Physics 1C Summer 2001 Final Exam (3 hours)

Instructions:

1. Use a new, empty Blue Answer Book. Write your Quiz code # (and optionally your name) on the front.

2. Write in blue or black ballpoint pen only - work in pencil will not be graded.

3. Label your answers clearly, and "X" out any rough work not to be graded - do not remove pages.

4. Attempt all 6 questions. Note that some questions may take more time than others even if they carry equal points.

5. This exam is "closed book" - no outside information allowed.

6. Some advice: even if you cannot come up with a numerical answer to a question, at least write down something *relevant* and *accurate* for partial credit.

$$\boxed{\begin{array}{l} h = 6.6\times10^{-34}\,\text{Js},\ \ c = 3\times10^{8}\,\text{m/s},\ \ e = 1.6\times10^{-19}\,\text{C}. \\ \dfrac{hc}{e} = 1.2375\times10^{-6}\ \text{eV}\cdot\text{m},\ \ m_e = 9.1\times10^{-31}\,\text{kg}. \end{array}}$$

1. A laser beam of vacuum wavelength 400nm is directed into a fiber-optic cable, which has a refractive index n=1.5 at this wavelength.

a. From the definition of refractive index, what is **(i)** the speed (m/s), and **(ii)** the wavelength (nm) of the beam inside the fiber?

b. Using Snell's Law ($n_i \sin\theta_i = n_t \sin\theta_t$), determine the critical angle of incidence for total internal reflection to confine the light inside the fiber, when surrounded by air (n=1.0).

The beam travels inside a horizontal piece of fiber at an angle just above the critical angle. The fiber then passes through an aquarium full of water (n=1.33).

c.(i) At what angle does the light emerge into the water? **(ii)** Does this beam emerge from the top surface of the water, or is it now "trapped" underwater due to total internal reflection at this surface? **(15 points)**

2. A camera with a f=+60mm converging lens is used to form the image of a child standing a distance s_o=2.5m away onto a piece of film. The child is 0.8m tall.

a. Draw a ray diagram (not to scale) showing the formation of this image and list its attributes (real/virtual, erect/inverted, magnified/minified).

b. Using the lens equation, calculate **(i)** the lens-film distance s_i in mm, required to produce a sharply focussed image, and **(ii)** the height of the image on the film.

c. Using a ray diagram, **(i)** show how a concave *mirror* with the same focal length could be used as a camera.

d. Giive two advantages of a mirror system over a lens system in this application. **(20 points)**

Lens/Mirror equation: $1/s_o + 1/s_i = 1/f$
Transverse Magnification $M_T = {}^{-s_i}/_{s_o}$

3. A discharge tube containing hydrogen gas (Z=1) produces an emission line spectrum, which is studied by transmitting the light through a diffraction grating.

a. Using the Bohr energy level formula $E_n = -13.6\frac{Z^2}{n^2}\,\mathrm{eV}$, show that the wavelength λ of the Hβ line (n=4→2) is about 486nm (give 4 significant figures in your answer).

b. If the grating spacing is a=1200nm, find the largest angle θ (degrees) at which the Hβ line is visible? (Hint: Use the grating equation $a\sin\theta = m\lambda$, and find the largest possible value of m first).

c. If the time interval for this atomic transition is about 100 ns, what is the corresponding "natural" width of the spectral line, in nm, due to Heisenberg's Uncertainty Principle $\Delta E\Delta t \geq h/4\pi$.

(15 points)

4a. Define a blackbody and briefly explain why a hole in a furnace is a good approximation to one. (A diagram may help).

