THEORY OF THE KENNICUTT-SCHMIDT LAW FOR STAR FORMATION IN GALAXIES

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Workshop on the Kennicutt-Schmidt Law

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REQUIREMENTS FOR A THEORY OF THE KS LAW

Two forms consistent with observation:

$$d\Sigma_*/dt \propto \Sigma_g^{1.4} \qquad d\Sigma_*/dt \propto \Sigma_g/t_{dyn}$$

Applies over 8 orders of magnitude in star formation rate $d\Sigma_*/dt$
from normal galaxies to starbursts

Star formation inefficient: gas depletion time » dynamical time t_{dyn}

Previous theories

Padoan (1995): Predicts SFR in turbulent GMCs, but no prescription for application to galaxies in which GMC properties are not observed

Silk (1997): SFR set by supernova feedback; depends on uncertain porosity of hot gas

Tan (2000): SFR set by cloud-cloud collisions; normalization set by comparison with obs.

Kravtsov (2003) & Li et al (2005): simulations show fraction of high-density gas $\propto \Sigma_g^{1.4}$ but definition of "high-density" arbitrary and rate of SFR not determined

Elmegreen (2002, 2003): SFR/volume = $\epsilon_{core} f_c (G\rho_c)^{1/2} \rho$ with $\epsilon_{core} = 1/2$

Fraction of gas in dense cores, f_c , determined from observed KS law

Corresponding density is $\rho_c/\rho = 10^5$, but it is not clear why this is the critical density, nor how it should vary in different galaxies

TURBULENCE-REGULATED STAR FORMATION (KM05)

Assume (1) that star formation occurs in GMCs

$$d\Sigma_*/dt = SFR_{ff}\ \Sigma_{GMC} / t_{ff}$$

$$= SFR_{ff}\ f_{GMC}\ \Sigma_g / t_{ff}$$

where t_{ff} is the free-fall time in a GMC

SFR_{ff} is the fraction of gas that goes into stars per free-fall time

f_{GMC} is the fraction of gas in GMCs

$\approx (1 + 0.025 / \Sigma_{g,2}^2)^{-1}$ from Rosolowsky & Blitz (2006),
where $\Sigma_{g,2} = \Sigma_g / 100\ M_{sun}\ pc^{-2}$

Objective: Determine SFR_{ff} and t_{ff} in terms of Σ_g and t_{dyn}

The Star Formation Rate per Free-Fall Time SFR_{ff}

Assume (2) that the probability distribution $p(x)$ of the density in GMCs is log normal as is appropriate for supersonically turbulent gas (e.g., Padoan & Nordlund 2002)

Let $x = \rho/\rho_0$ where ρ_0 is the average density in the GMC

Then $dp(x) \propto \exp [- (\ln x - \langle \ln x \rangle)^2 / 2\, \sigma_\rho^{\,2}]$

where $\sigma_\rho^{\,2} \approx \ln (1 + 0.75\, \mathcal{M}^2)$ and $\mathcal{M} = \sigma/c_s$ is the Mach number

Assume (3) that gas above some critical density ρ_{cr} forms stars with an efficiency ε_{core} at a rate corresponding to some number (ϕ_t) of free-fall times:

$$dM_*/dt = (M\, \varepsilon_{core}/\, \phi_t\, t_{ff}) \int_{xcr} x \, dp(x)$$

where $x_{cr} = \rho/\rho_{cr}$

$$\Rightarrow \qquad SFR_{ff} = (\varepsilon_{core}/\, \phi_t) \int_{xcr} x \, dp(x)$$ (can be evaluated analytically)

For numerical evaluation, we take $\varepsilon_{core} \approx 1/2$ (Matzner & McKee 2000)

What is the Minimum Density for Star Formation, ρ_{cr} ?

Assume (3′) that stars of average mass form in cores dominated by thermal, not turbulent pressure --- valid in Galactic GMCs

Sonic length: Scale at which turbulent motions match thermal motions:

Line-width size relation $\sigma_l = c_s (l / \lambda_s)^{1/2} \Longrightarrow$

$\lambda_s = 2R\,(c_s / \sigma_{2R})^2 = 2R/\mathcal{M}^2$ where 2R is the diameter of the GMC

Gravitational collapse possible in a thermal core only if the sonic length > Jeans length (see Padoan 95; equivalent to mass inside sonic length > ~ Bonnor-Ebert mass)

Can show this implies $\rho_{cr} = 0.8\,\alpha_{vir}\,\mathcal{M}^2\,\rho_0$

where $\alpha_{vir} = 5\sigma^2 R/GM \sim 1$ is the virial parameter

Notes: (1) this corresponds to $P_{cr} = \rho_{cr}\, c_s^{\,2} \approx \rho_0\, \sigma^2$: the critical thermal pressure is comparable to the turbulent pressure in the GMC (Padoan 95)

