

# Overview of KS-Law: Interpretations, Cloud and Star Formation Structures

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San Diego, December 19-20, 2006

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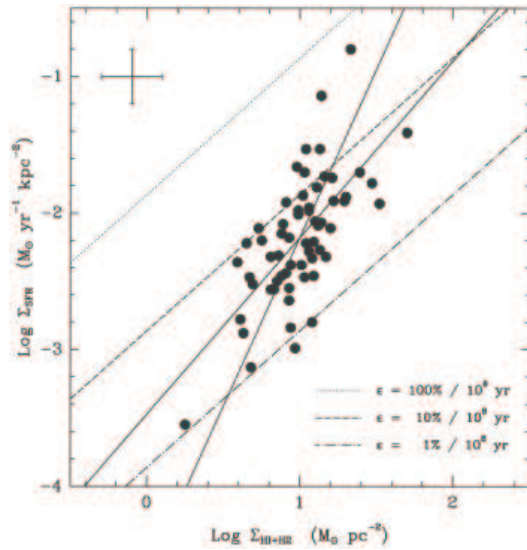
San Diego, December 19-20, 2006

1. Observations
2. Theory
3. Reflections

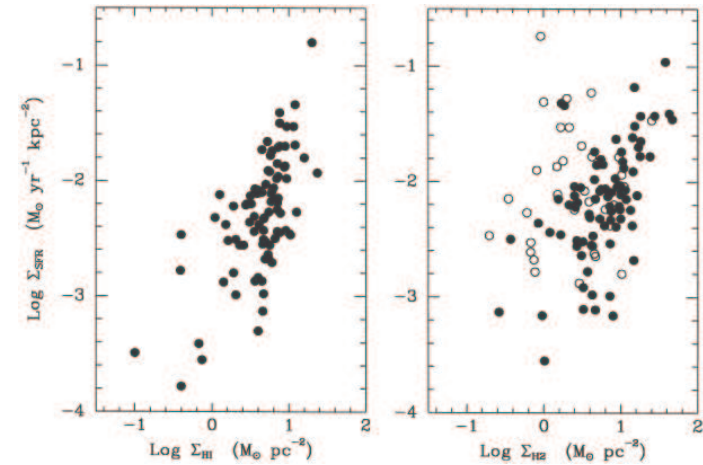
# The Schmidt (1959) paper

- Derived SFR over history of galaxy
  - assuming a constant initial luminosity function  $\Psi(M_v)$
  - a lifetime function  $T(M_v)$ , and gas return/star of  $M^{-0.7}$
  - a rate  $f(t)$  that scales with a power of gas mass  $M_G(t)$ 
    - $f(t) \sum_{M_v} \Psi(M_v) = C [M_G(t)]^n$
  - gave analytical solutions for  $n=0, 1, 2$
  - used scale height for HI of 144 pc, scale height for Cepheids of 80 pc, and for clusters of 58 pc to give  $n=2$  to 3.
    - whitedwarf count:  $n>2$ , He abundance:  $n=2$ , uniform HI:  $n\geq 2$ , cluster MF:  $n=1-2$
  - noted that  $n=2$  suggests dense galaxies (ellipticals) should now have less gas than low-density galaxies (LMC)
- final comment: “It is hoped to study the evolution of galaxies in more detail in the future”

# The Kennicutt 1998 result, normal gal.



H $\alpha$  SFR vs  
HI+H $_2$ ,  
 $n=1.29, 2.47$

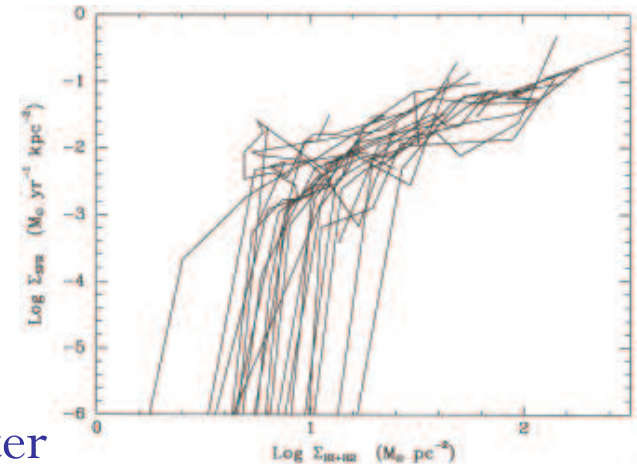


H $\alpha$  SFR vs HI and H $_2$

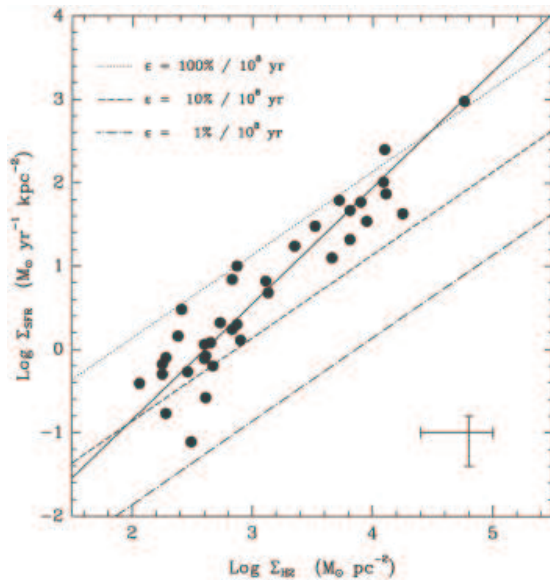
Assumptions:

1. 61  $i < 75^\circ$  galaxies with CO, HI, H $\alpha$
2. constant extinction correction & H $_2$ /CO
3. Salpeter IMF, 0.1-100  $M_{\odot}$
4. Radial size from RC2

local H $\alpha$  SFR vs HI+H $_2$   
with gal. radius a parameter



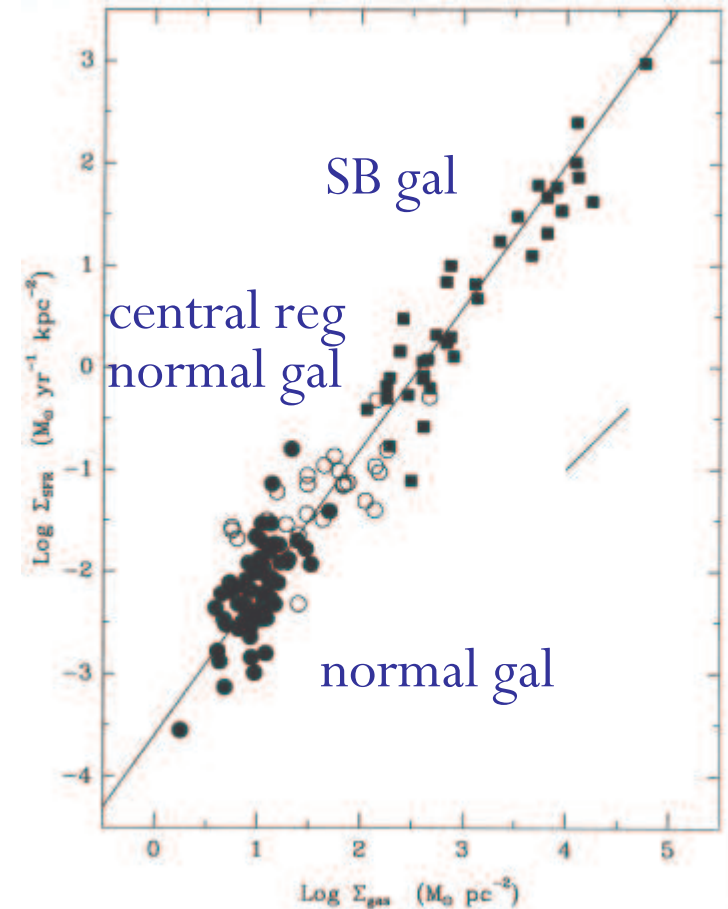
# The Kennicutt 1998 result, starburst gal



FIR SFR  
vs  $\text{H}_2$   
 $n=1.28, 1.4$

Assumptions:

1. 36 galaxies with high-res CO, FIR (ignore HI)
2. constant  $\text{H}_2/\text{CO}$
3. 10-100 My continuous bursts (Leitherer et al)
4. Radial size from burst region (CO or IR)

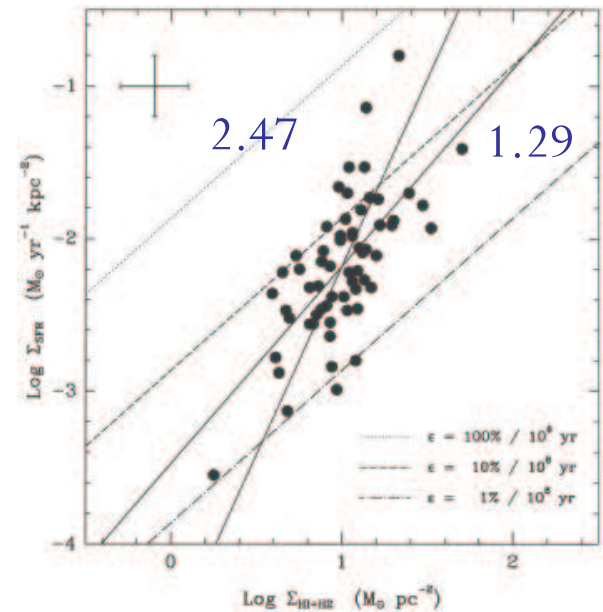
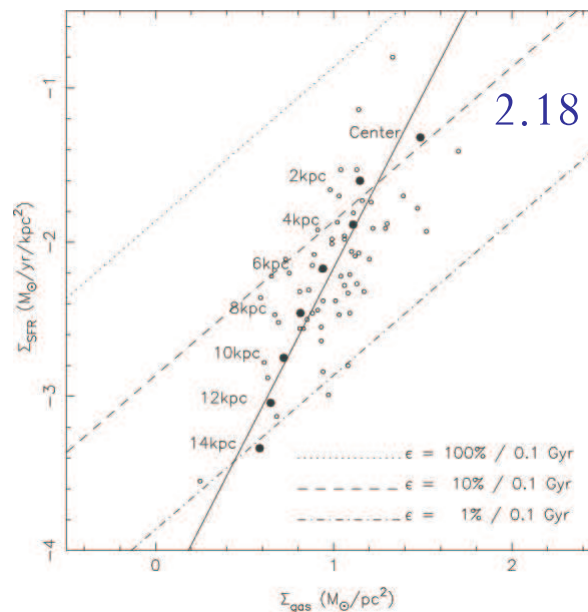


combined: normal & SB  
 $n=1.40 \pm 0.05$

# K-S Law in the Milky Way

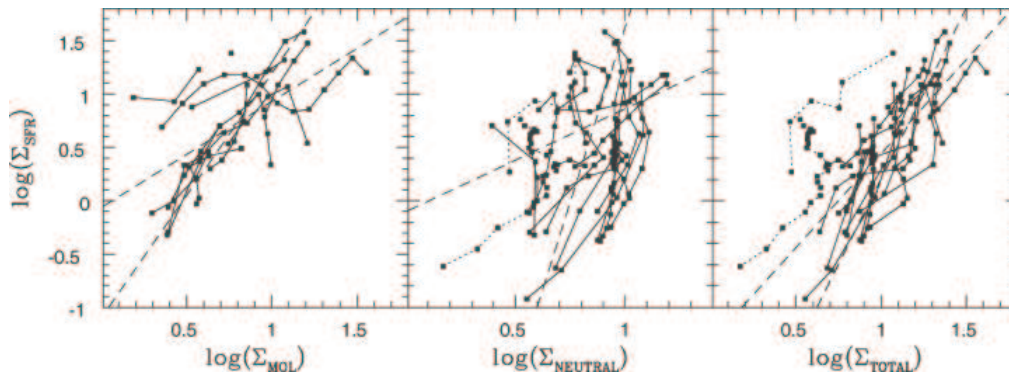
- Misiriotis et al. 06 use COBE/DIRBE to get both the gas/dust distribution and SFR distribution
  - find K-S slope of 2.18 ± 0.20
  - claim similar to Kennicutt '98  $n=2.47$  (from bivariate fit)