The tungsten filament in a certain light bub is also a good approximation to a blackbody radiator. The filament emits a power of 25W at a temperature of 2000K; Wien's Law tells us that the wavelength of peak spectral intensity is then 1500nm. A student slowly increases voltage across the bulb, which increases the filament temperature to 3000K.

b. Before the temperature is increased, if the 25W filament's power is spread uniformly over a sphere, what is the irradiance measured by a light meter at a distance of 2m?

c. Using Wien's Law and Stefan's Law respectively, find the new values of **(i)** peak wavelength (nm) and **(ii)** total emitted power (W) from the bulb.

d. To approximate scale, sketch the form of the spectral intensity I_λ of the blackbody for these two cases, using the same axes. Pay attention to **(i)** the peak wavelengths, and **(ii)** the areas under the curves using your answers to part (c).

(20 points)

$$\boxed{\text{Wien's law: } \lambda_{max}T = \text{constant. Stefan's law: } P = \sigma A T^4}$$

$$I = \frac{P}{4\pi d^2}$$

5. In a photoelectric effect experiment, electrons are ejected from a metal surface when it is irradiated by an unpolarized light source of wavelength 486nm. The stopping potential V_s is given by the equation $\boxed{eV_s = hf - \varphi}$.

a. The light source has an irradiance of 0.05 W/m^2. How many photons per second are incident on the metal surface of area 0.002 m^2?

b. For the 486nm light the stopping potential is measured to be 0.25V. What is **(i)** the threshold energy (work function) ϕ of the metal, in eV, and **(ii)** the corresponding cutoff wavelength (in nm) above which no photo-electrons are observed.

c. A sheet of polaroid is now placed between the light source and the metal. Briefly describe the effect of this change (if any) on **(i)** the irradiance, **(ii)** the photocurrent produced at zero voltage, and **(iii)** the stopping potential.
(15 points)

6. X-rays are generated by bombarding a copper target with 10 keV electrons.

a. What is the shortest X-ray wavelength produced, in nm?

b. These shortest-wavelength X-rays are then reflected off a crystal surface, which is slowly rotated from θ=90 degrees (grazing incidence) to θ=0.0 degrees (perpendicular to the beam). Using the Bragg diffraction formula $\boxed{2d\cos\theta = m\lambda}$, show that if the first diffraction peak is observed at θ=60 degrees, the atomic spacing d of the crystal must be about 0.12nm (use 3 decimal places in your answer).

c. Using the deBroglie wavelength $\lambda = \frac{h}{(mv)}$, find the kinetic energy, in eV, of a beam of electrons striking this crystal, which would also produce a first-order Bragg diffraction peak at θ=60 degrees.
(15 points)

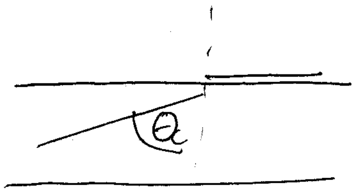
1. Refractive index $n \equiv \frac{c}{v}$

a) For $n = 1.5$, (i) speed $v = \frac{c}{n} = \frac{3\times10^8 \text{m/s}}{1.5} = \underline{2\times10^8 \text{ m/s}}$

(ii) Wavelength $\lambda = \frac{v}{f}$ with f the same, $f = c/\lambda_0$

$\therefore$ λ inside fiber $= \frac{\lambda_0}{n} = \frac{400}{1.5} = \underline{266.67 \text{ nm}}$

b)



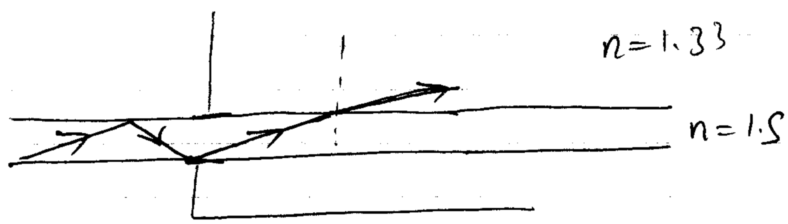
$n_i \sin\theta_i = n_t \sin\theta_t$

with $n_i = 1.5$, $n_t = 1.0$ (air)

and $\theta_i \equiv \theta_c$ when $\theta_t = 90°$

$\Rightarrow \sin\theta_c = \frac{n_t}{n_i} = \frac{1}{1.5} = 2/3$, or $\underline{\theta_c = 41.81°}$

c)



i) From Snell's law with $n_i = 1.5$, $\theta_i \geq \theta_c = 41.81°$, $n_t = 1.33$

