Reading Quiz 4

1. Doubling the speed of a particle (e.g. neutron) :

a) doubles its energy $E \; \frac{1}{2}mv^2$

$\lambda = \frac{h}{p} = \frac{h}{mv}$

b) doubles its wavelength

c) halves its wavelength

d) none of the above.

2. In an atom, the electron's wavefunction is found to be certain wavefunction $\psi_{n l m_l m_s}$ The quantity $|\psi|^2$ tells us:

a) the electron's charge density X

b) the electron's momentum

c) the electron's position

d) the electron's probability distribution.

3. The Pauli exclusion principle holds that no 2 electrons in an atom may have the same:

a) energy X

b) spin X

c) wavefunction Ψ_{nlm_s}

d) all of the above X

4. Heisenberg's Uncertainty Principle prevents us from precisely measuring a particle's:

a) momentum

b) momentum and energy together

c) momentum and position together

d) energy and position together

$\Delta p \Delta x \geq \hbar/2$

$\Delta E \Delta t \geq \hbar/2$

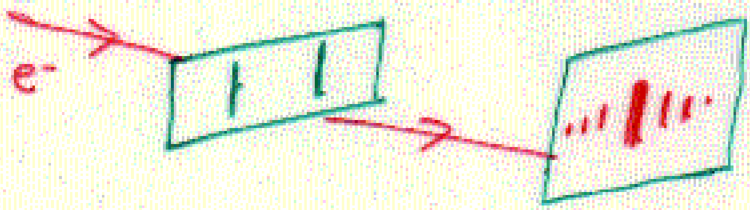
Matter Waves = Probability Waves

1926: Schrödinger : A "wave function" $\Psi(x,t)$ describes evolution of a matter wave, e.g. in electron diffraction

$$|\Psi(x)|^2 = \text{probability of observing particle at position } x.$$

$\therefore$ As for photon, particles also show non-local (wave) behavior !

i.e. a particle must "see" both slits in Young's expt.



(Proof: block one slit → destroys fringe pattern)

We can watch interference pattern build up electron-by-electron

⇒ non-deterministic, cannot predict where e^- will end up!

cf. Newtonian mechanics, where

identical initial conditions

→ identical results.

Electrons in Atoms: Quantum Probability States

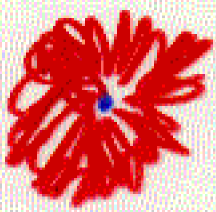
In general, total energy $E = K + U(r)$ with $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

i.e. $$\frac{p^2}{2m} + U(r) = E,$$

Using $p = h/\lambda$, solution is a set of wave functions

$\Psi(r, \theta, \phi) \Rightarrow$ "cloud" of probability around atom

Bohr radius



(Fig 29.5)

$|\Psi|^2$ gives probability, a_0 now just "averag[e] radius"

Solutions Ψ_{nlms} identify quantum state with 4 quantum numbers

n : determines energy E_n (c.f. Bohr model)

l : angular momentum (c.f. elliptical orbits)

m : azimuthal quantum number, projection L_z of $\vec{L}$

s : e^- has "spin" (c.f. Earth's spin) $= \pm\frac{1}{2}\hbar$

- either up $\uparrow$ or down $\downarrow$.

Pauli Exclusion Principle: Only 1 electron can occupy a particular (n, l, m, s) state

$\rightarrow$ electronic (chemical) structure!

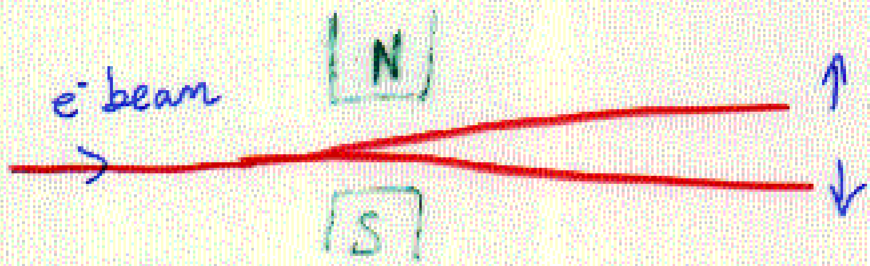
Superposition of States

Schroedinger's Wave function Ψ : solution to wave eqn

$\therefore$ Superposition $\Psi = \alpha\Psi_1 + \beta\Psi_2$ also a solution
(or $\alpha\Psi_1 - \beta\Psi_2$)

$|\Psi|^2 \Rightarrow$ prob. of observing particle in either state

e.g. pass electron through mag. field to measure spin



Before measurement, $\Psi = \Psi_\uparrow + \Psi_\downarrow$ for any electron (50/5

After measurement, Ψ "collapses" to either $\Psi = \Psi_\uparrow$ (100%)
or $\Psi = \Psi_\downarrow$ (100%.