(Kennicutt 98)

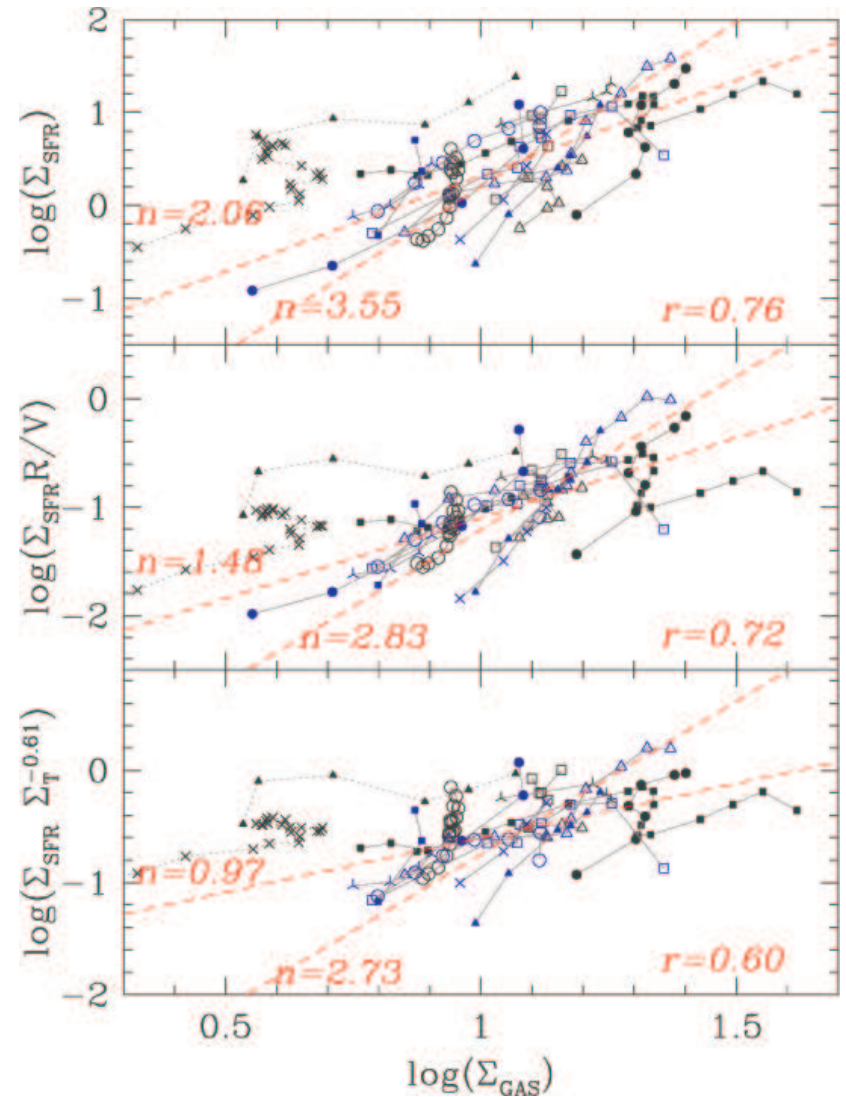


# K-S law variations

- Boissier et al. 03 compared SFR vs radius in 16 galaxies with three expressions, best fits:
  - Kennicutt/Schmidt:  $\Sigma_{\text{gas}}^{2.06}$
  - Boissier & Prantzos '99:  $\Sigma_{\text{gas}}^{1.48} V/R$
  - Dopita & Ryder '94:  $\Sigma_{\text{gas}}^{0.97} / \Sigma_{\text{tot}}^{0.61}$
 – assumed  $\text{H}_2/\text{CO}$  varied with radius as the metallicity (Boselli et al 02)
- concl: 3 laws are equally good; K-S  $n > 1.4$



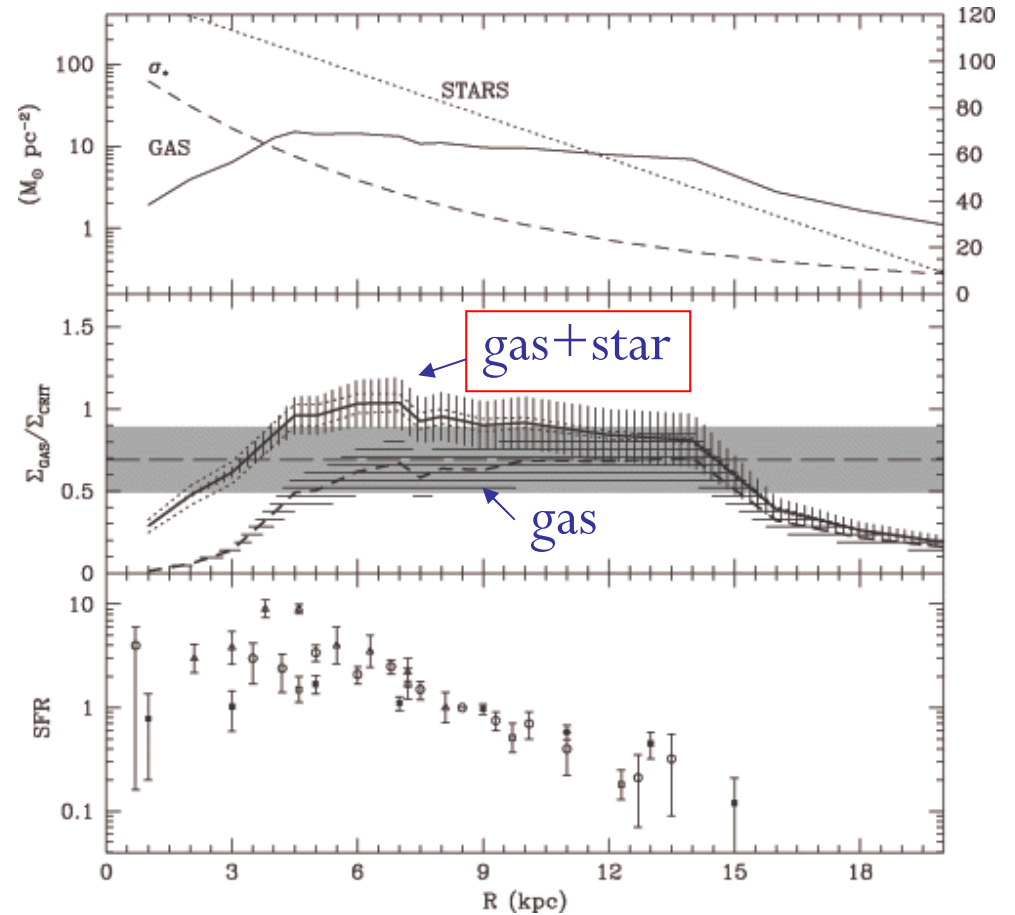
molecular, atomic, total gas comparison





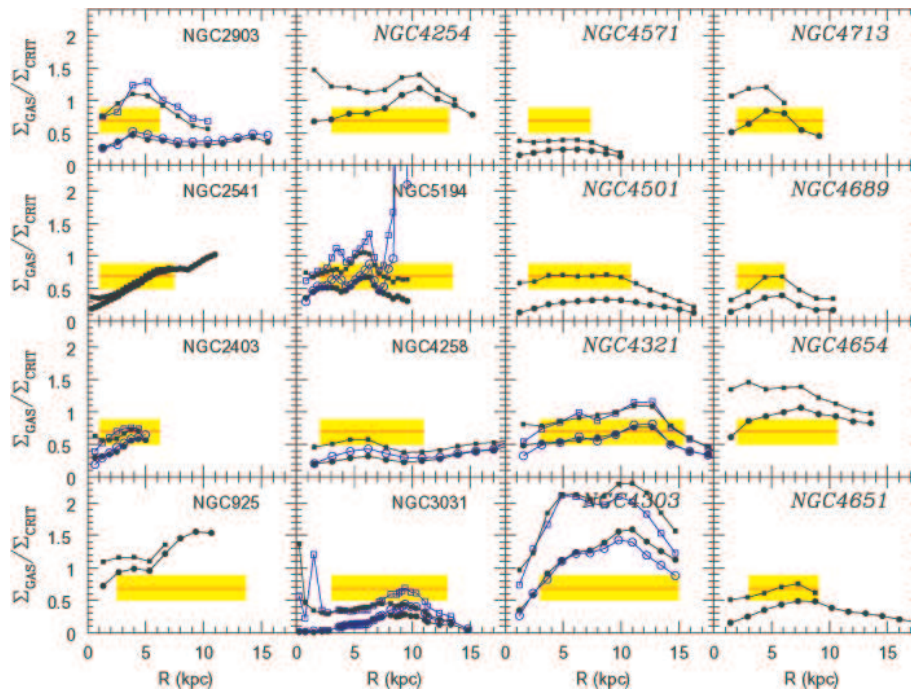
# K-T Threshold in the Milky Way

- Boissier et al. 03 determined  $\Sigma/\Sigma_{\text{crit}}$  in the MW using pure-gas  $\Sigma_{\text{crit}}$  or gas+star  $\Sigma_{\text{crit}}$  (Wang&Silk94)
  - gas+star gives best threshold



# K-T Threshold Variations

- Boissier et al. 03 also determined  $\Sigma/\Sigma_{\text{crit}}(R)$  for 16 galaxies
  - assumed  $\text{H}_2/\text{CO}$  varies radially
  - for pure-gas  $\Sigma_{\text{crit}}$ , 8/16 of disks are subcritical
  - for gas+star  $\Sigma_{\text{crit}}$ , 2/16 subcritical
  - biggest difference: M&K get higher  $\text{H}_2$  density using constant  $\text{H}_2/\text{CO}$



filled circles: gas only  
 filled squares: gas+stars  
 open symbols: high-res RC

$\Sigma_{\text{GAS}}/\Sigma_{\text{crit}}$  versus Radius  
 (yellow – Martin & Kennicutt '01)

# Threshold determines gas fraction

Zasov & Smirnova 06:

