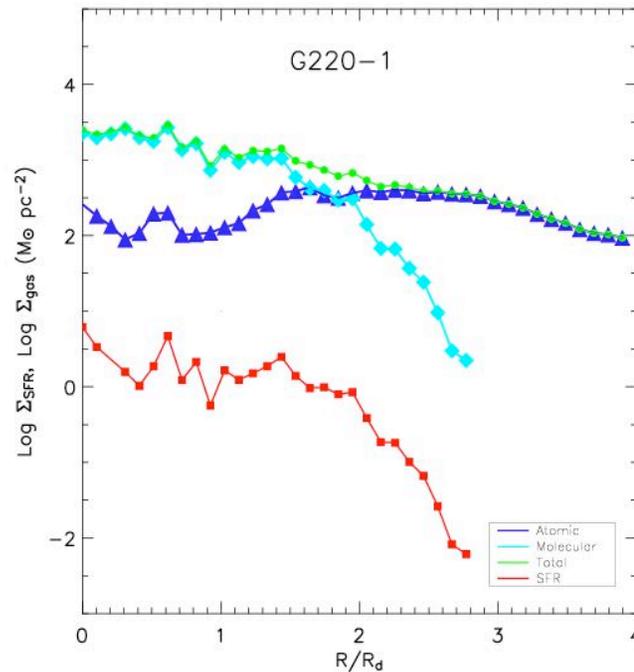
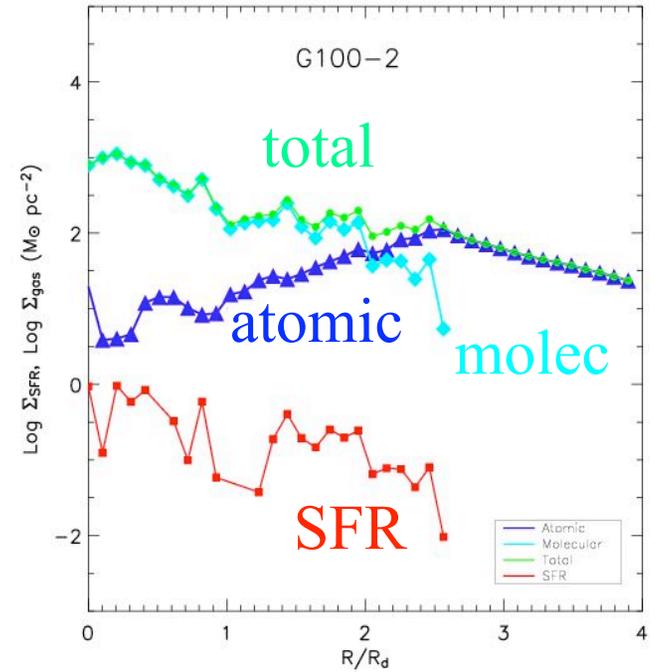
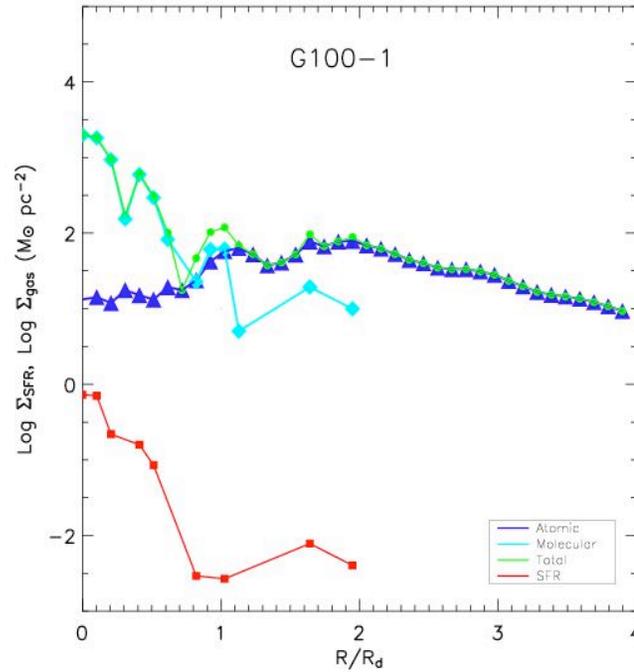
Wong & Blitz 2002

Model distributions,  
assuming sink  
particle mass goes

- 30% to stars
- 70% to molecules

Li, Mac Low, & Klessen 2006

Also cf. breaks in  
stellar profiles  
seen at threshold  
(Pohlen &  
Trujillo 2006,  
Elmegreen &  
Hunter 2006)



## *Conclusions*

- ❖ Nonlinear development of gravitational instability appears sufficient to explain Kennicutt-Schmidt law.
- ❖ However, consistency between our models and the observed behavior of molecular clouds must still be tested.