Physics 11 Homework III Solutions
Ch. 3 - Problems 2, 15, 18, 23, 24, 30, 39, 58.

Problem 2

So, we fly 200km due west from City A to City B, then 300km 30° north of west from City B to City C.

(a) We want the distance between City C and City A. If we define north as $+\hat{\mathbf{y}}$ and east as $+\hat{\mathbf{x}}$, then we can write each leg of the trip as a vector (in km). Thus, $\vec{r}_{A\to B}$ = (-200, 0) and $\vec{r}_{B\to C}$ = (300cos(150), 300sin(150)) = (-150$\sqrt{3}$, 150), where all angles are measured positively (counterclockwise) from the $+\hat{\mathbf{x}}$ axis. Then, simple vector addition gives $\vec{r}_{A\to C} = \vec{r}_{A\to B} + \vec{r}_{B\to C}$, which is simply:

$$\begin{aligned}\vec{r}_{A\to C} &= (-200, 0) + (-150\sqrt{3}, 150), \\ &= (-200 + (-150\sqrt{3}))\hat{\mathbf{x}} + (0 + 150)\hat{\mathbf{y}}, \\ &\approx (-460, 150).\end{aligned}$$

The distance between City C and City A is just the magnitude of the vector, namely,

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-460)^2 + 150^2} \approx 484km.$$

(b) In order to determine the relative direction, we need the vector $\vec{r}_{A\to C}$ which was found above. Then,

$$\theta = \pi + \arctan(\frac{r_y}{r_x}) = \pi + \arctan(\frac{150}{-460}) = 161.9°$$

or π - θ = 18.1° north of west. The reason for the addition of π above is to take into account the convention that arctan is defined on the interval (-$\frac{\pi}{2}$, $\frac{\pi}{2}$).

Problem 15

A hurricane travels 60° north of west with a speed of 41.0 $\frac{km}{h}$. Three hours later, it suddenly changes heading to due north slowing down to 25.0 $\frac{km}{h}$. Suppose the hurricane starts at the origin of our coordinate system, (0, 0). After three hours, the eye of the hurricane is at (3*41*cos(120), 3*41*sin(120)) = (-61.5, 106.5), in units km. Then, the hurricane changes direction and heads due north. Thus, 4.5 h from start of measurements, we find the hurricane at (-61.5, 106.5) + (0, 1.5*25) = (-61.5, 144). The distance from the origin (Granda Bahama Island) after this interval is r = $\sqrt{(-61.5)^2 + 144^2}$ = 157 km.

Problem 18

(a) Using axes as defined in the text (the same used above in the previous two problems), write each force in terms of its components. So, force $\vec{F_1}$ can be written (in units N), (120cos(60), 120sin(60)) = (60, 60$\sqrt{3}$). Also, force $\vec{F_2}$ = (80cos(105), 80sin(105)) = (-20.7, 77.3). A brief aside to discuss terminology, the resultant force is the single force that represents the sum of multiple individual forces. So, the resultant force is:

$$\begin{aligned}
\vec{F_r} &= (60, 60\sqrt{3}) + (-20.7, 77.3) = (39.3, 181.2), \\
|\vec{F_r}| &= \sqrt{39.3^2 + 181.2^2} = 185.4N, \\
\theta &= \arctan(\frac{181.2}{39.3}) = 77.8°.
\end{aligned}$$

So, the resultant force has magnitude 185.4 N and points 77.8° measured counterclockwise from the $+\hat{\mathbf{x}}$ axis.

Problem 23

We have the following information:

$$\begin{aligned}
v_x &= (200\frac{mi}{h})(1609\frac{m}{mi})(\frac{1h}{3600s}) = 89.4\frac{m}{s}, \\
h_i &= 100m, \\
x_i &= 0m, \\
x_f &= 100m, \\
a_x &= 0, \\
a_y &= g.
\end{aligned}$$

Now, the time it takes the falcon to travel 100 m (horizontally) is t = $\frac{x_f - x_i}{v_x} = \frac{100m}{89.4\frac{m}{s}}$ = 1.12s. Thus, in the same time interval, the falcon free falls (from rest) a distance x = $\frac{1}{2}(9.8\frac{m}{s^2})(1.12s)^2$ = 6.13 m.

