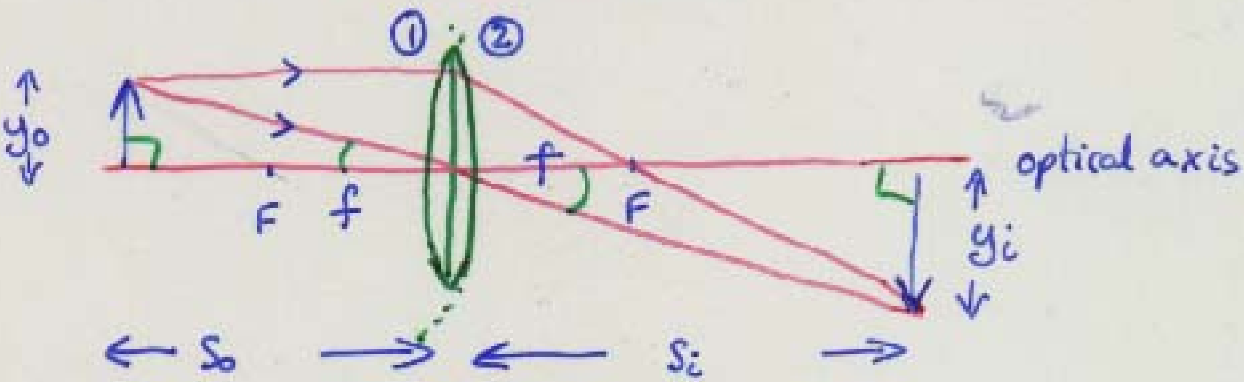
Spherical Thin Lenses (Review)

Focal length f: $\boxed{\frac{1}{f} = \left(\frac{n_g}{n_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$



Gaussian Lens Eqn: $\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}$

Sign Conventions: (Light enters from left)

R_1, R_2 : +ve if center on the right (surface bulges left)

s_o : +ve for real object on left

s_i : +ve for real image on right
  -ve for virtual image on left

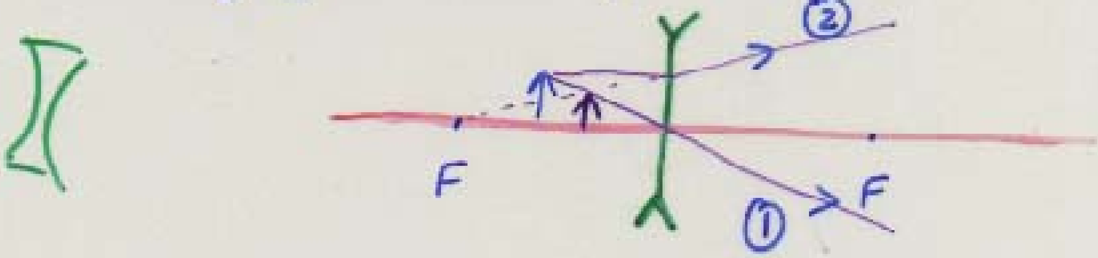
f : +ve for converging lens (convex shape)
  -ve for diverging lens (concave ").