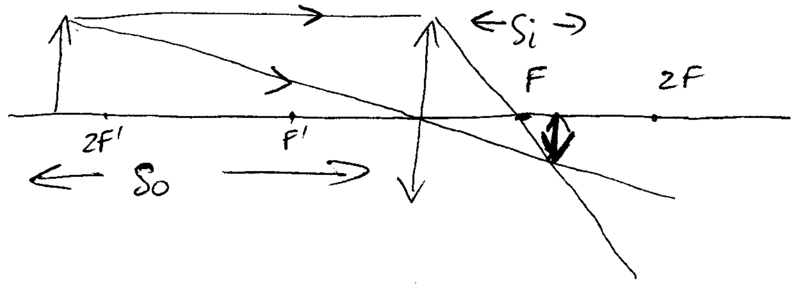
$\sin\theta_t = \frac{n_i \sin\theta_i}{n_t} = \frac{1.5\times 2/3}{1.33} = \frac{1}{1.33}$

$\Rightarrow \underline{\theta_t = 48.61°}$

ii) Since $n_i \sin\theta_i \geq 1$, the beam is trapped inside the water.

2.

a)



any 2 rays can locate image

Object beyond $2f \Rightarrow$ image between f and $2f$

Image is real, inverted, minified

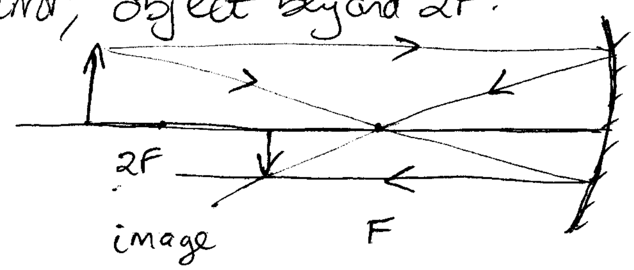
b) i) From lens equation, $\frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} = \frac{1}{+60\,mm} - \frac{1}{2500\,mm}$

$\Rightarrow S_i = 61.475\,mm$

ii) Image height $\propto$ image distance, or $y_i = M_T y_o$ with

$M_T = -\frac{S_i}{S_o} \Rightarrow$ height $y_i = y_o \underbrace{\left(\frac{61.475\,mm}{2500\,mm}\right)}_{0.0246} = \underline{19.67\,mm}$

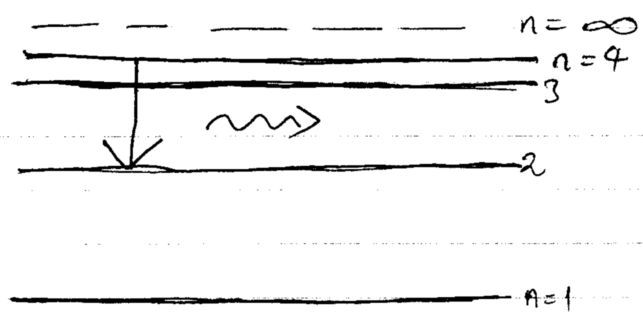
c) For a mirror, object beyond 2F:



film plane is between F and 2F.

d) Mirror advantages - any 2 of
- achromatic (all colors have same focus)
- can be supported from back, not just edges
- only one surface to clean
- works identically underwater (same focus $f = R/2$, independent of outside medium)

3.