All galaxies have  $\Sigma(\text{HI})$  at critical  $\Sigma_c$

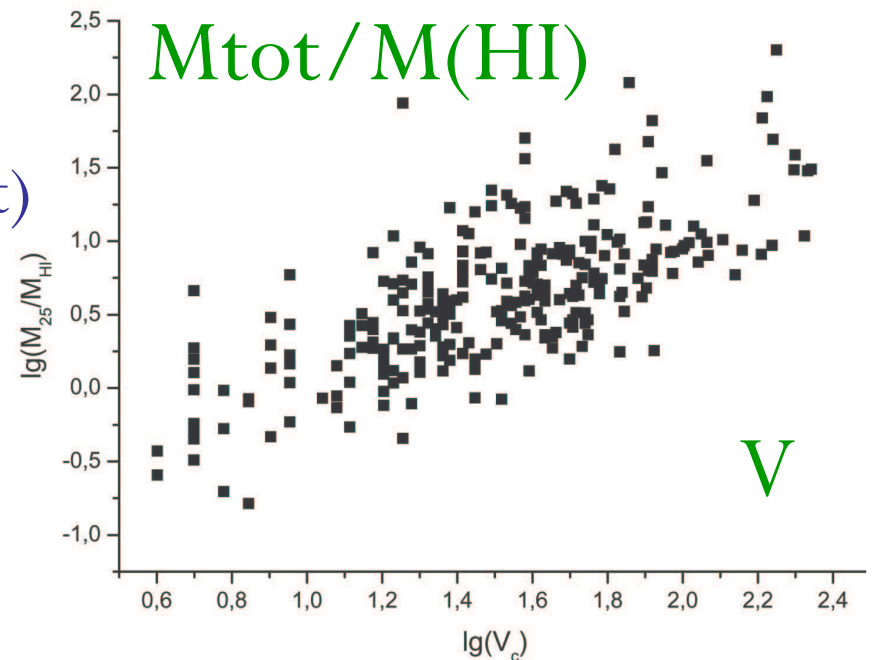
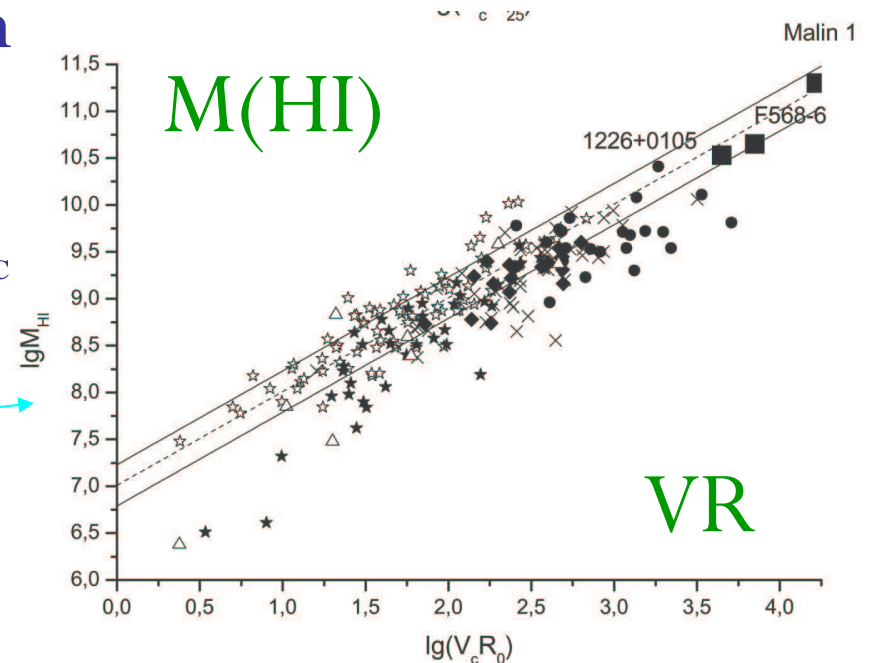
$$\Sigma_c = a\kappa c / \pi G \sim V/R \text{ from } \kappa$$

$$\text{so } M_{\text{gas}} = \int^R 2\pi R \Sigma_c dR \sim VR \quad \rightarrow$$

$$\text{but } M_{\text{tot}} \sim V^2 R$$

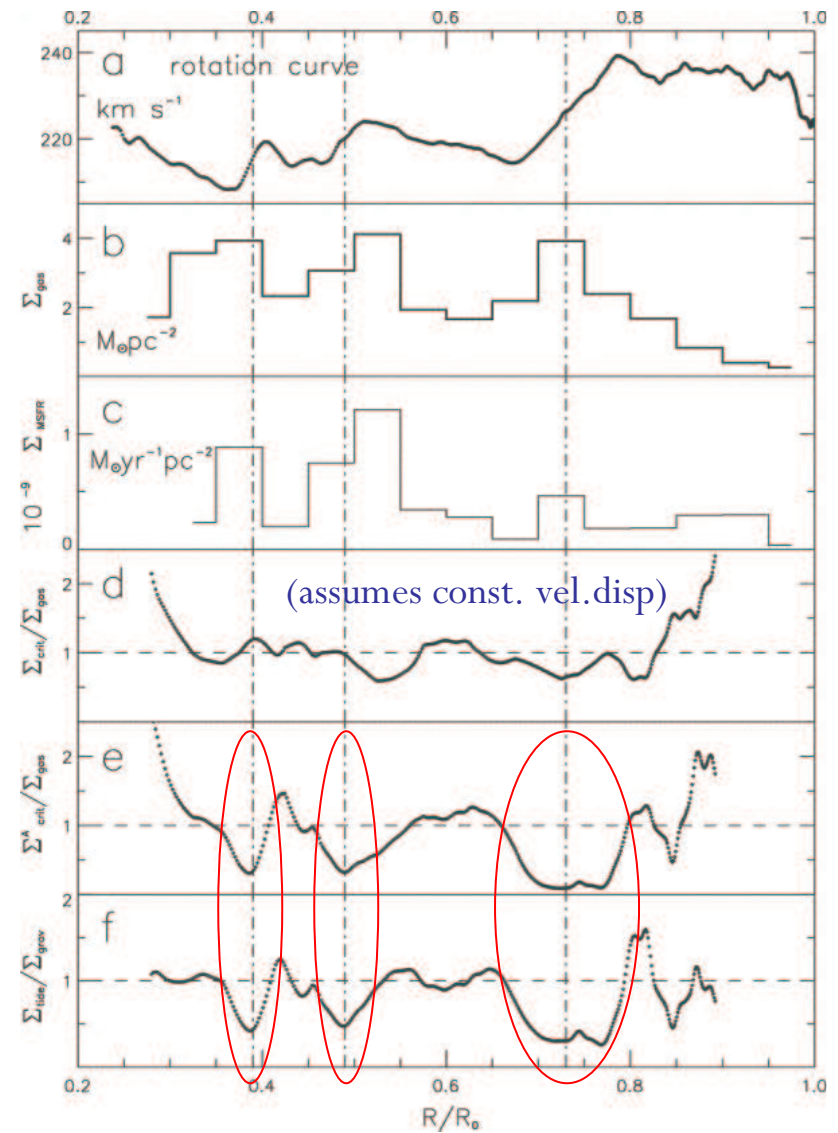
$$\text{so } M_{\text{tot}} / M_{\text{gas}} \sim V \quad \rightarrow$$

-small galaxies are more gas-rich  
because  $\Sigma \sim \Sigma_c$  (not  $n > 2$  ala Schmidt)



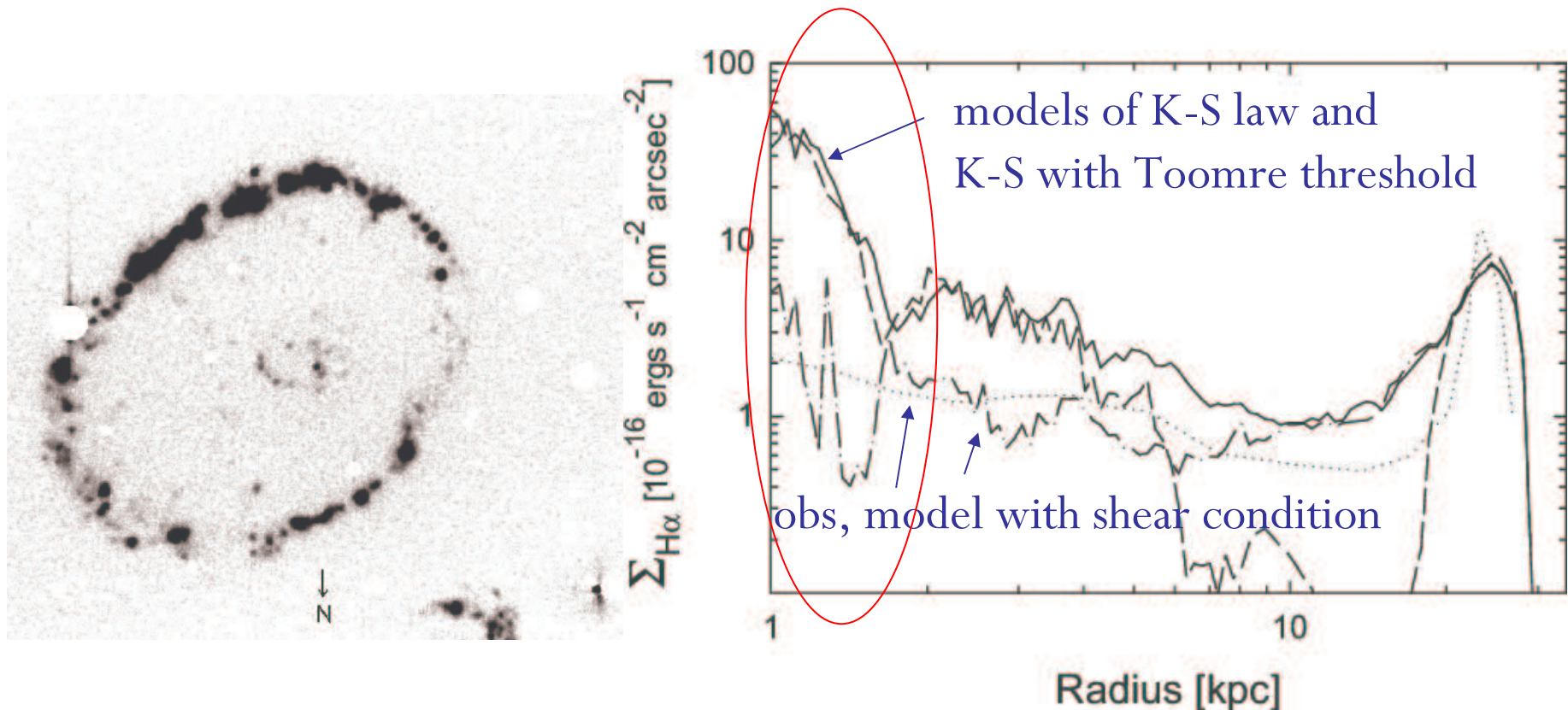
# A Shear Dependence?

- Luna et al. 06: MW SF from IRAS point sources, CO from new southern survey (const.  $H_2/CO$ )
  - finds massive SF in low-shear spiral arms, and overall,  $SFR \sim \Sigma_{\text{gas}}^{1.2 \pm 0.2}$ .



# Another Shear Dependence

- Vorobyov 03: Cartwheel galaxy inner ring too faint for pure K-S law, need high shear to limit SF there.