$\therefore$ measurement changes Ψ itself !

Heisenberg Uncertainty Principle

Cannot observe both position and momentum of particle simultaneously with infinite precision (c.f. Newton)

Momentum $\quad p = \frac{h}{\lambda}$ — wavelength extended in x $\Rightarrow$ "fuzzy" location.

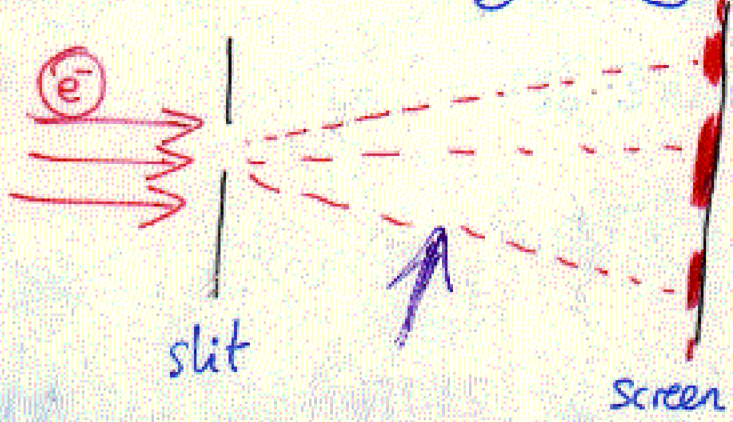
Heisenberg: $\quad \Delta p \Delta x \geq \frac{\hbar}{2} = \frac{h}{4\pi}$

Similarly, from $\quad E = hf$ — freq. extended over cycles in time

$$\Rightarrow \quad \Delta E \Delta t \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$

$\therefore$ If we localize particle in space, as $\Delta x \downarrow$, $\Delta p \uparrow$
$\rightarrow$ unable to predict future position.

e.g. Constrain Δx by passing beam through a slit



As $\Delta x \downarrow$, pattern width $\propto \Delta v_x \propto \Delta p_x$

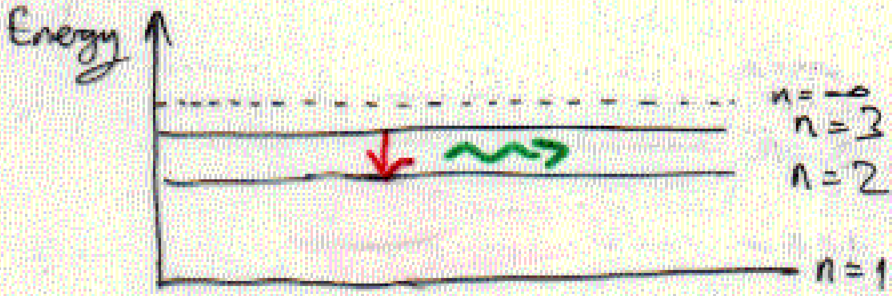
(Note: any attempt to measure p_x for particle destroys pattern

Heisenberg cont'd:

$\Delta E \, \Delta t \gtrsim \frac{\hbar}{2}$ ⇒ finite widths of spectral lines

i.e. if transition takes a time $\Delta t \sim$ few ns,

⇒ uncertainty in photon energy $\Delta E = E_2 - E_1 \gtrsim \frac{\hbar}{2\Delta t}$



e.g. for the Hα ($n = 3 \rightarrow 2$) line of hydrogen:

$$E = E_3 - E_2 = 13.6\,\text{eV}\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.88\,\text{eV}$$

Av. Lifetime of $n=3$ state $= 10^{-8}$ s

$$\Rightarrow \text{spread in } \Delta E \gtrsim \frac{\hbar}{2\Delta t} = \pm 3.3\times10^{-8}\,\text{eV}$$

So since $\lambda = \frac{hc}{E} = 656.3$ nm

$$\text{Spread } \Delta\lambda \approx \frac{d\lambda}{dE}.\Delta E = \frac{hc}{E^2}\Delta E = \underline{1.15\times10^{-5}\,\text{nm.}}$$

"natural line width"

(Thermal Doppler effect broadens line even more).

Consequences of Heisenberg U. P.

e.g. ① Confining a particle within Δx

$\rightarrow$ "spread" in momentum $\Delta p \geq \frac{\hbar/2}{\Delta x}$

e.g. If $\Delta x \approx$ Bohr radius a_0, can show that

$p = mv$
$K = \frac{1}{2}mv^2$

min. KE of electron $\frac{(\Delta p)^2}{2m_e} \geq \frac{\hbar^2}{4a_0^2} \cdot \frac{1}{2m_e}$