Problem 24

We have the following information:

$$
\begin{aligned}
v_x &= 18.0\frac{m}{s}, \\
h_i &= 50.0m, \\
h_f &= 0m, \\
a_x &= 0\frac{m}{s^2}, \\
a_y &= g, \\
(v_y)_i &= 0\frac{m}{s}.
\end{aligned}
$$

The stone strikes the beach at t = $\sqrt{\frac{2*h}{g}}$ = 3.19 s. Its final velocity is given by:

$$
\begin{aligned}
\vec{v} &= (18.0, g*t) = (18.0, 31.3)\frac{m}{s}, \\
|\vec{v}| &= \sqrt{18.0^2 + 31.3^2} = 36.1\frac{m}{s}, \\
\theta &= \arctan(\frac{31.3}{18.0}) = 60.1^\circ, \text{ measured clockwise from the -}\hat{\mathbf{x}}\text{ axis.}
\end{aligned}
$$

So, the stone strikes the beach at a speed of 36.1 $\frac{m}{s}$ at an angle of 60.1° measured from the -$\hat{\mathbf{x}}$ axis.

Problem 30

We have the following information:

$$
\begin{aligned}
\Delta x &= 36.0m, \\
\Delta h &= 3.05m, \\
v_i &= 20.0\frac{m}{s}, \\
\theta &= 53^\circ, \\
a_x &= 0\frac{m}{s^2}, \\
a_y &= g.
\end{aligned}
$$

(a) We want to determine if the ball clears the crossbar, and if so, by how much. Traveling at a constant horizontal velocity v_x = 20.0cos(53) = 12.0 $\frac{m}{s}$. A ball traveling at this speed would take t = $\frac{\Delta x}{v_x}$ = 3.00s. At this time, the y-position of the ball is

$$y(t = 3.00) = (20.0 \sin(53))(3.00) + \frac{1}{2}(-9.8)(3.00)^2 = 3.82m.$$

Since y(t=3.00) $>$ Δh, the ball clears the goal post. It clears the post by y(t=3.00) - Δh = 0.77m. If we solve the y equation when y = 0, we find

$$\begin{aligned} 0 &= 20.0 \sin(53)t - \frac{1}{2}(-9.8)t^2, \\ &= 16t - 4.9t^2, \\ &= t(16 - 4.9t), \end{aligned}$$

which has solutions t = 0, 3.27s. Thus, the football lands a distance x(t=3.27) = (20.0cos(53))(3.27) - Δx = 3.36m past the goal post.

(b) To determine whether the ball is rising or falling, we must determine where (horizontally) the (vertical) apex of ball's trajectory occurs. Recall that at this point, $v_y = 0$. This occurs at time t = $\frac{20.0 \sin(53)}{9.8}$ = 1.63s. At this time, the horizontal position is given by

$$x(t = 1.63) = (20.0 \cos(53))(1.63) = 19.6m.$$

Since x(t=1.63) $<$ Δx, the ball crosses the goal post while falling.

Problem 39

A boat crosses a river 0.505 mi wide with a velocity of 3.30 $\frac{mi}{h}$ at an angle 62.5° north of west. The river carries an eastward current of 1.25 $\frac{mi}{h}$. How far upstream is the boat when it reaches the opposite shore?

Let's define perpendicular and parallel directions in relation to the river-bank (as well as the current). Then, the perpendicular component of velocity is given by $v_\perp$ = 3.30sin(117.5) = 2.93 $\frac{mi}{h}$. Thus, it takes the boat t = $\frac{0.505}{2.93}$ = 0.173 h to reach the opposite side. Thus, the boat ends up x = (3.30cos(117.5))(0.173) + (1.25)(0.173) = (-0.047 mi)($\frac{5280ft}{1mi}$) = 250 ft. upstream.

Problem 58

We have the following information:

$$\begin{aligned} y_i &= 2.00m, \\ y_f &= 3.05m, \\ \theta &= 40.0°, \\ x &= 10.0m, \\ a_x &= 0, \\ a_y &= g. \end{aligned}$$

We want to determine the initial velocity of the basketball so that it goes in the hoop without touching the backboard. Let's call it v_0. Then, with this velocity, the time it takes the ball to reach the hoop is given by

$$t = \frac{x}{v_0 \cos(\theta)}.$$

Substituting this value for t into the expression for y as a function of t yields y as a function of v_0.

$$\begin{aligned} y_f &= y_i + v_0 \sin(\theta)(\frac{x}{v_0 \cos(\theta)}) + \frac{1}{2} a (\frac{x}{v_0 \cos(\theta)})^2, \\ &= y_i + x \tan(\theta) + \frac{g x^2}{2 v_0^2 \cos^2(\theta)}. \end{aligned}$$

Solving the above for v_0 yields

$$\begin{aligned} v_0 &= \sqrt{\frac{g x^2}{y_f - y_i - x \tan(\theta)}}, \\ &= \sqrt{\frac{(-9.8)(10.0)^2}{3.05 - 2.00 - 10.0 \tan(40)}}, \\ &= 11.6 \frac{m}{s}. \end{aligned}$$