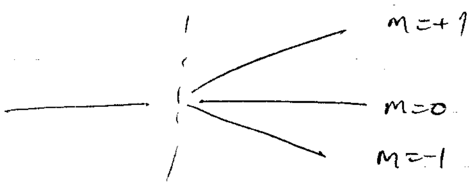
$$E_n = -13.6\,\mathrm{eV}\,\frac{Z^2}{n^2}$$

For H_β $(4 \to 2)$ line, energy difference $E_4 - E_2$ = photon energy

i.e. $\dfrac{hc}{\lambda_{42}} = -13.6\,\mathrm{eV}\left(\dfrac{1}{4^2} - \dfrac{1}{2^2}\right) = +2.55\,\mathrm{eV}$

$\therefore \quad \lambda_{42} = \dfrac{hc}{2.55\,\mathrm{eV}} = \dfrac{hc}{e \times 2.55} = \dfrac{1.2375 \times 10^{-6}}{2.55} = \underline{485.3\,\mathrm{nm}}$

b)



$$\sin\theta_m = \frac{m\lambda}{a} = m \times \frac{485.3}{1200} = m \times 0.4044$$

So for $|\sin\theta| < 1$, $\quad m < \dfrac{1}{0.4044} = 2.47$

i.e. $m = 2$ gives the largest angle, $\theta_2 = \sin^{-1}\left(\dfrac{2 \times 485.3}{1200}\right)$

$\Rightarrow \underline{\theta_2 = 53.98^\circ}$

c) For $\Delta t = 100 \times 10^{-9}\,\mathrm{s}$, $\quad \Delta E \geq \dfrac{h}{4\pi \Delta t} = 5.25 \times 10^{-28}\,\mathrm{J}$ or $3.28 \times 10^{-9}\,\mathrm{eV}$

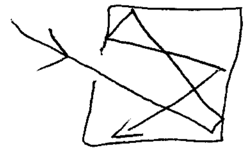
line width $\Delta\lambda \approx \dfrac{d\lambda}{dE}\Delta E$ with $\lambda = \dfrac{hc}{E}$ for photon

i.e. $\Delta\lambda \approx \dfrac{hc\,\Delta E}{E^2}$ or $\dfrac{\Delta\lambda}{\lambda} = \dfrac{\Delta E}{E} = \dfrac{3.28 \times 10^{-9}\,\mathrm{eV}}{2.55\,\mathrm{eV}} = 1.28 \times 10^{-}$

$\Rightarrow \Delta\lambda \approx 1.28 \times 10^{-9} \times 485.3\,\mathrm{nm} = \underline{6.25 \times 10^{-7}\,\mathrm{nm}}$

4a. A blackbody absorbs all EM radiation incident upon it

For a hole in an oven:



light incident on hole suffers multiple reflections + absorption in oven walls, hardly ever gets back out.

$\Rightarrow$ hole is a "perfect" absorber.

b) Irradiance $I = \dfrac{P}{4\pi d^2}$ with $d = 2.0\,m$

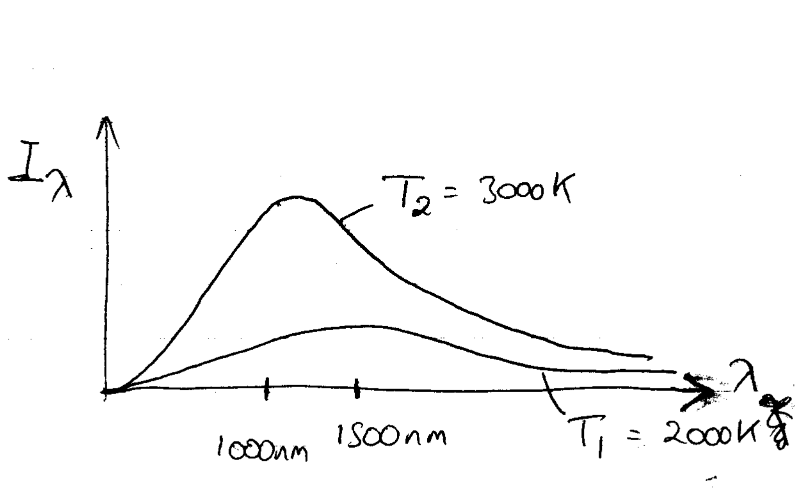
$$\Rightarrow \quad I = \frac{25\,W}{4\pi(2)^2} = 0.497\ W/m^2$$

c) Wien's law $\Rightarrow$ $\lambda_{max} T = \text{constant}$ so for $T_1 = 2000\,K$, $\lambda_1 = 1500\,nm$

i) $\dfrac{\lambda_1}{\lambda_2} = \dfrac{T_2}{T_1}$ i.e. $\lambda_2 = 1500\,nm \times \left(\dfrac{3000\,K}{2000\,K}\right)^{-1} = 1000\,nm.$ (shorter)