(2) low-mass cores have $\alpha_{vir} \sim \mathcal{M} \sim 1$ and are therefore at the critical density

Evaluation of the Star Formation Rate per Free-Fall Time, SFR_{ff}

Recall $SFR_{ff} = (\epsilon_{core}/\ \phi_t) \int_{xcr} x\ dp(x)$

We now know x_{cr} and we adopt $\epsilon_{core} = 1/2$

Vazquez-Semadeni et al. (2003) carried out hydrodynamic simulations and showed that the star formation rate depends on the sonic length. Fitting to their results gives $\phi_t = 1.9$ as the number of free-fall times (evaluated at ρ_0) required for core collapse.

A power-law fit to our results yields

$$SFR_{ff} \approx 0.017\ \alpha_{vir}^{-0.7}\ (\mathcal{M}/100)^{-0.3}$$

$\Rightarrow$

Star formation is inefficient (a few percent), in agreement with observation.

The rate depends only weakly on the Mach number $\mathcal{M}$; note that $\alpha_{vir} \sim 1$ in GMCs

New Form of the KS Law

We now have $d\Sigma_*/dt = SFR_{ff}\ f_{GMC}\ \Sigma_g / t_{ff}$

$$= 0.017\ \alpha_{vir}^{-0.7}\ (\mathcal{M}/100)^{-0.3}\ f_{GMC}\ \Sigma_g / t_{ff}$$

What are the Mach number $\mathcal{M}$ and the free-fall time $t_{ff} \propto \rho_0^{-1/2}$?

Assume star-forming disk is marginally stable so that $Q \approx 1$

$\Rightarrow \sigma_g = \pi G \Sigma_g Q / 2^{1/2} \Omega$ in disk, where Ω is the angular velocity

Density in the disk is given by $\rho_g = P_g / \sigma_g^2$ where $P_g \approx (\pi/2) G \Sigma_g \Sigma_{tot}$

Pressure in GMC $\approx$ (2-10) P_g and density in GMC $\approx$ (2-7) $\rho_g \Rightarrow \sigma_{GMC} \approx \sigma_g$

and $t_{ff} \propto \rho_0^{-1/2} \propto \Omega$ (details in KM05)

$\Rightarrow d\Sigma_*/dt \approx 0.16\ \mathcal{M}^{-0.3}\ f_{GMC}\ \Sigma_g\ \Omega$ similar to KS law except for $\mathcal{M}^{-0.3}$

$\approx 9.5\ f_{GMC}\ \Sigma_{g,2}^{0.7}\ \Omega_6^{1.3}$ $M_{sun}\ yr^{-1}\ kpc^{-2}$, where $\Omega_6 = \Omega \times 10^6$ yr

Star Formation Threshold

Star formation cuts off in outer regions of galaxies

Generally attributed to Toomre Q rising above 1 => stable

Predicted SFR varies as $Q^{-1.3} f_{GMC}$: declines rapidly in outer regions

since Q increases and the molecular fraction decreases;

in addition, a smaller fraction of the molecular gas is in GMCs at large radii.

Test of New Form: $d\Sigma_*/dt \approx 9.5\ f_{GMC}\ \Sigma_{g,2}^{0.7}\ \Omega_6^{1.3}\quad M_{sun}\ yr^{-1}\ kpc^{-2}$

Should apply to individual galaxies as well as sample of galaxies

Does not apply to individual GMCs since expect large fluctuations in the star formation rate (Krumholz, Matzner & McKee 2006)

Milky Way: use f_{GMC} and $\Sigma_{g,2}$ from observation

calculate Q(r) in spiral arms (suppressed in above expression)

Predict SFR between 3 and 11 kpc of 4.5 $M_{sun}\ yr^{-1}$

Consistent with observed rate ≈ 3 $M_{sun}\ yr^{-1}$ (McKee & Williams 1997)

Comparison with Classical Forms of KS Law

There are two forms of the KS law because Σ_g and Ω are correlated in the data:

$$\Omega_6 \approx 0.06\, \Sigma_{g,2}^{0.5} \quad \text{for } \Sigma_g > 1\ M_{sun}\, pc^{-2}$$

First form:

Observed: $d\Sigma_*/dt = 0.16\, \Sigma_{g,2}^{1.4}$ $M_{sun}\ yr^{-1}\ kpc^{-2}$ (Kennicutt 1998)

Theory: $d\Sigma_*/dt = 0.19\, f_{GMC}\ \Sigma_{g,2}^{1.3}$ $M_{sun}\ yr^{-1}\ kpc^{-2}$