# The Molecular Cloud Connection

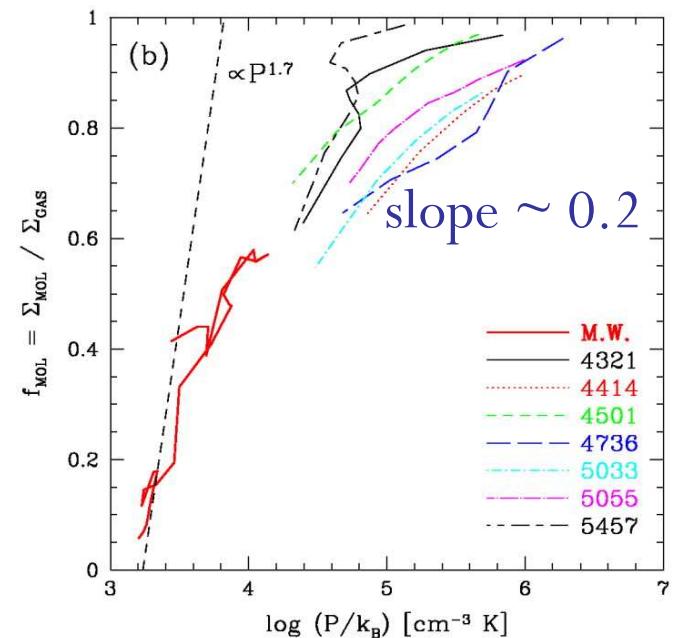
- Wong & Blitz 02: find SFR in direct proportion to molecular cloud density ( $n=1$ ), and suggest K-S law from changing  $f_{\text{mol}}$ 
  - assumed  $\text{H}_2/\text{CO}$  const.
  - combined index  $n=n_{\text{mol}}(1+\text{dln}f_{\text{mol}}/\text{dln}\Sigma_{\text{gas}})$
  - where  $n_{\text{mol}}=1$  (e.g., Rownd & Young 99) and  $f_{\text{mol}}$  increases with  $P$

measure  $\text{dln}f_{\text{mol}}/\text{dln}P \sim 0.2$

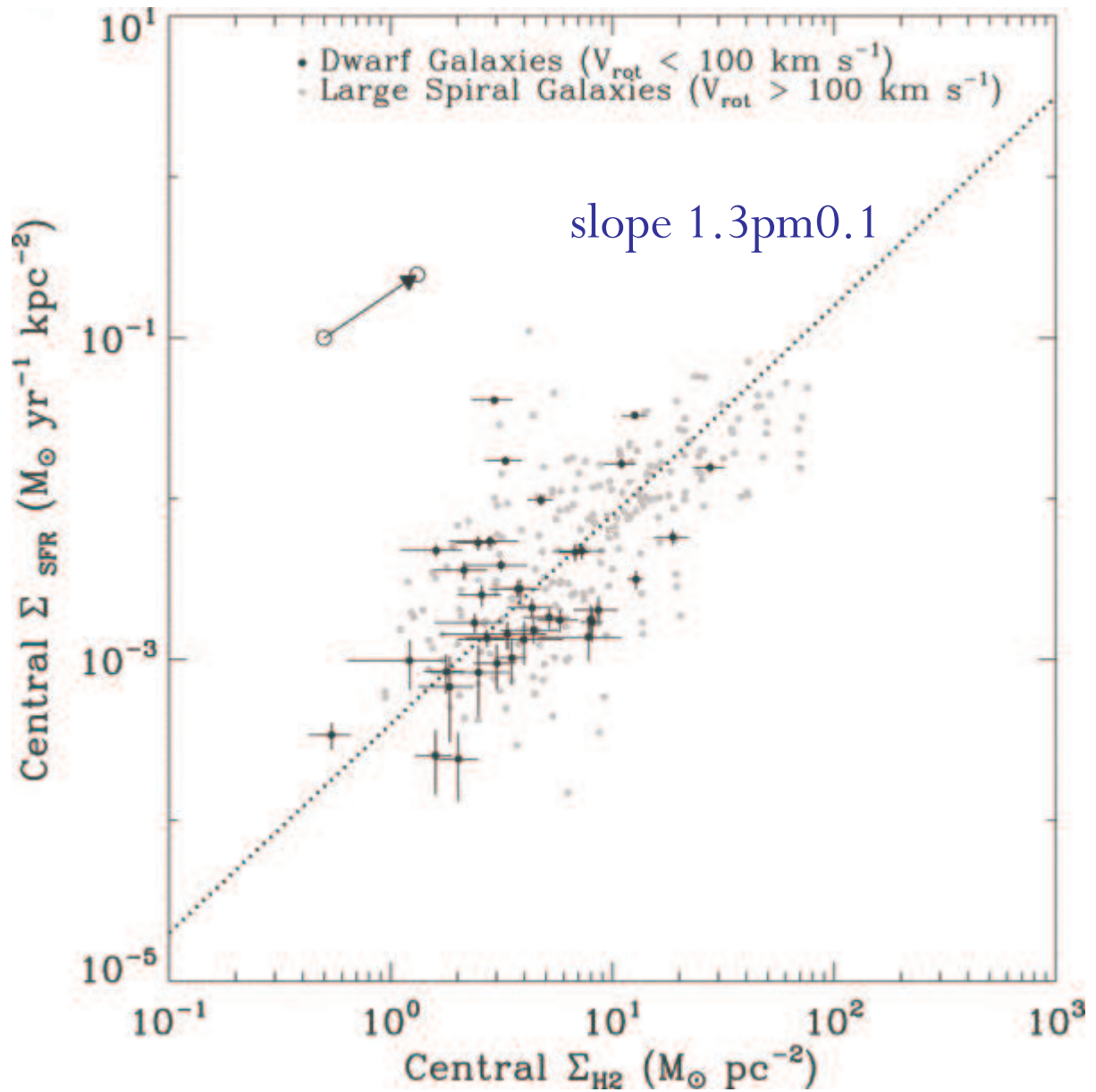
and if  $P \sim \Sigma_{\text{gas}}^2$

then  $\text{dln}f_{\text{mol}}/\text{dln}\Sigma_{\text{gas}} \sim 0.4$  😊

Suggest  $Q$  not a good measure of SFR,  
but a better measure of gas fraction  
(high  $Q$  corresponds with low  $\Sigma_{\text{gas}}/\Sigma_{\text{tot}}$ )



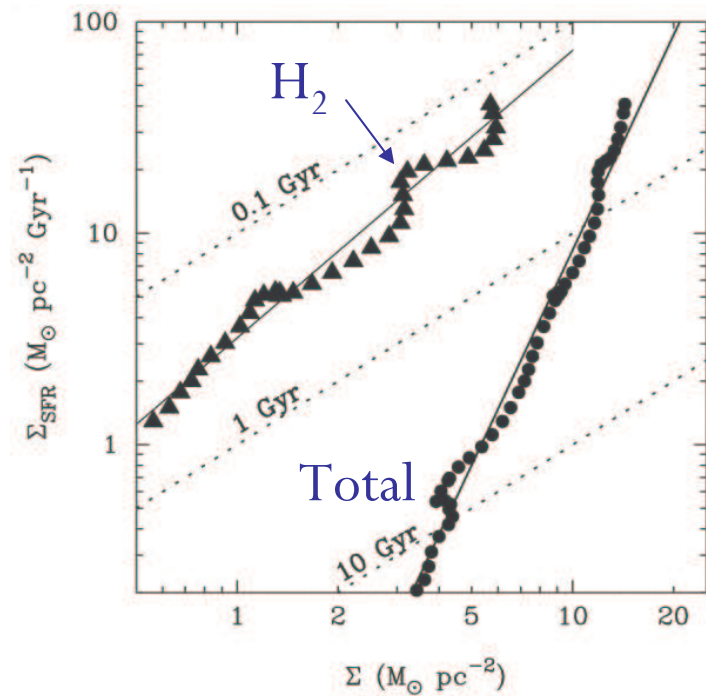
Dwarf galaxies  
continue the  
correlation  
of spirals



Leroy et al. 05

# M33 Molecular K-S law

- Heyer et al. 04 found  $n=1.36$  for  $\Sigma(\text{H}_2)$ 
  - In M33,  $f_{\text{mol}}$  is small, P small

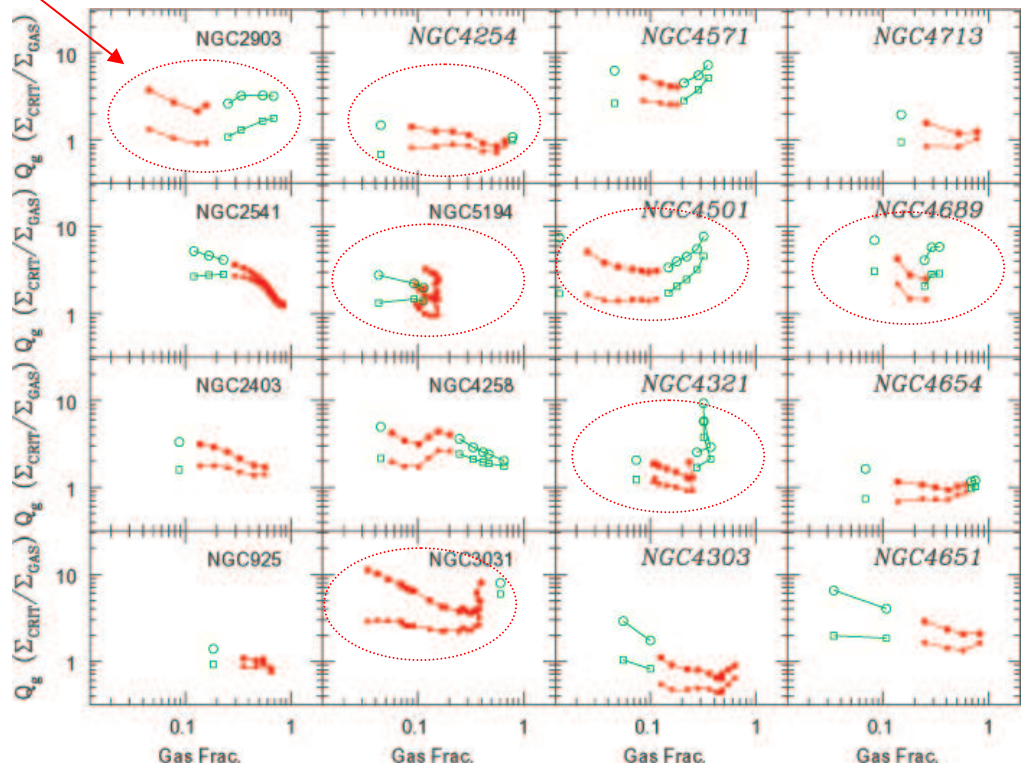




# A Molecular Cloud Connection?

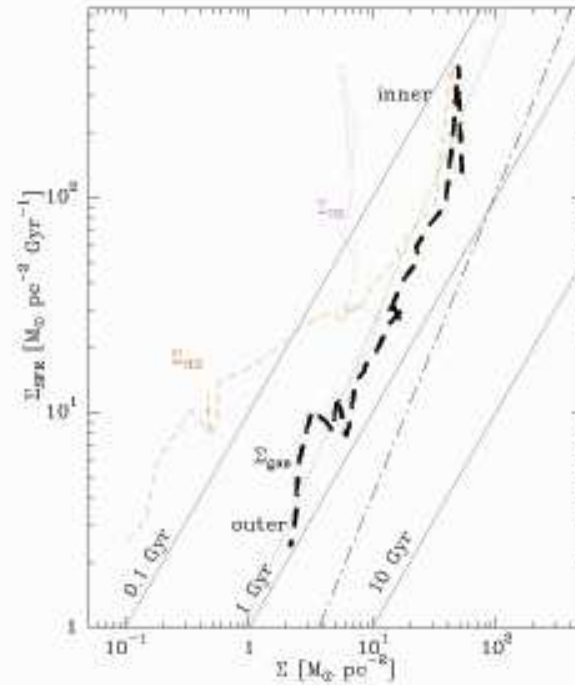
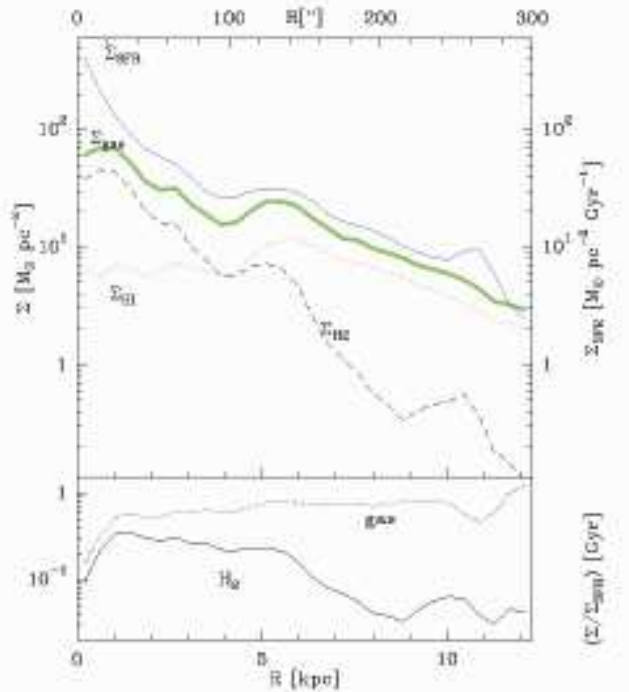
- Boissier et al. 03 found no anticorrelation between  $f_{\text{gas}}$  &  $Q$ 
  - assumed variable  $\text{H}_2/\text{CO}$
- 7/16 low  $Q$  & low  $f_{\text{gas}}$