Definitions: optical axis, optical center, focal length

Examples with 5cm focal length lenses

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

1. Diverging lens with $f = -5$cm

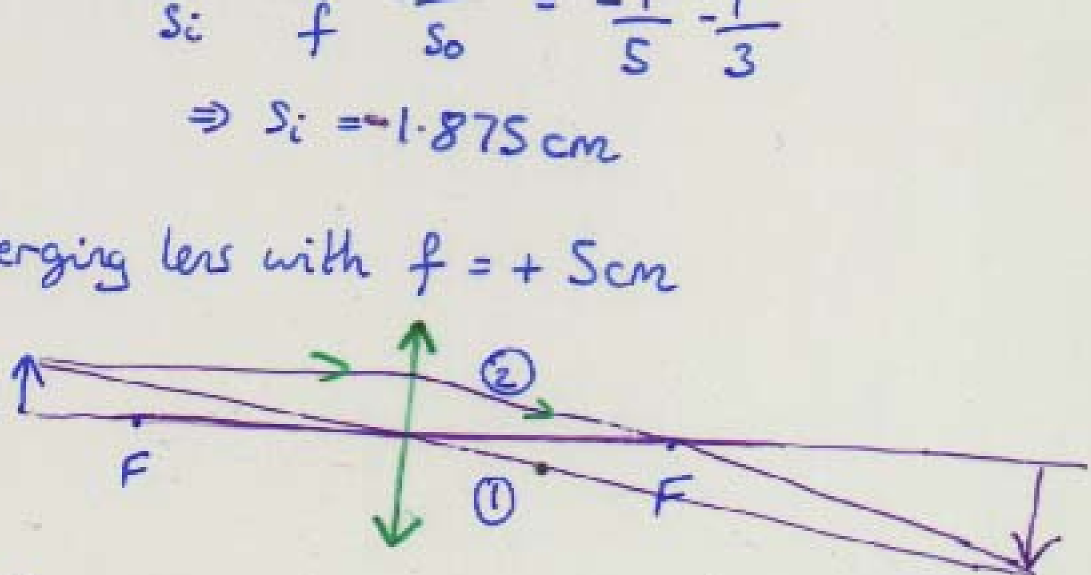


If $S_o = 3$cm, image is virtual, erect, minified

and $$\frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} = \frac{-1}{5} - \frac{1}{3}$$

$$\Rightarrow S_i = -1.875 \text{ cm}$$

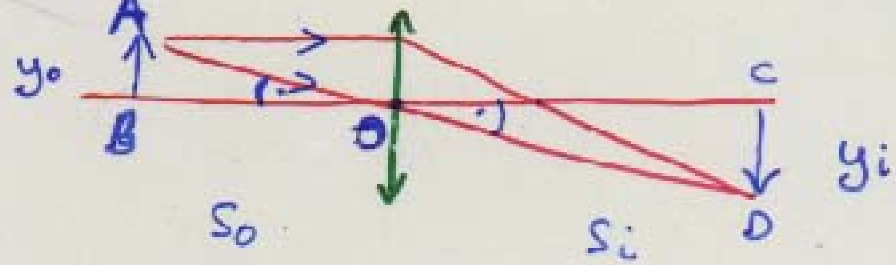
2. Converging lens with $f = +5$cm



If $S_o = 8$cm, image is real, inverted, <u>magnified</u>

with $$\frac{1}{S_i} = \frac{1}{5} - \frac{1}{8} \Rightarrow S_i = 13.33\text{cm}.$$

Transverse Magnification M_T



In dimension ⊥ to optical axis, image can be magnified ($y_i > y_o$) or minified ($y_i < y_o$) - or stays the same size.

From similar Δs AOB, COD : $\dfrac{y_o}{S_o} = \dfrac{y_i}{S_i}$

Define $M_T \equiv \dfrac{y_i}{y_o} = \dfrac{\text{image height}}{\text{object height}}$

$$\boxed{\therefore\ M_T = -\frac{S_i}{S_o}}$$

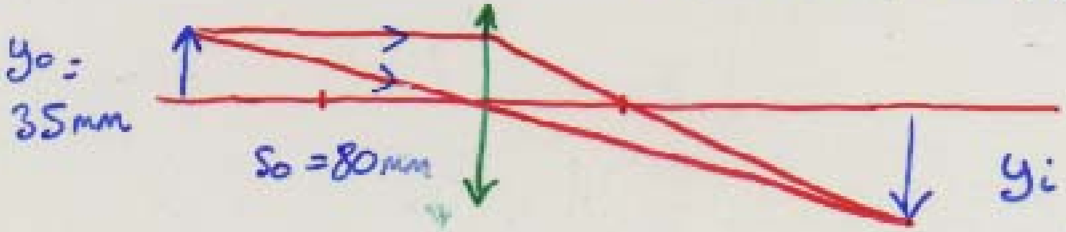
M_T negative : inverted image (always real)

positive : erect image (always virtual)

Example: Slide Projector with $f = 60\,mm$

Place a 35mm wide slide at $s_o = 80\,mm$

- what is the distance and size of the projected image?



$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{60} - \frac{1}{80} \Rightarrow s_i = 240\,mm$$

∴ screen should be placed 240 mm away.

