Physics 11 Homework IV Solutions
Ch. 4 - Problems 6, 11, 16, 20, 30, 36, 37, 58, 62, 65.

Problem 6

We have the following information:

$$\begin{aligned} m &= 1.5e7\ kg, \\ F &= 7.5e5\ N, \\ v_0 &= 0\ \frac{m}{s}, \\ v_f &= (80\ \frac{km}{h})(\frac{1000\ m}{1\ km})(\frac{1\ h}{3600\ s}) = 22.2\ \frac{m}{s}. \end{aligned}$$

The acceleration is given by

$$a = \frac{F}{m} = \frac{7.5e5N}{1.5e7kg} = 0.05\ \frac{m}{s^2}.$$

Using the equation $v_f = v_0 + at$, the time it takes to reach final velocity is

$$t = \frac{v_f}{a} = \frac{22.2\ \frac{m}{s}}{0.05\ \frac{m}{s^2}} = 444\ s.$$

Problem 11

We have the following information:

$$\begin{aligned} \vec{F}_p &= 2000\ N\ \hat{\mathbf{x}}, \\ \vec{F}_r &= -1800\ N\ \hat{\mathbf{x}}, \\ m &= 1000\ kg. \end{aligned}$$

(a) The acceleration is given by

$$a = \frac{\sum \vec{F}}{m} = \frac{200\ N}{1000\ kg} = 0.2\ \frac{m}{s^2}.$$

(b) Assume $v_0 = 0$ and $t = 10$ s. How far does the boat travel in this time interval?

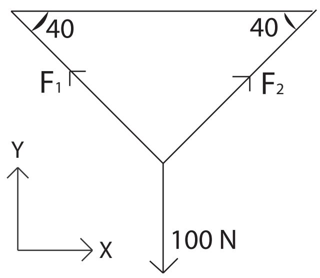
$$x = \frac{1}{2}at^2 = \frac{1}{2}(0.2\ \frac{m}{s^2})(10\ s)^2 = 10\ m.$$

(c) What is its velocity at the end of this time interval?

$$v_f = at = (0.2\ \frac{m}{s^2})(10\ s) = 2\ \frac{m}{s}.$$

Problem 16

The free-body diagram appears below.



Problem 16

Our force equations are as follows.

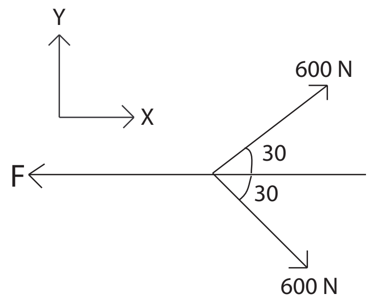
$$\sum \vec{F}_x \ :\ F_1 \cos(40) - F_2 \cos(40) \rightarrow F_1 = F_2 \equiv F,$$
$$\sum \vec{F}_y \ :\ 2F \sin(40) - W = 0 \rightarrow F = \frac{W}{2\sin(40)} = \frac{100}{2\sin(40)} = 77.8\ N.$$

Problem 20

The free-body diagram appears below.

This problem is the same as the previous one. If the boat moves with constant velocity, the acceleration is zero which implies that the sum of the forces is as well. Thus, we find that

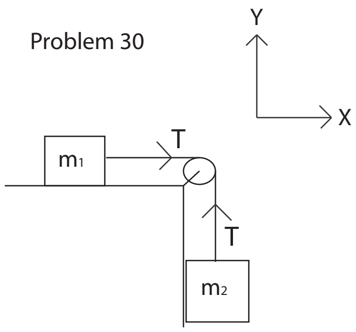
$$F = 2(600)\cos(30) = 1039\ N.$$



Problem 20

Problem 30

The free-body diagram appears below.



The relevant forces equation are:

$$\begin{aligned} N - W &= 0, \\ m_2 g - T &= m_2 a, \\ T &= m_1 a. \end{aligned}$$

Summing the last two equations and solving for a yields

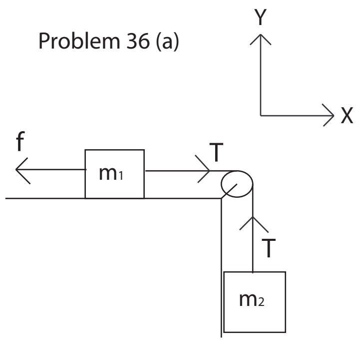
$$\begin{aligned} m_2 g &= (m_1 + m_2) a, \\ a &= \frac{m_2}{m_1 + m_2} g. \end{aligned}$$

Note that the acceleration is the same for both masses m_1 and m_2 since they are connected by cord which we assume to be ideal, i.e. massless and inextensible. Substituting our expression for a into our second force equation above gives T as:

$$T = m_1 a = \frac{m_1 m_2}{m_1 + m_2} g.$$

Problem 36

(a) The free-body diagram appears below.