gas only = circles  
 gas+stars = squares  
 red symbols = active SF



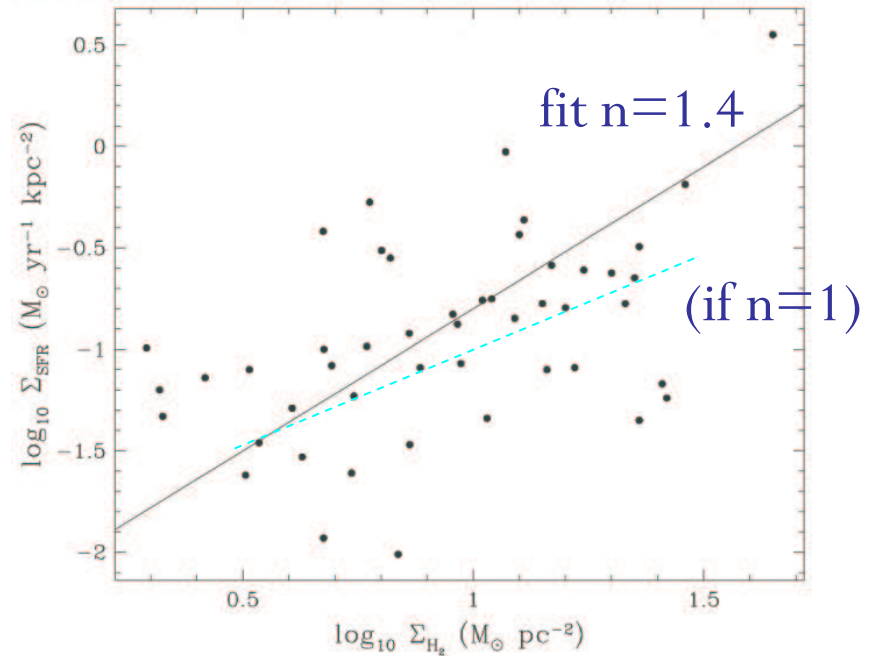
# No Molecular K-S Law in M51?

- Schuster et al. 06 use IRAM 30-m map of CO(2-1), VLA of 21 cm, and SFR from VLA 20 cm continuum; resolution 450 pc.
  - assume constant  $H_2/CO$
  - find total gas  $n=1.4pm0.6$ , poor correlation with  $H_2$



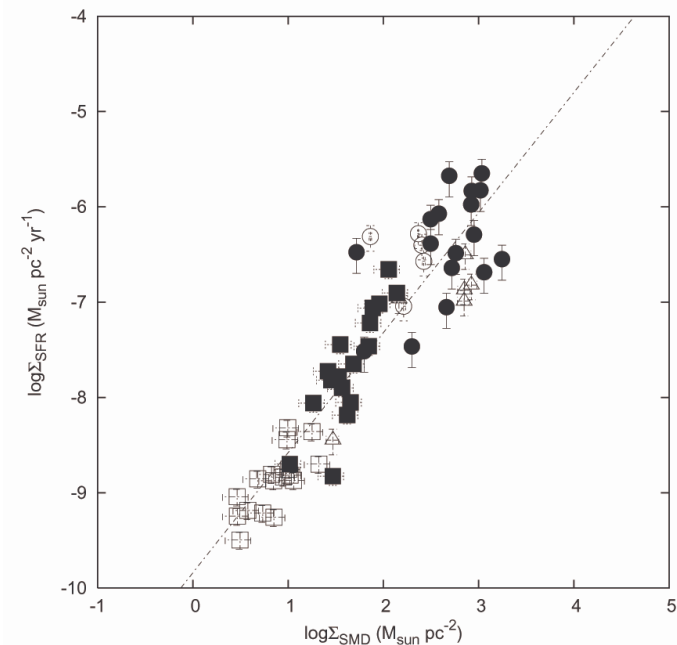
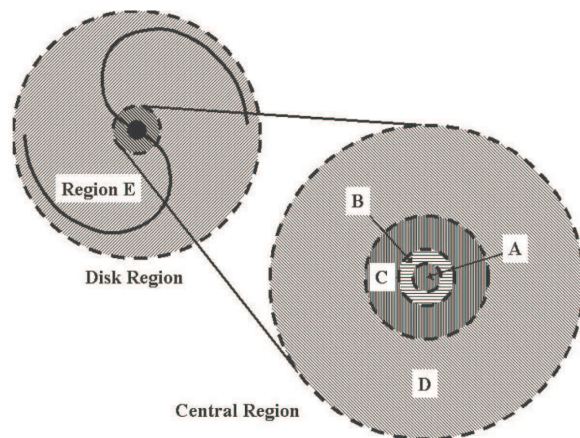
# K-S Law at High Molecular Densities

- Yao et al. 03 measure CO in SCUBA IR luminous galaxies. SFR from FIR.  $H_2/CO$  constant.
  - find  $n=1.4 \pm 0.3$
- These are highly molecular galaxies; should be no variation in  $f_{\text{mol}}$



# K-S Law at High Molecular Densities

- Komugi et al. 06 obs CO with 5 beam sizes in 17 best-case galaxies.
  - SFR from H $\alpha$  (extinction corr.), assumed constant H<sub>2</sub>/CO
  - finds K-S relation,  $n=1.33 \pm 0.08$ , radial dependence
  - K-S valid from 1 to 1000 M<sub>O</sub>pc<sup>-2</sup> (using a single tracer)
- Their result (with Yao et al's) suggests that  $n=1.4$  is not from variations in the molecular fraction but is intrinsic to the total gas.



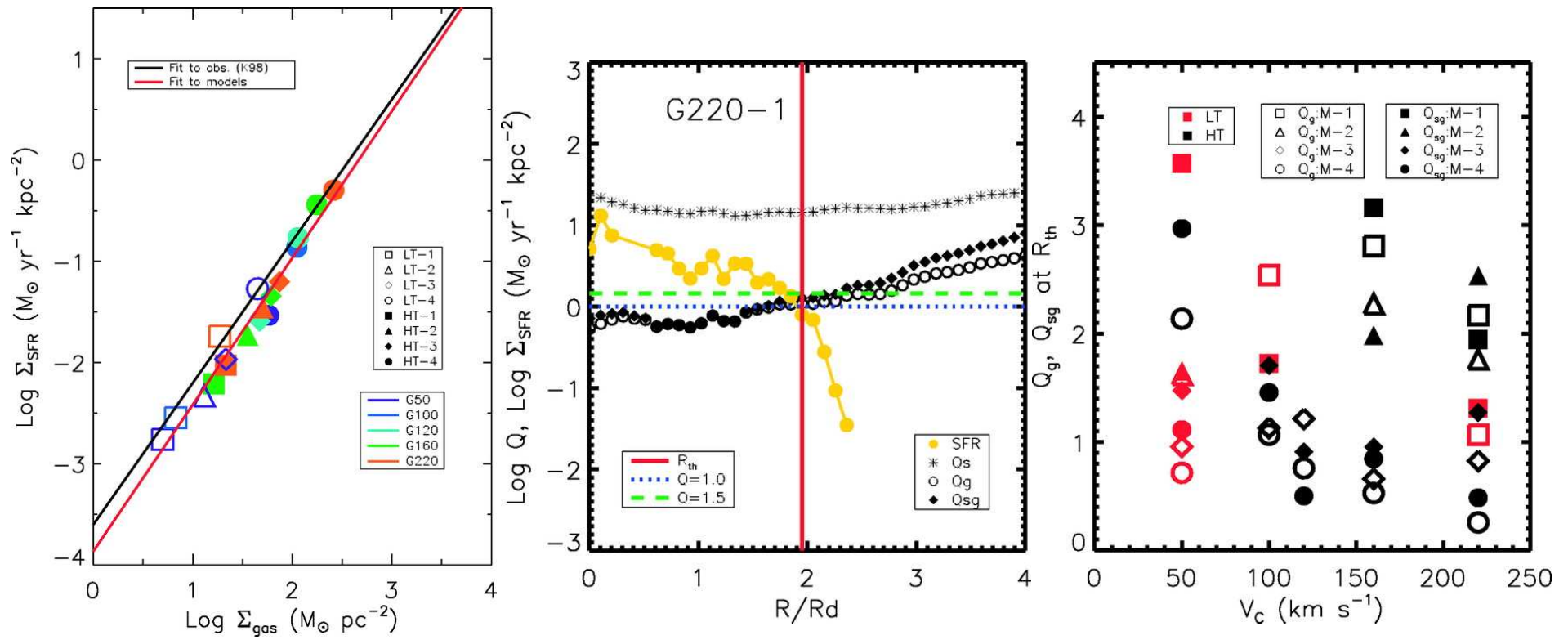
# Observations Summary

- K-S slope depends on assumptions (variable  $\text{H}_2/\text{CO}$ , extinction, ...) and varies from 1.2 to 3 in different studies
- K-T threshold may require stars+gas, and sometimes shows shear sensitivity
- molecular K-S law still debated
  - it may be independent of SF that  $f_{\text{mol}} \sim \Sigma_{\text{gas}}^{0.4}$  at low  $f_{\text{mol}}$