Second form:

Observed: $d\Sigma_*/dt = 1.7\, \Omega_6\, \Sigma_{g,2}$ $M_{sun}\ yr^{-1}\ kpc^{-2}$ (Kennicutt 1998)

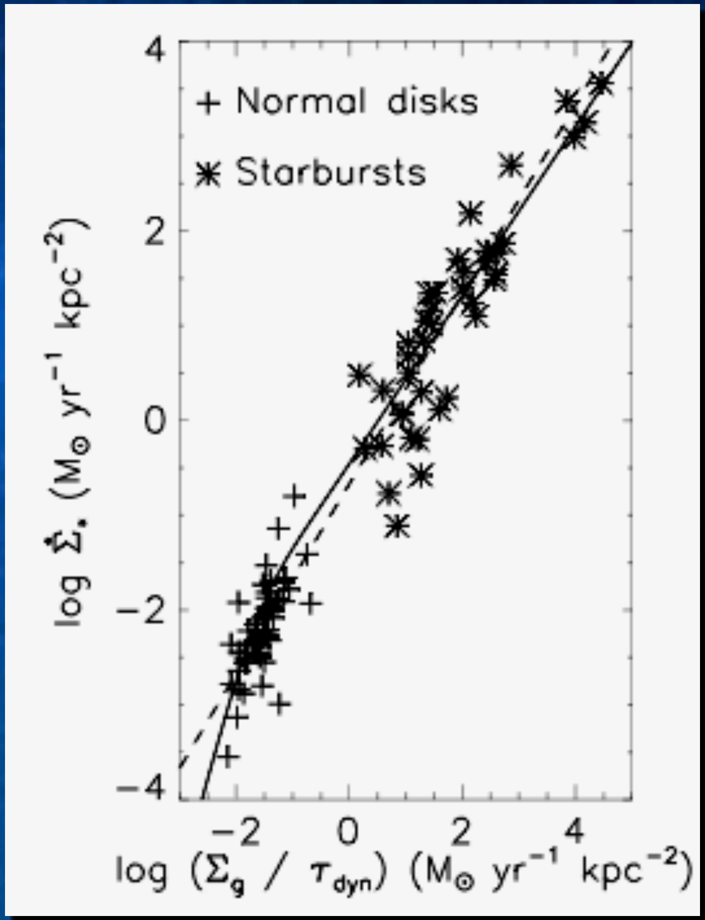
Theory: $d\Sigma_*/dt = 3.2\, f_{GMC}\, (\Omega_6 \Sigma_{g,2})^{0.9}$ $M_{sun}\ yr^{-1}\ kpc^{-2}$

Comparison with Second Form of KS Law

$\tau_{dyn} = 4\pi / \Omega$

- - - Power-law fit (Kennicutt 98)

—— Theory



Tests of Theory

OBSERVATIONAL:

* Test SFR_{ff} from observations of a sample of GMCs

* Test $d\Sigma_{*}/dt$ in annular rings in galaxies

* Increase the sample size to break the degeneracy between the two forms of the KS law and between observation and theory

THEORETICAL:

+ Predicts time scale for star cluster formation of 3-4 dynamical times, consistent with observation (Tan, Krumholz, & McKee 2006)

+ When used in a dynamical model for GMC evolution, successfully predicts GMC lifetimes and column densities (Krumholz, Matzner, & McKee 2006)

Extending the Theory

* Determine the GMC fraction f_{GMC} theoretically

    Particularly important for low-metallicity galaxies and high-redshift galaxies

* Determine the effects of magnetic fields

    Could alter density PDF and slow rate of star formation

    Observations of fields in the Galaxy suggest effects are modest

* Predict the level of turbulence in GMCs (i.e., predict α_{vir})

    Understand the driving mechanisms that counter turbulent decay

    Particularly puzzling in starbursts, where σ larger than given by plausible momentum sources other than self-gravity

* Show how the massive stars that are observed are related to the low-mass stars predicted by the theory (i.e., understand the IMF)

These are some of the fundamental questions of star formation

CONCLUSION

The assumptions that

- Stars form in virialized GMCs that are supersonically turbulent

- The density distribution is log normal, as expected for such turbulence in isothermal gas

- Gas dense enough that thermally supported cores that are gravitationally unstable forms stars with an efficiency $\varepsilon_{core} \sim 1/2$

imply a star formation law that should apply when averaged over a large number of GMCs, whether in a single galaxy or many galaxies:

$$d\Sigma_*/dt \approx 9.5 \;\; f_{GMC}\, \Sigma_{g,2}^{0.7}\, \Omega_6^{1.3} \quad M_{sun}\, yr^{-1}\, kpc^{-2}$$

This result is consistent with existing observations