Transverse magnification $M_T = \frac{-s_i}{s_o} = -\frac{240}{80} = \underline{-3}$

∴ image height $y_i = (-)3y_o = (-)3 \times 35\,mm$

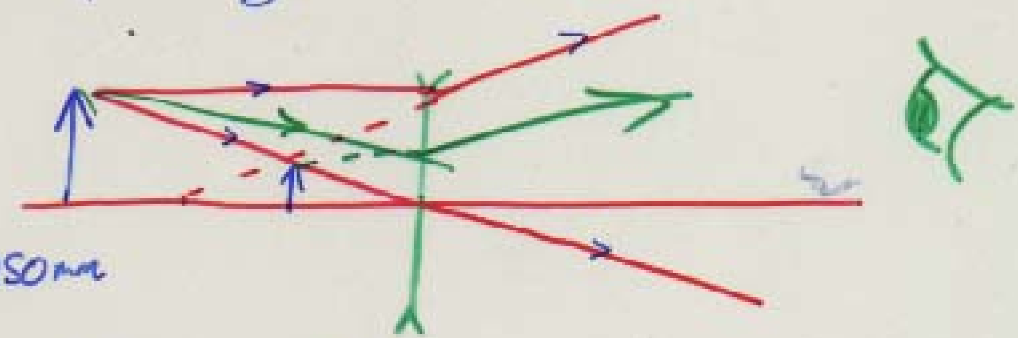
$= \underline{(-)\,105\,mm}$ (inverted)

Note: As object (slide) brought closer to focus,
rays become more parallel $\Rightarrow s_i \to \infty$
and $M_T \uparrow$; cannot change image height + distance separately
with a single (fixed focal length) lens.

$$\left[\ s_i = \frac{f s_o}{s_o - f}\ ,\quad M_T = \frac{-f}{s_o - f}\ \right]$$

Example: A diverging lens used in a door's peephole is required to reduce (minify) a person's face by a factor of 4 when they stand 250mm from the door.

- What should focal length be?



$S_o = +250\,mm$

Image: erect, virtual, minified

We need $\underline{M_T = \frac{-S_i}{S_o} = +1/4}$ when $S_o = 250\,mm$

i.e. $S_i = -62.5\,mm$

$\therefore$ Focal length $\frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{250} + \frac{1}{-62.5}$

$\Rightarrow f = -83.33\,mm$

Note: Since image virtual, cannot be recorded on a screen (unless another lens used; e.g. eye).

- Can show $M_T = \frac{-f}{S_o - f}$: changes with object distance ...

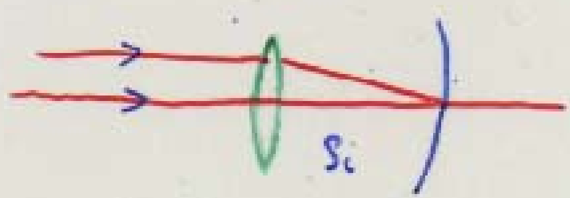
As $S_o \to \infty$, $M_T \to 0$

$S_o \to 0$, $M_T \to +1$

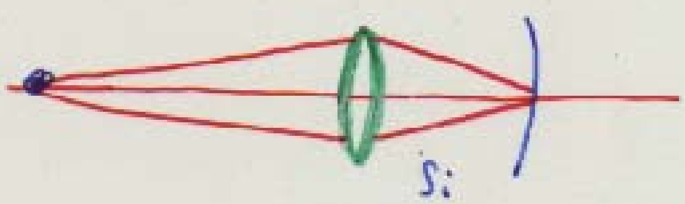
Single Lens Applications

1. Human Eye: (actually a combination of cornea, aqueous humor, lens, vitreous humor)

Eye functions as a <u>camera</u> with image distance S_i (lens to retina) fixed, variable focal length



eye lens relaxed, $f = S_i$



lens muscles contract to reduce f

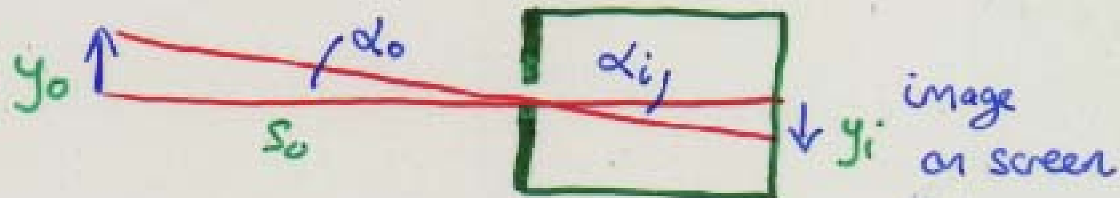
far point: furthest distance seen clearly

near point: shortest " " " ~ 25 cm

but can be ~ few meters by age 60!