# Theory

# K-S Law by Gravitational Instability

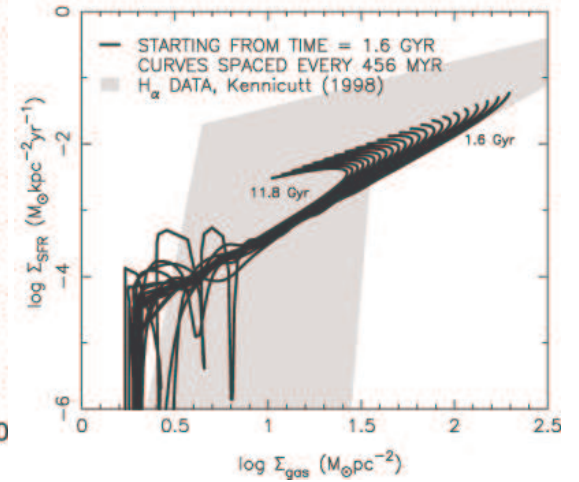
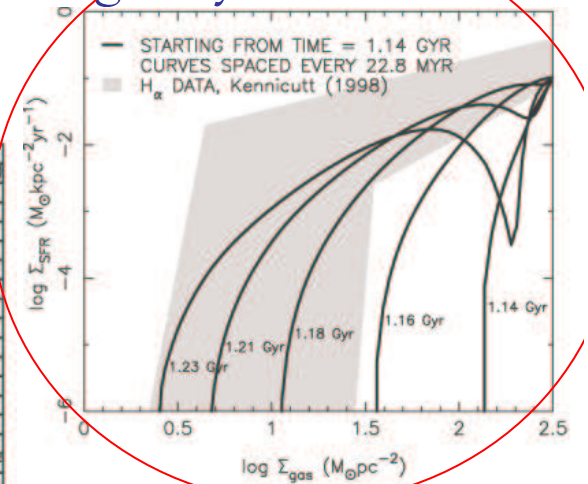
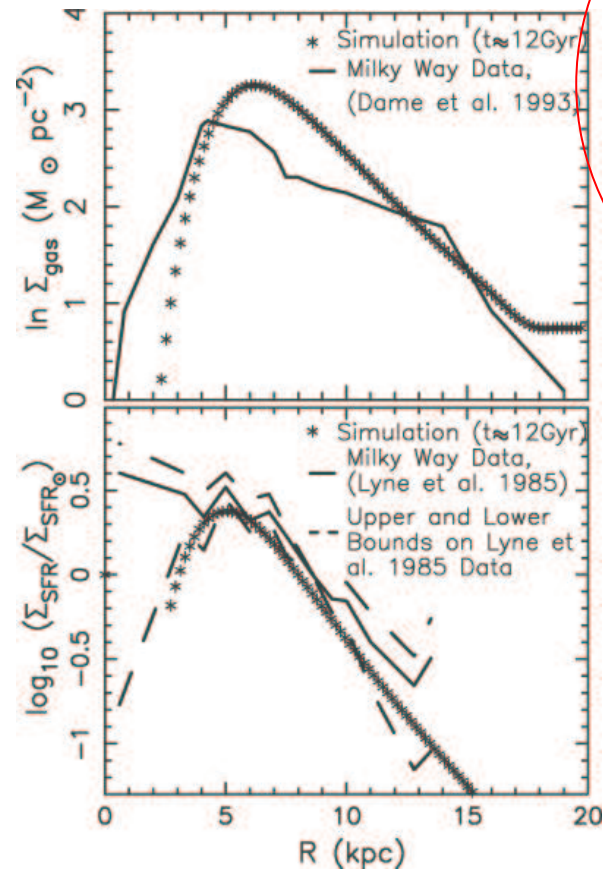
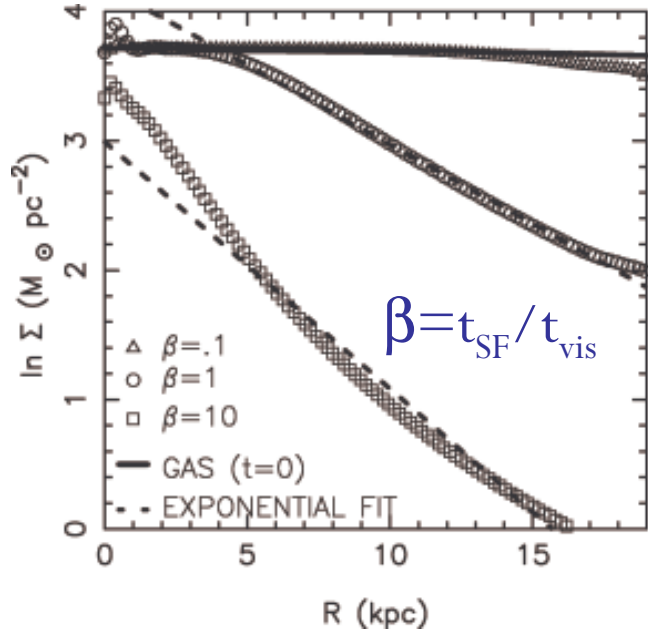
- Li, Mac Low & Klessen 05: SPH of galaxy disk with self-gravity forming sink particles at  $n > 10^3 \text{ cm}^{-3}$





# Viscous Disk Evolution

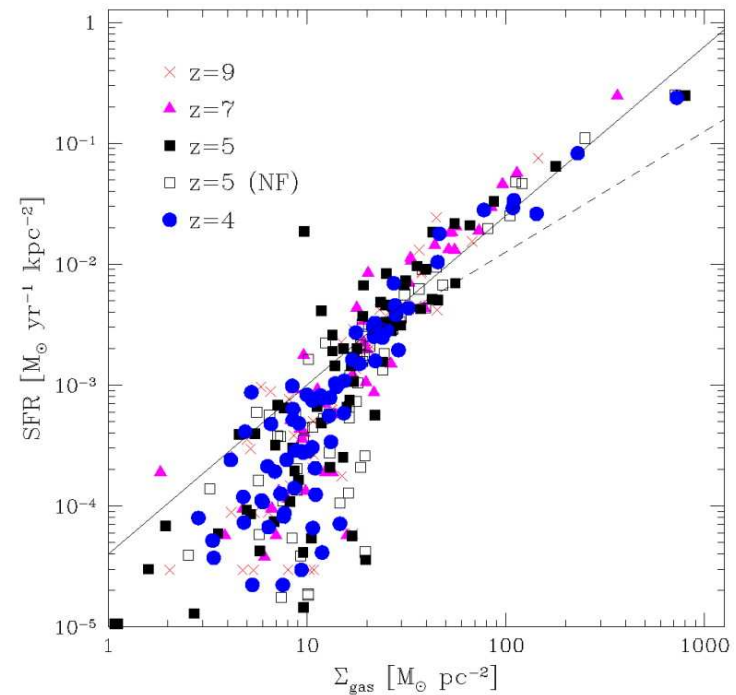
- Slyz et al. 02 model disk evolution with:
  - $\text{SFR} \sim \Sigma/t$  where  $t =$  viscous time, large in outer galaxy





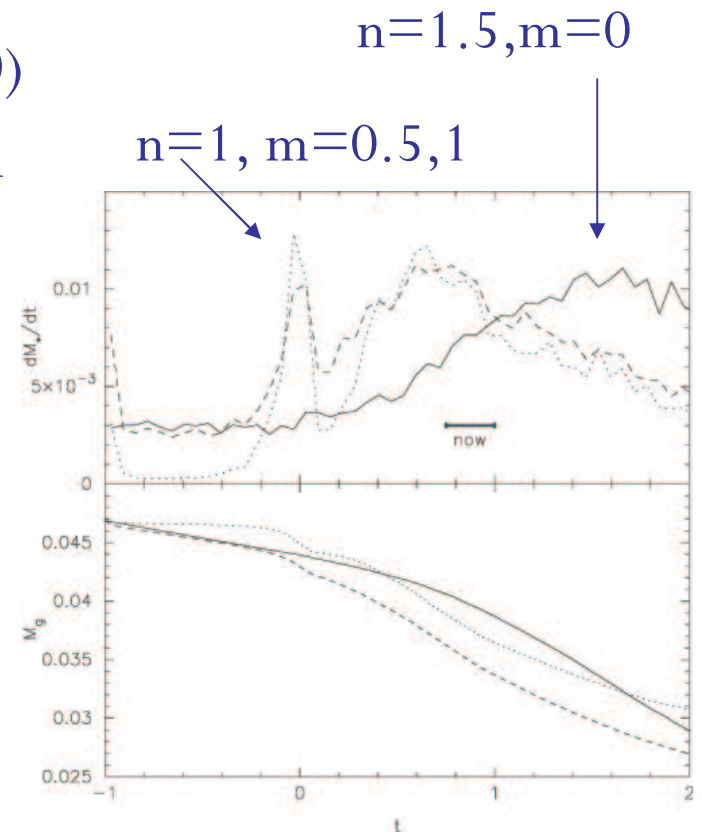
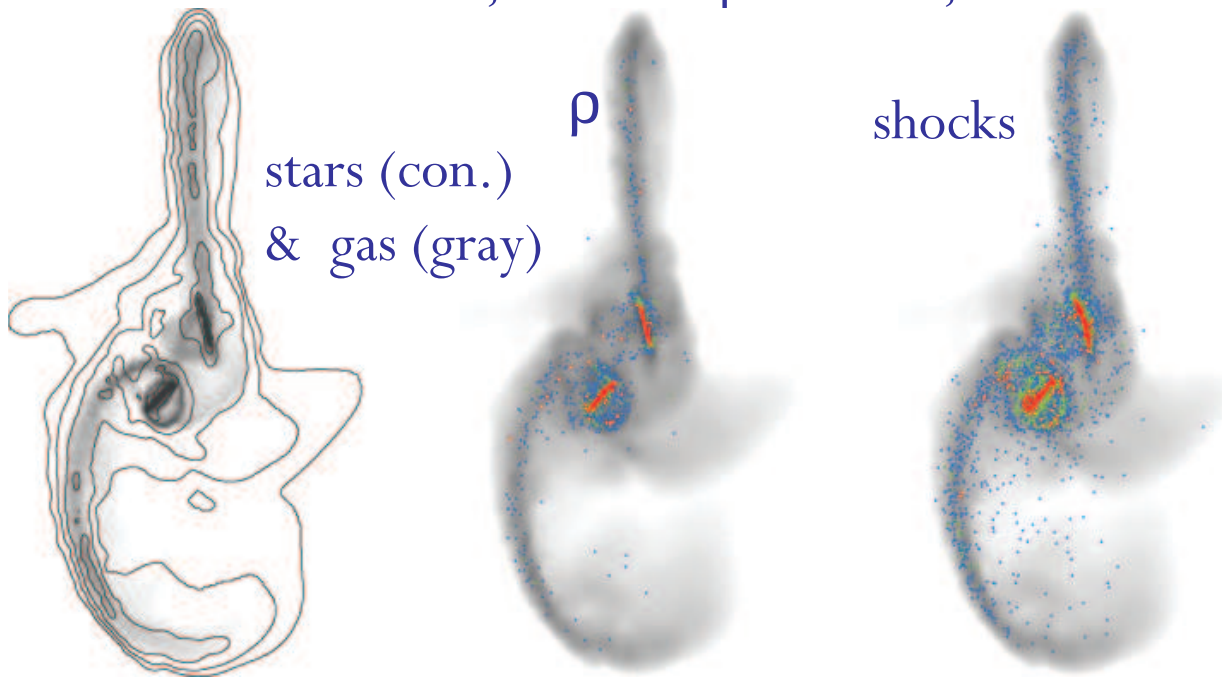
# K-S from Tail of Density pdf

- Kravtsov 03: cosmol. simulations, N-body in Eulerian adaptive mesh.
  - assume constant efficiency at high gas density
  - and SF only in dense regions ( $n > 50 \text{ cm}^{-3}$  – resol. limit)
  - SF in tail of density pdf (cf. Krumholz & McKee 05)