$$= \frac{h^2}{16\pi^2 a_0^2}$$

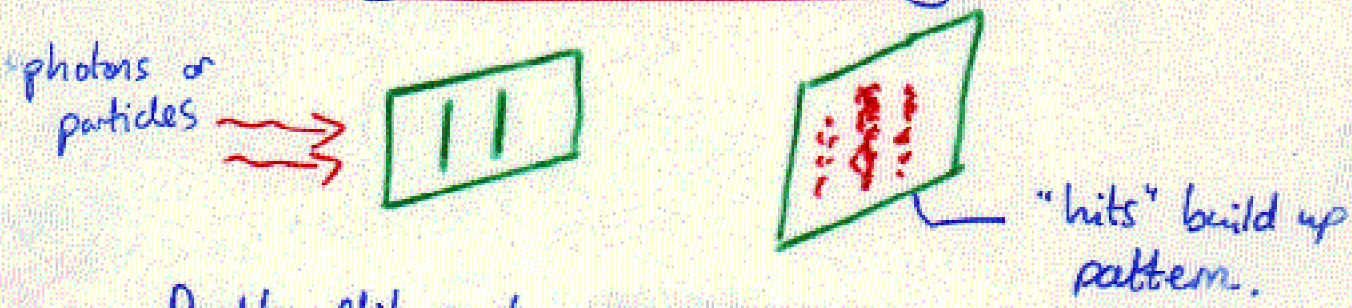
$$\geq 3.38\,eV$$

.... which is true since $K = -E = 13.6\,eV$ for hydrogen.

$\therefore$ lowest state of H atom is a min. energy state <u>and</u> ~ a "minimum uncertainty" state.

② Any particle confined to known Δx has $\Delta p > 0$, even at zero Kelvin : "zero-point energy"
(even if we cannot extract this energy from system)

Probability Waves and Intensity



Double-slit expt.

Use a faint source, e.g. 1 photon or particle/s, watch "hits" on screen build interference pattern.

- Cannot predict where $\left\{ \begin{matrix} \text{photon} \\ \text{particle} \end{matrix} \right\}$ ends up

  i.e. identical start conditions $\not\Rightarrow$ identical outcomes
  (non-deterministic)

- Pattern builds up gradually; each $\left\{ \begin{matrix} \text{photon} \\ \text{particle} \end{matrix} \right\}$ "self-interferes" to produce pattern: must pass through <u>both</u> slits!

- Irradiance across screen

$$I(x) = c\varepsilon_0 \underbrace{(E(x))}_{\text{electric field}}^2 = \underbrace{N(x)}_{\text{\# photons/s at } x} hf$$

$\Rightarrow$ Probability of photon at x $\quad P(x) \propto (\text{amplitude})^2$
particle " " $\quad P(x) \propto |(\psi)|^2$.

Einstein hated non-deterministic nature of Q.M.

Proposed instead: particle or photon has <u>definite</u> position and momentum, just "inaccessible" to us, i.e. a "<u>Hidden Variable theory</u>"...

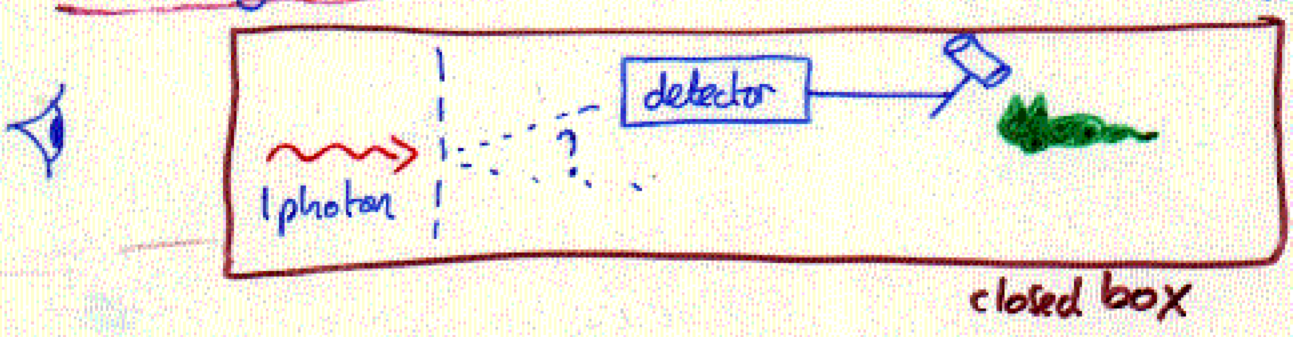
"Gott würfelt nicht" – God does not play dice!

Einstein was wrong!!! We now know that particles + photons have <u>no</u> definite location until we observe them.

Then, $|\psi(x)|^2 \Rightarrow$ prob. of observing particle at x.

$\Rightarrow \psi(x)$ <u>changes</u> after observation, i.e. observer is now linked to experiment's outcome

<u>Schrödinger's Cat</u>: $\psi_{cat} = 0.5\,\psi_{alive} + 0.5\,\psi_{dead}$



Open box $\Rightarrow$ "collapses" wave function ψ!