Now, if we consider the forces acting on mass m_1, we find:

$$\begin{aligned} \sum \vec{F}_y &: \; N - W = 0 \rightarrow N = m_1 g \\ \sum \vec{F}_x &: \; T - \mu_s m_1 g = m_1 a. \end{aligned}$$

Careful examination of this equation and comparison to the corresponding equations in the previous problem reveals that the block system only moves if $f_s < T$, where $f_s = \mu_s m_1 g$. If we were to write the force equation for mass m_2,

$$m_2 g - T = m_2 a,$$

and solve this together with the force equations for mass m_1 for the acceleration and tension, we would find,

$$\begin{aligned} m_2 g - \mu_s m_1 g &= (m_1 + m_2) a, \\ a &= \frac{m_2 - \mu_s m_1}{m_1 + m_2} g, \end{aligned}$$

and

$$\begin{aligned} T &= m_1(\mu_s g + a), \\ &= m_1 g(\mu_s + \frac{m_2 \mu_s m_1}{m_1 + m_2}), \\ &= m_1 g(\frac{m_1\mu_s + m_2\mu_s + m_2 - m_1\mu_s}{m_1 + m_2}), \\ &= \frac{m_1 m_2}{m_1 + m_2}(1 + \mu_s)g. \end{aligned}$$

So, in order for the system to begin moving, we must have

$$\begin{aligned} T &> f_s, \\ \frac{m_1 m_2}{m_1 + m_2}(1 + \mu_s)g &> \mu_s m_1 g, \text{or} \\ \frac{m_2}{m_1 + m_2}(1 + \mu_s) &> \mu_s, \\ m_2 + m_2\mu_s &> m_1\mu_s + m_2\mu_s, \\ \mu_s &< \frac{m_2}{m_1}. \end{aligned}$$

With m_1 = 10 kg and m_2 = 4.0 kg, this means that $\mu_s < \frac{4.0\ kg}{10\ kg} = 0.40$. Since $\mu_s = 0.50 > 0.40$, the system does not move. Alternately, plugging in the given values we find that a = -0.07g < 0. Clearing, this result cannot be physical, as it tells us that, without an externally applied force, mass m_2 moves upward against the force of gravity. (Although it might be kind of interesting to see, and if you can figure out how to do it, you will go on to fame and fortune.) Nevertheless, our everyday experience tells us this is nonsensical - the only reasonable interpretation being that the system simply does not move. However, it is a good idea to always compare the forces as we did above.

(b) The free-body diagram is the same as in part (a). With a coefficient of kinetic friction $\mu_k = 0.30$ and using the expression for acceleration found above (it carries over perfectly well here, as all we have done is make the substitution $\mu_s \to \mu_k$), we find

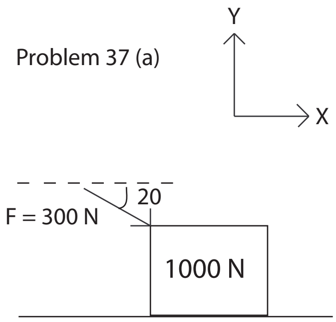
$$a = \frac{m_2 - \mu_s m_1}{m_1 + m_2} g = \frac{(4.0 - (0.30)(10))kg}{(10 + 4.0)kg} g \approx 0.07g = 0.7\frac{m}{s^2}.$$

Problem 37

We have the following information:

$$\begin{aligned} F &= 300\ N, \\ \theta &= 20.0^{\circ}, \\ W &= 1000\ N. \end{aligned}$$

(a) Note that v $\equiv$ constant $\rightarrow$ a = 0. The free-body diagram appears below.



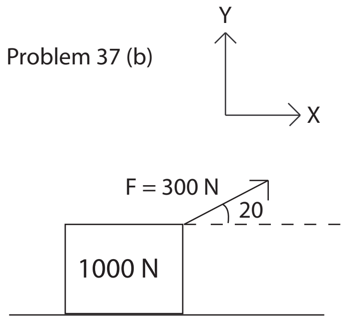
The sum of the forces in the $\hat{\mathbf{y}}$ direction yields

$$\begin{aligned} 0 &= N - W - F\sin(\theta), \\ N &= W + F\sin(\theta), \\ &= 1000 + 300\sin(20), \\ &= 1103\ N. \end{aligned}$$

The sum of the forces in the $\hat{\mathbf{x}}$ direction yields

$$\begin{aligned} 0 &= F\cos(\theta) - \mu_s N, \\ \mu_s &= \frac{F\cos(\theta)}{N}, \\ &= \frac{300\cos(20)}{1000 + 300\sin(20)}, \\ &= 0.256. \end{aligned}$$

(b) The free-body diagram appears below.



The sum of the forces in the $\hat{\mathbf{y}}$ direction yields

$$
\begin{aligned}
0 &= N - W + F\sin(\theta), \\
N &= W - F\sin(\theta), \\
&= 1000 - 300\sin(20), \\
&= 897\ N.
\end{aligned}
$$

The sum of the forces in the $\hat{\mathbf{x}}$ direction yields

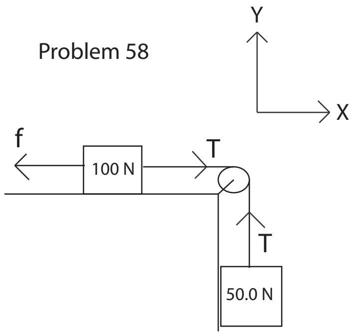
$$
\begin{aligned}
ma &= \frac{W}{g}a = F\cos(\theta) - \mu_s N, \\
a &= \frac{g}{W}(F\cos(\theta) - \mu_s N), \\
&= \frac{9.8}{1000}(300\cos(20) - (0.256)(1000 - 300\sin(20))), \\
&= 0.511\ \frac{m}{s^2}.
\end{aligned}
$$

Problem 58

The free-body diagram appears below.