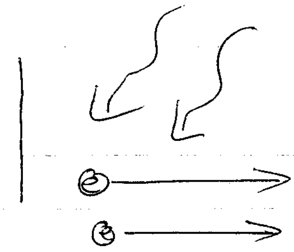
ii) Stefan's law $\Rightarrow$ power $\propto T^4$ so $P_2 = P_1 \underbrace{\left(\dfrac{T_2}{T_1}\right)^4}_{5.0625} = \underline{126.56\,W}$

d)



5.

$I = 0.05 W/m^2$



a) Energy arriving at plate/s $= I_1 \times area = 0.05 \times 0.002 = \underline{100 \mu W}$

Energy of single photon $= hf = \frac{hc}{\lambda} = \frac{6.6\times10^{-34} \times 3\times10^{8}}{486\times10^{-9} m} = 4.07 \times 10^{-19} J$

$\therefore$ # of photons per second is given by $\frac{100\times10^{-6} J/s}{4.07\times10^{-19} J} = \underline{2.45 \times 10^{14}/s}$

b) From conservation of energy, $\varphi = \frac{hc}{\lambda} - eV_s$

In eV, $\frac{hc}{\lambda} = 2.55 eV$ (cf question 3), so for $V_s = 0.25 eV$

i) $\varphi = 2.55 - 0.25 = 2.3 eV$

ii) Cutoff wavelength: $\frac{hc}{\lambda_c} = \varphi$ when $V_s = 0$

i.e. $\lambda_c = \frac{hc}{\varphi} = \frac{hc}{e \times 2.3} = \frac{1.2375\times10^{-6}}{2.3} = \underline{538.0 nm}$

c) A sheet of polaroid will

i) reduce the irradiance I to $I/2$

ii) Since 1 photon $\rightarrow$ 1 electron, the photocurrent will halve

iii) Stopping potential $\propto$ electron K.e. = photon energy $- \varphi$

- unaffected by the number of photons changing.

6.

a) Shortest wavelength photon has energy $E = hf = \frac{hc}{\lambda} = \frac{1}{2} m_e v^2$

where electron k.e. $\frac{1}{2} m_e v^2 = e V_a$

$$\text{i.e.} \quad \frac{hc}{\lambda} = eV_a, \quad \text{or} \quad \lambda = \frac{hc}{e} \cdot \frac{1}{V_a}$$

$$= \frac{1.2375 \times 10^{-6}}{10 \times 10^3} = 0.1237\,\text{nm} \quad \underline{(0.124\,\text{nm})}.$$

b) For $d = \frac{m\lambda}{2\cos\theta}$ with $m = 1$, $\theta = 60°$

crystal spacing $d = \frac{1.\lambda}{2.\frac{1}{2}} = \lambda = \underline{0.124\,\text{nm}}$.

c) Using electrons directly on crystal, if we require

de Broglie $\lambda = \frac{h}{p} = 0.124\,\text{nm}$ also,

$$\text{k.e.} \quad \frac{p^2}{2m_e} = e V_a \quad \text{so} \quad e V_a = \frac{h^2}{2 m_e \lambda^2}$$

$$\text{or accel. voltage } V_a = \frac{h^2}{2 e m_e \lambda^2} = \frac{(6.6 \times 10^{-34})^2}{2 \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-31} \lambda^2}$$

$$\Rightarrow V_a = 97.3\,V$$

so electron k.e. = $\underline{97.3\,\text{eV}}$.