# A SFR dependent on shocks, not $\rho$

- Barnes 04 modeled the interacting pair “Mice” with SPH, replacing the  $\rho$  dependence with a dependence only on  $dU/dt$  from PdV work in shocks.  
 $SFR \sim \rho^n (dU/dt)^m$
- The result was better with shocks ( $n=1, m>0$ )
  - earlier SF, more dispersed SF, as observed



# K-S from cloud collisions

- Tan 00 suggested all SF from colliding clouds and derived

$$- \text{SFR} \sim 1.5\varepsilon \Sigma f_{\text{SF}} f_{\text{G}} \Omega (1 - 0.7\beta) / Q \quad \boxed{\sim \Sigma \Omega / Q}$$

- $\varepsilon$  = fraction of cloud turned into stars
- $f_{\text{SF}}$  = fraction of collisions which form stars
- $f_{\text{G}}$  = collision probability ( $\sim 0.5$ )
- $\beta$  = power in power law rotation curve
- $\Omega$  = rotation rate

# Theory Summary

- GI, pdf tails, shocks, cloud collisions all similar in a  $Q \sim 1$  ISM
  - different ways of identifying the dense gas
  - origin of the dense gas the same:
    - gravity + turbulence (+ stellar compressions)
- Viscous model OK too for K-S-T, but seems to fail in dwarf galaxies, which have little shear and still have exponential disks

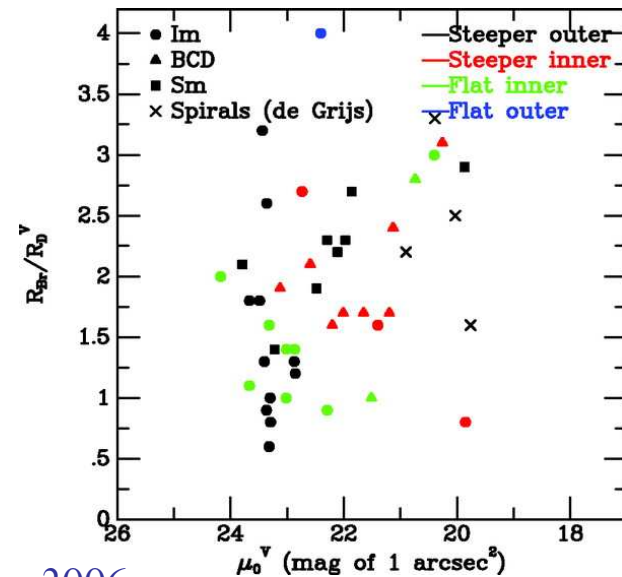
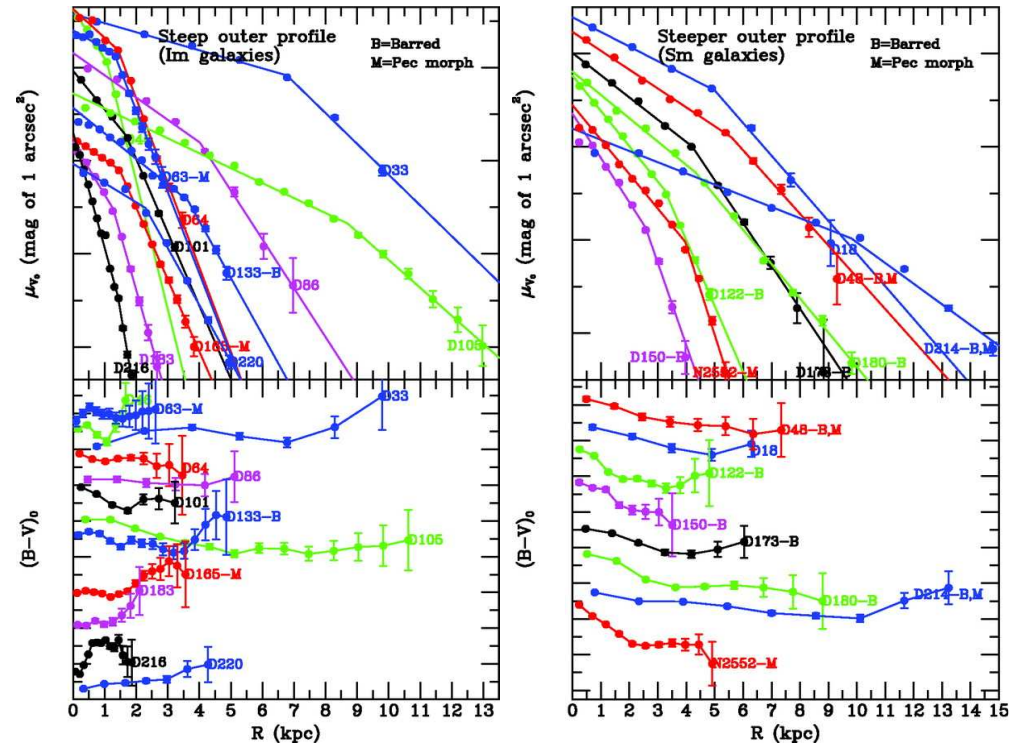
# Reflections

# The Threshold:

- Smooth, not sharp (H $\alpha$  edge sharper than V-band)
- Three main SF processes all have a Q threshold
  - gravitational instability, shell collapse, turbulence
- No normal thresholds with B or viscosity
  - shear limits time for gas condensations
- Islands of high  $\Sigma$  unstable when average  $\Sigma$  is stable
- Threshold also from cool/warm phase transition

# Smooth, not Sharp

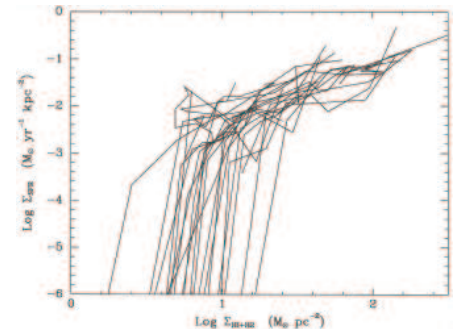
- There is rarely a sharp outer cutoff in galaxy disks
  - second exponential or slow taper is more common (Naslund & Jorsater 1997; de Grijs, Kregel, & Wesson 2001; Pohlen et al. 2002; Hunter et al. 2006)
  - $R_{\text{break}}/R_{\text{innerdisk}} \sim 4$  for spirals,  $\sim 2$  for dwarfs



Hunter & Elmegreen 2006

# Expect H $\alpha$ edge sharper than V or uv

- H $\alpha$  emission measure for a fixed Lyman continuum luminosity varies as  $n^{4/3}$ 
  - For constant scale height, EM is 200 times fainter at 4 scale lengths than in inner disk ( $=e^{-16/3}$ )
- If the disk flares, the EM drops even faster:
  - for a self-grav disk at constant vel. disp.,  $n \sim \Sigma^2$  and EM drops by a factor of  $4 \times 10^4$  in 4 scale lengths ( $=e^{-32/3}$ )
- Galaxy edges look sharper in H $\alpha$  than they are in total SFR
  - e.g., Thilker et al. 2005 GALAX observations

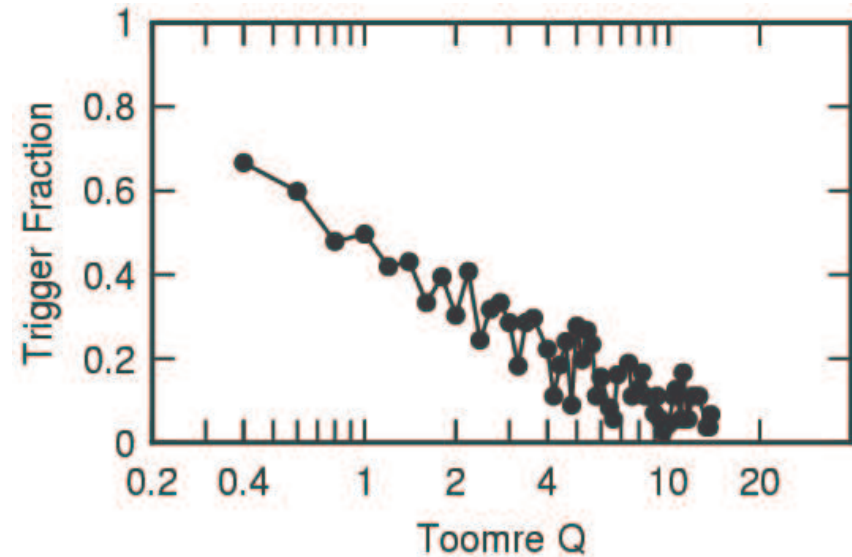




# Q threshold for Sequential Triggering

- Shells expand and go unstable when accumulated gas is cold and massive enough, & induced rotation ( $\kappa$ ) is small enough

$$Q = \kappa c / \pi G \Sigma \quad \underline{\text{important}}$$



Elmegreen, Palous & Ehlerova 02

Also,

$$t_{\text{frag}} \sim \left( \frac{c_{\text{sh}} / [GL]^{0.2}}{2\pi G \rho} \right)^{0.5} \sim 0.3 t_{\text{dynamical}}$$

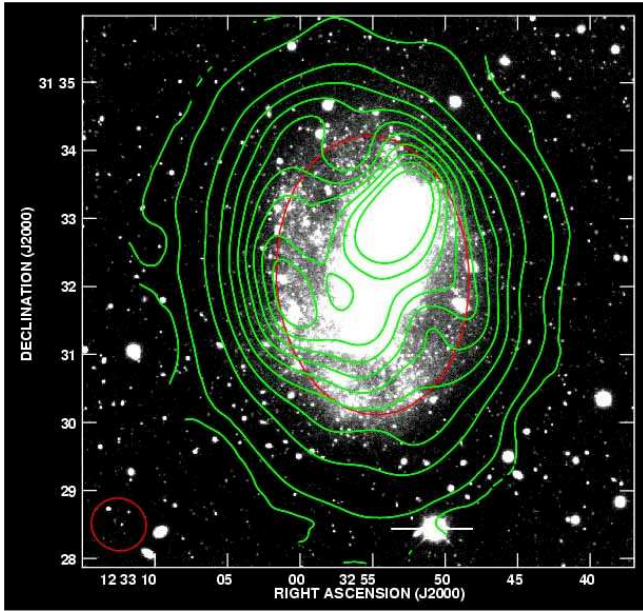
(Numerator  $\sim 0.1$  for  $c_{\text{sh}} \sim 1$  km/s and  $L = 10^4 L_{\odot}$ )