(a) For the mass on the table, the sum of the forces in the $\hat{\mathbf{y}}$ direction yields

$$
\begin{aligned}
0 &= N - W_t, \\
N &= W_t = 100\ N.
\end{aligned}
$$



For the same mass, the sum of the forces in the $\hat{\mathbf{x}}$ direction yields

$$\begin{aligned} 0 &= T - f_s, \\ T &= f_s, \end{aligned}$$

For the hanging mass, the sum of the forces in the $\hat{\mathbf{y}}$ direction yields

$$\begin{aligned} 0 &= W_h - T, \\ T &= W_h, \\ &= 50.0\ N. \end{aligned}$$

Substituting this value above we find

$$f_s = T = 50.0\ N.$$

(b) Recall that the force due to static friction is defined as

$$f_s = \mu_s N.$$

Thus, in order to ensure static equilibrium, the coefficient of static friction must be at least

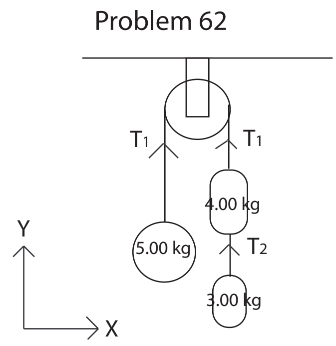
$$\mu_s = \frac{f_s}{N} = \frac{f_s}{W_t} = 0.500.$$

(c) The coefficient of kinetic friction is $\mu_k = 0.250$. Thus, in order for the system to move with constant speed, the total force must be zero. With the definitions found in the previous two parts, this means that

$$\mu_k W_t = W_h, \text{or}$$
$$W_h = (0.250)(100) = 25.0 \; N.$$

Problem 62

The free-body diagram appears below.



(a) The force equations can be written as

$$5a = T_1 - 5.00g,$$
$$7a = 7.00g - T_1.$$

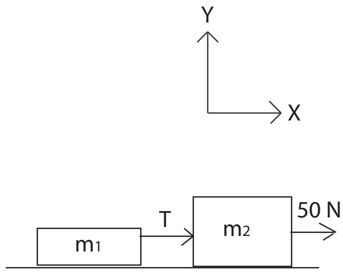
Solving for the acceleration yields a $= \frac{g}{6} = 1.63 \; \frac{m}{s^2}$.

(b) The tension T_1 is easily found from above to be $T_1 = 5(a + g) = \frac{35}{6}g = 57.2$ N. The tension T_2 is just the force acting on the 3.00 kg mass, and thus can be found easily by equating this force with the acceleration the mass experiences. Thus, $T_2 = 3(a + g) = \frac{21}{6}g = 34.3$ N.

Problem 65

The free-body diagram appears below.
We have the following information:

$$m_1 = 10 \; kg,$$
$$m_2 = 20 \; kg,$$
$$F = 50 \; N.$$



Problem 65 (a)

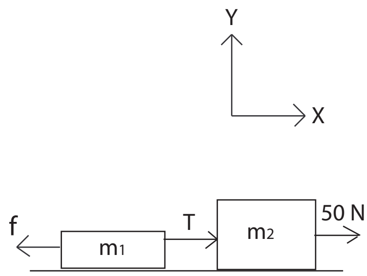
(a) A 10 kg box is attached to a 20 kg box by an ideal string. a 50 N force is applied to the 20 kg box. As can easily be seen in the free-body diagram, the force is actually applied to the system that is both masses. Thus, acceleration of each box is the same and is given by

$$a = \frac{F}{m} = \frac{50N}{(10+20)kg} = 1.7\ \frac{m}{s^2}.$$

The tension in the connecting ideal string is given by

$$T = m_1 a = (10\ kg)(1.67\ \frac{m}{s^2}) = 17\ N.$$

(b) The free-body diagram appears below.



Problem 65 (b)

Let's consider the same scenario, but this time there exists a coefficient of kinetic friction, $\mu_k = 0.10$ between each box and the surface. Then, the force equation for the two box system is

$$
\begin{aligned}
(m_1 + m_2)a &= 50 - f_k, \\
&= 50 - \mu_k(m_1 + m_2)g, \\
a &= \frac{50 - \mu_k(m_1 + m_2)g}{m_1 + m_2}, \\
&= 0.69\ \frac{m}{s^2}.
\end{aligned}
$$

The tension can be found from

$$
\begin{aligned}
m_1 a &= T - f_k, \\
&= T - \mu_k m_1 g, \\
T &= m_1(a + \mu k g), \\
&= 17\ N.
\end{aligned}
$$