2. Cameras, Angular Size

Pin hole camera just uses small hole to focus light rays:



Magnification y_i/y_o depends on hole-screen distance s_i.

Note: all object distances are in focus!
(infinite depth of field)

Angular Size of object, image defined as angle subtended ~~by~~ at the camera

i.e. ~~α_o~~ given by $\tan\alpha_o = \frac{y_o}{s_o}$

If angle α_o is small ($\frac{y_o}{s_o} << 1$), $\alpha_o \approx \tan\alpha_o = \frac{y_o}{s_o}$

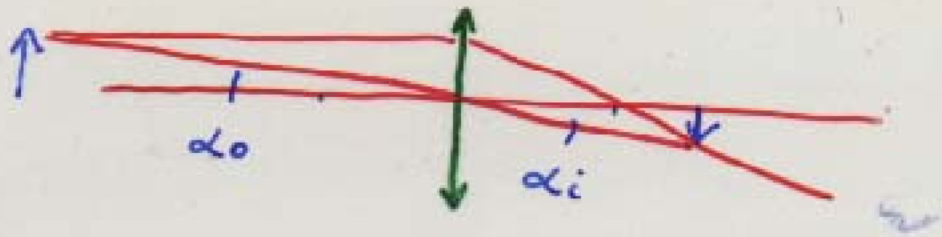
must use radians

eg. Sun and moon both have $\alpha_o = \frac{1}{2}^\circ = \frac{\frac{1}{2}\pi}{180}$ radians

$= 8.7 \times 10^{-3}$ rad.

Cameras cont/d

Use lens to focus more light onto film



- Object is beyond $2f$, $s_o > 2f$

  $\Rightarrow$ image between f and $2f$, so can change lens-film distance to accommodate focus (cf. eye)

- Angular sizes of object, image $\alpha_o = \alpha_i$

Image distance $\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{s_o - f}{f s_o}$

i.e. $s_i = \frac{s_o f}{s_o - f} \Rightarrow M_T = -\frac{s_i}{s_o} = \frac{-f}{s_o - f}$

e.g. for sun, moon $s_o \to \infty$ so $M_T \to 0$, not useful

<u>But</u> can use angular size to find image height

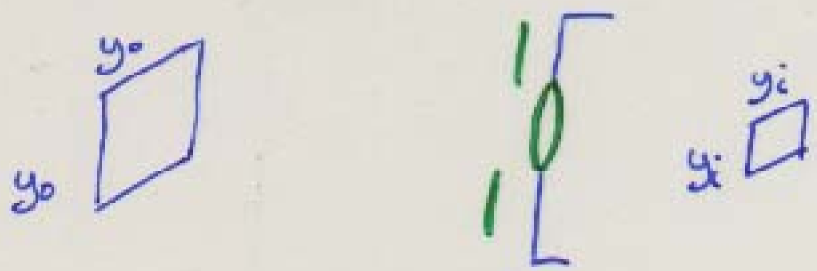
$\alpha_i = \alpha_o \simeq \tan\alpha_i = \frac{y_i}{s_i} = 8.7 \times 10^{-3}$ rad

e.g. for eye with $s_i = 17.1$mm, moon has $y_i = 0.2$mm !

f-number, irradiance, exposure

Cameras concentrate light from a large object (e.g. a square of side y_o) into a smaller area on film.

c.f. burning glass!



energy / unit area arriving on film $\propto \frac{1}{y_i^2}$

For objects beyond $\gg 2f$, $y_i \propto f$

also, energy passing through lens, diameter $D \propto \frac{\pi D^2}{4}$

$\therefore$ irradiance onto film $\propto \frac{D^2}{f^2}$

F-number is defined as $\frac{\text{focal length}}{\text{lens diameter}} = \frac{f}{D}$

$\Rightarrow$ irradiance ("brightness") on film $\propto \frac{1}{(\text{f-number})^2}$