# Q Threshold for Turbulence Triggering

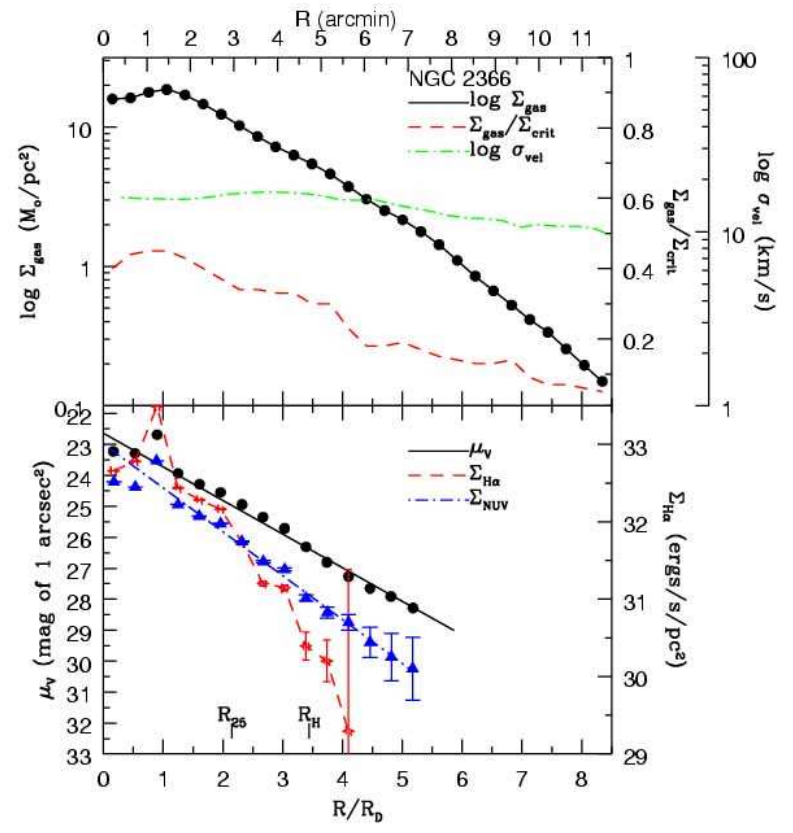
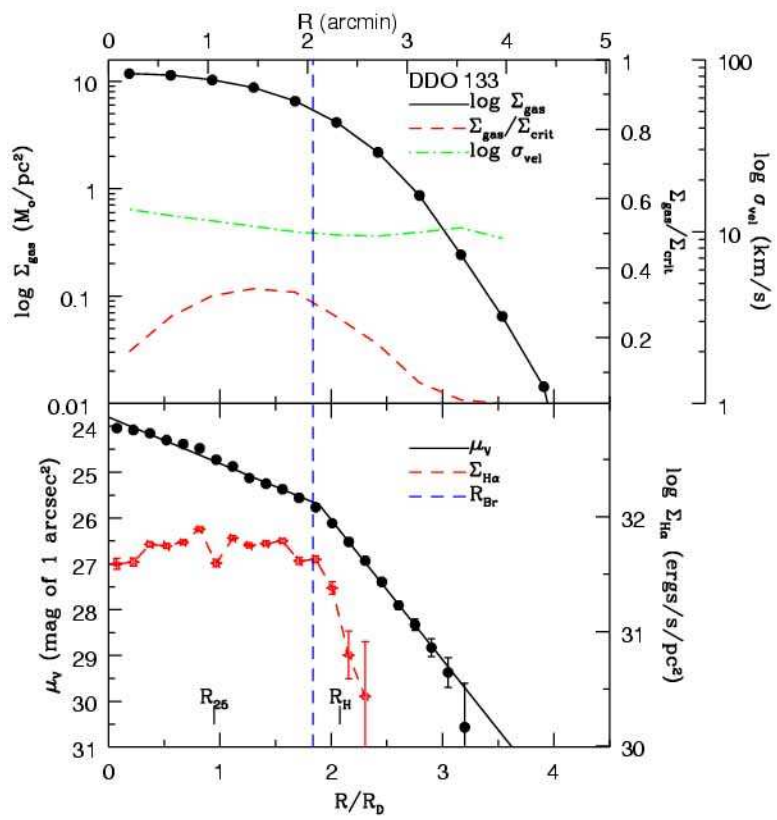
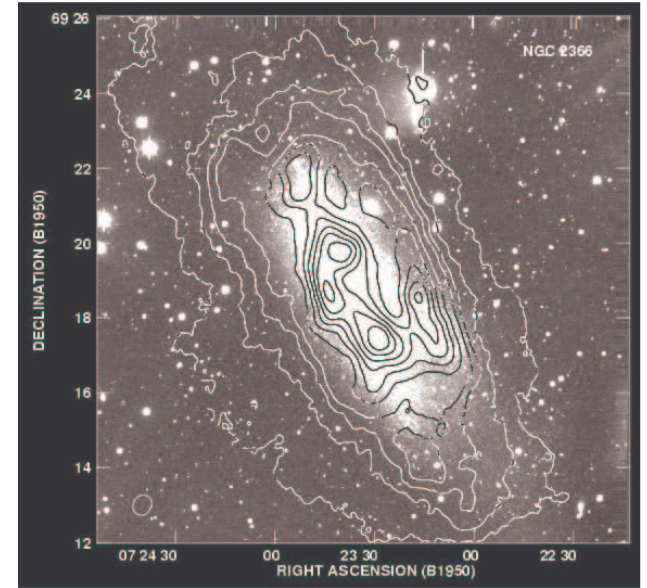
- Isothermal compression has to include the ambient  $M_{\text{Jeans}}$  to trigger instabilities
- Turbulent outer scale comparable to  $L_{\text{Jeans}}$ , which is  $\sim$ scale height
- But if compress distance  $>$  epicyclic length, Coriolis forces spin up compression, leading to centrifugal force resistance.
- so need  $L_{\text{Jeans}} < R_{\text{epicycle}}$ , which means  $Q < 1$ 
  - $L_{\text{Jeans}} \sim H \sim a^2 / \pi G \Sigma$ ,  $R_{\text{epic}} \sim a / \kappa$ , so  $L_{\text{Jeans}} / R_{\text{epi}} = Q$

# GI with B or viscosity

- Q: centrifugal force (from Coriolis spin-up) balances self-gravitational force over a Jeans length.
- If angular momentum is not conserved then the disk can be always unstable
  - B and viscosity strip angular momentum, removing threshold
  - Magneto-Jeans instability can dominate in low-shear environments like spiral arms and some inner disks
    - Elmegreen 87, 91,94; Kim & Ostriker 01.02



- Hunter et al 06
- double exponentials
  - very stable outer disks
  - sudden drop in H $\alpha$
  - SF in “islands”



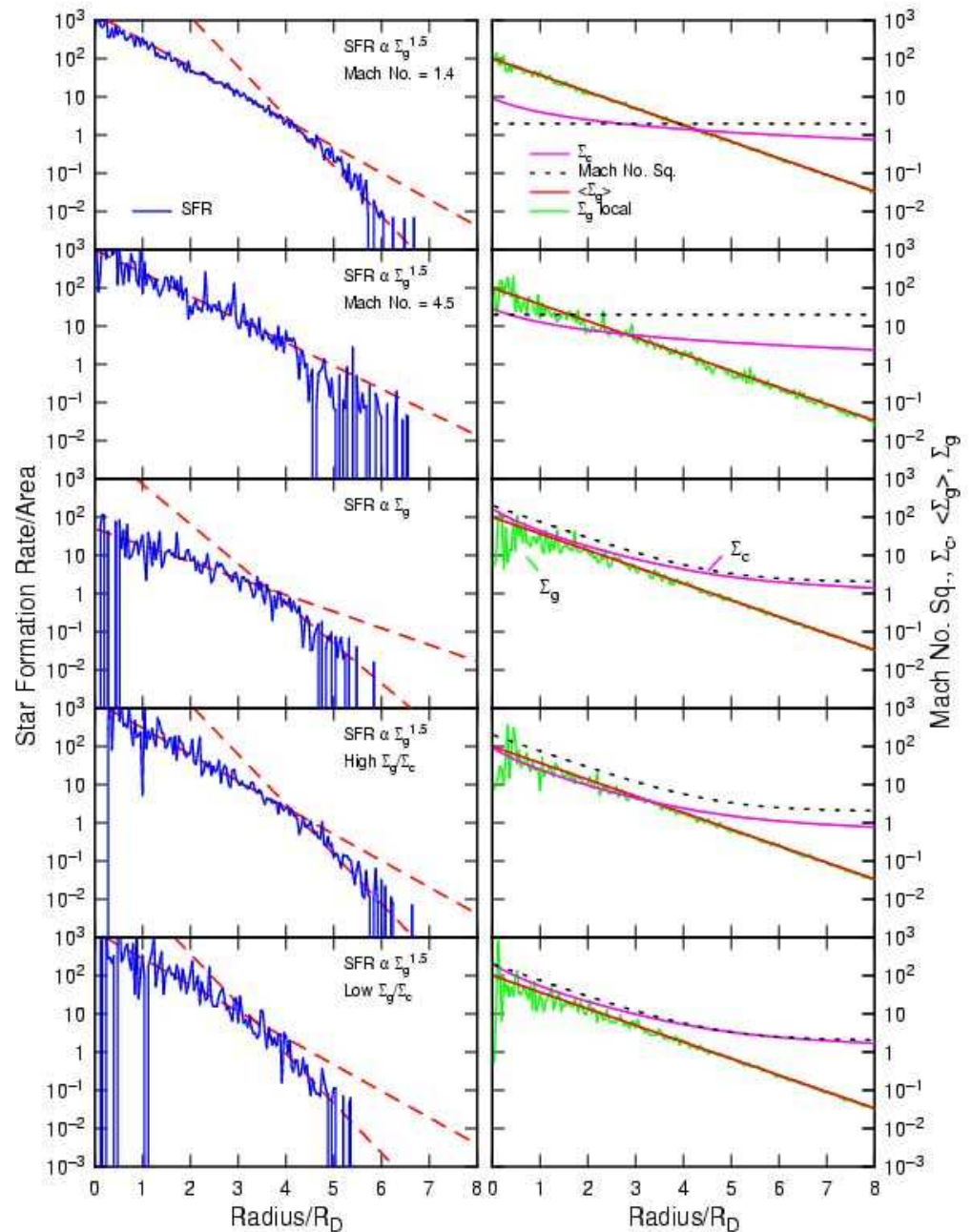
Inner/Outer disk transition:

1. exponential gas disk
2. turbulent Mach,  $M(R)$
3.  $\Sigma$  distribution log-normal
4. SF locally where  $\Sigma > \Sigma_c$
5. rising and then flat RC
6.  $\text{SFR} \sim \Sigma^1$  or  $\Sigma^{1.5}$

SF saturated in the inner disk

Continues in the  $\Sigma$  peaks in the outer disk because of turbulence ( $M \sim 1$ ) even though average  $\Sigma < \Sigma_c$ .

Outer profile is steeper exp.



Elmegreen & Hunter 06



# Threshold from a warm/cool phase transition: outer disks, dwarfs, LSB gal

- SF needs a cool phase; a cool phase needs moderate P
  - Elmegreen & Parravano 94; Gerritsen & Icke 97; Hunter et al. 98, 01; Billett et al. 02; Wolfire et al. 03
  - Observe outer disk high warm fraction (Braun 97) and high warm fraction in dw Irr (Young & Lo 96,97; de Blok & Walter 06)
  - P high (cool gas possible) in self-gravitating “island” clouds, spirals, cloud collision shocks, turbulence-compressed regions, ...
- Schaye 04: modeled the SF threshold, considering where the cold phase becomes important.

# The Power Law

- Three models: GI, shocks, pdf tail, are all similar
- SF is from a combination of processes (sequential triggering, GI, turbulence compression, SDW compression, ...) so the power law is probably from a combination of processes too.
  - there is no single “theory of the K-S law”

# Summary

- Significant variations from galaxy to galaxy, within a galaxy, and from observer to observer in the form of the SFR- $\Sigma$  law
- There is a threshold for SF
  - from disk gravity, cooling limitations, shear, ...
- The power law has several explanations, most related to a  $\sim\sqrt{\Sigma}$  dependence of dynamical rate on column density, or a  $\sqrt{\Sigma}$  dependence of the relevant column density (molecular, shocked, pdf tail, etc) on the total density for a fixed rate inside that relevant column density.