Homework VI

Ch. 7 - Problems 15, 19, 22, 25, 35, 43, 51.

Problem 15

(a) The centripetal acceleration of a point on the equator of the Earth is given by $\frac{v^2}{r}$. The velocity of the earth can be found by taking the ratio of the circumference of the earth to its rotational period. Thus, we find:

$$v = \frac{2\pi r}{T} = \frac{2\pi(6373\ km)}{24h(3600\ \frac{s}{h})} = 0.463\ \frac{km}{s} = 463\ \frac{m}{s}.$$

Thus, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(463\ \frac{m}{s})^2}{6373\ km} = 0.034\ \frac{m}{s^2}.$$

(b) Assuming the rotational axis runs through the north pole, the centripetal acceleration is simply a = 0 $\frac{m}{s^2}$.

Problem 19

(a) The force is simply given by

$$F = m\frac{v^2}{r} = (55.0\ kg)\frac{(4.00\ \frac{m}{s})^2}{0.800\ m} = \ 1100\ N.$$

(b) The skater's weight is simply mg = 539 N. Thus,

$$\alpha = \frac{F}{W} = \frac{m\frac{v^2}{r}}{mg} = \frac{v^2}{rg} = 2.04.$$

Thus, the force exerted on the skater is 2.04 times her weight.

Problem 22

The maximum lateral acceleration is simply

$$\begin{aligned} a_{max} &= \frac{v^2}{r}, \\ &= \frac{(86.5\ \frac{km}{h})^2}{61.0\ m}, \\ &= 1.23e8\ \frac{m}{h^2}, \\ &= 9.46\ \frac{m}{s^2}, \\ &= 0.97g. \end{aligned}$$

Problem 25

(a) The tension is easily found from

$$\begin{aligned} T - W_b &= 0, \\ T &= m_b g, \\ &= 9.8\ N. \end{aligned}$$

(b) The horizontal force acting on the puck is simply the tension in the string, namely, 9.8 N.

(c) The speed of the puck can be found by from the force equation. So,

$$\begin{aligned} T &= \frac{m_t v^2}{r}, \\ v &= \sqrt{\frac{m_b}{m_t} g r}, \\ &= 6.3\ \frac{m}{s}. \end{aligned}$$

Problem 35

(a) The satellite's orbital speed can be found from the force equation. That is,

$$\begin{aligned} \frac{GMm}{r^2} &= m\frac{v^2}{r}, \\ r &= \frac{GM}{v^2}, \\ &= 15940\ km. \end{aligned}$$

Now, recall that when using Newton's law of gravitation, the force equation measures the radius from the center of the Earth. Thus, to find the altitude above the surface of the Earth, we must subtract off the radius of the Earth, leaving us with an altitude of 9570 km above the surface of the Earth.

(b) The period of the satellite's orbit is simply given by

$$\begin{aligned} T &= \frac{2\pi r}{v}, \\ &= \frac{2\pi(15940000\ m)}{5000\ \frac{m}{s}}, \\ &= 20030\ s, \\ &= 5.56\ h. \end{aligned}$$

Problem 43

(a) The tangential speed of the ball is simply

$$v = 2\pi\omega r = 2.51\ \frac{m}{s}.$$

(b) The centripetal acceleration is

$$a = (2\pi\omega)^2 r = 7.90\ \frac{m}{s^2}.$$

(c) Recall the above force equation

$$T = \frac{mv^2}{r}.$$

Then, the maximum tangential speed that can be attained without breaking the rope is

$$v = \sqrt{\frac{Tr}{m}} = 4.0\frac{m}{s}.$$

Problem 51

See free body diagram below.
From the diagram, we see that the normal force is given by

$$F_N = mr\omega^2 = 75m.$$

Then, the vertical force equation reads f - W = 0. Thus, the minimum coefficient of friction is

$$\mu = \frac{W}{F_N} = \frac{mg}{75m} = \frac{g}{75} = 0.131.$$

List the four fundamental forces of nature. Compare their basic properties (range, strength, ...) in quantitative fashion. How do we see these forces demonstrated in nature?

1. Gravity is weakest of the four fundamental forces. All objects in the universe are acted upon by the gravitational interaction. The gravitational interaction is responsible for apples falling to the earth and planets orbiting the sun. Although it is so weak, gravity dominates on large scales. It is responsible for the dynamics of galaxies and the large scale structure and evolution of the universe. It has an infinite range, and thus a massless carrier particle, the graviton. In the limit of small masses, the force is described well by Newton's law of gravitation. This states that the gravitational interaction is of the form $F_g = \frac{GMm}{r^2}$, where G = 6.67e-11 $\frac{N\ m^2}{kg^2}$ is the gravitational constant. For an accurate description of gravity involving large masses, one must go to Einstein's theory of General Relativity which is governed by the Einstein equations,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu},$$

where $G_{\mu\nu}$ is the Einstein tensor and is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R,$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, and R is the scalar curvature. $T_{\mu\nu}$ is the stress-energy tensor and Λ is the cosmological constant. For the purposes of this discussion, let's set the strength of gravity to 1.

2. The electromagnetic interaction was the first for which a complete theory was formulated. It is actually the unification of two observed interactions, the electric and magentic. The electric force is responsible for the attraction and repulsion of charges. In particular, like charges repel while opposite charges attract. The magnetic force is due to moving charge. Much experimental work in this area was performed by Michael Faraday while theoretical unification was completed by James Clerk Maxwell in 1856. The electric force is of the form $F_e = \frac{kQq}{r^2}$, where $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 = 8.85e-12 $\frac{C^2}{N\ m^2}$ is the permittivity of free space. Together with the magnetic force, the

Lorenz force equation takes the form F = q(E + v x B), where E and B are the electric and magnetic fields respectively. A consequence of electromagnetic theory is the existence of electromagnetic radiation, of which visible light and radio waves are two such examples. The electric force, like gravity, is an inverse-square law force. Thus, it too has infinite range and a massless carrier particle (the photon). The relative strenth of the electric force to the gravitational force between a proton and an electron is given by

$$\begin{aligned} \frac{F_e}{F_g} &= \frac{ke^2}{Gm_pm_e}, \\ &= 2.3e39! \end{aligned}$$

The electric force is 39 orders of magnitude stronger than the gravitational force!

3. The weak interaction is a nuclear force responsible for certain types of nuclear decay. Much of the pioneering work in the area was carried out by Enrico Fermi, amongst others. The relative strength of the weak force is approximately $\frac{1}{137}F_e$, making it 37 orders of magnitude stronger than gravity. It turns out that the electromagnetic and weak interactions are related. Such a unification was accomplished by Sheldon Glashow, Steven Weinberg, and Abdus Salaam, for which they were awarded the 1979 Nobel Prize in physics. The weak force is of very short range, approximately 10^-18 m, giving it the shortest range of any of the four fundamental forces. Corresponding to such a short range, the weak force has very massive carrier particles, the positively and negatively charged W bosons and the neutral Z boson. The weak interaction is tied in with particles known as neutrinos, for which Fred Reines, a professor at UC Irvine, received the 1995 Nobel Prize in physics for his work on these particles.

4. The final fundamental force is the strong force. As suggested by its name, it is the strongest of the fundamental forces. It is a very short range force, acting over the dimensions of the nucleus, approximately 10^{-15} m, and is responsible for holding the nucleus together, thus overcoming the electric repulsion of the protons. The strong force takes the form

$$F \sim \frac{e^{-\frac{r}{r_0}}}{r^2},$$

where $r_0 = 10^{-15}$ m. Relative to the gravitational force, the strong force is over 45 orders of magnitude stronger!