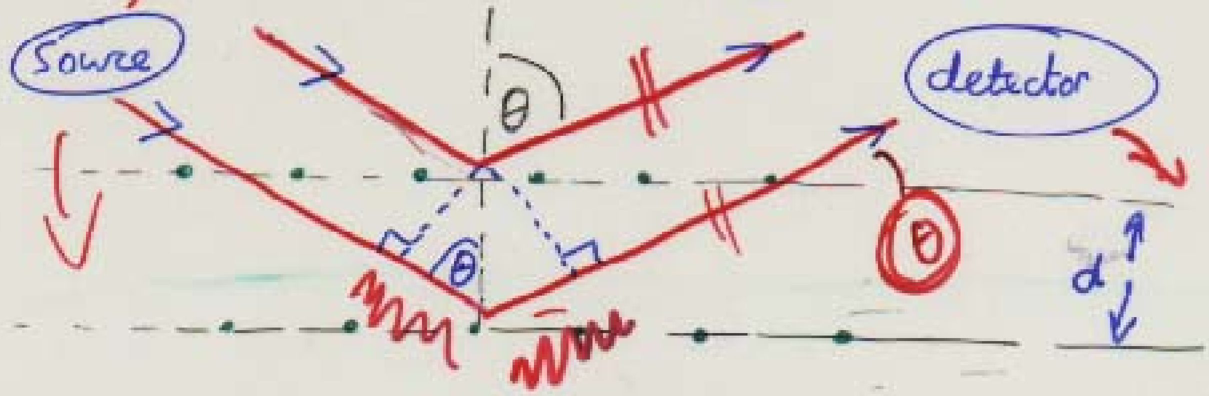
Measuring X-rays : Bragg Diffraction

Since X-rays have $\lambda \sim 0.1\,nm$, atoms in crystals planes act like a diffraction grating :



Waves reflected from adjacent planes have $\theta_i = \theta_R \; (= \theta)$

But also have path length diff. $= 2d\cos\theta$

$\therefore$ For const. interference, must have

$$2d\cos\theta = m\lambda \quad : m = 1, 2, 3, \ldots$$

(Note: Hecht uses different definition for θ)

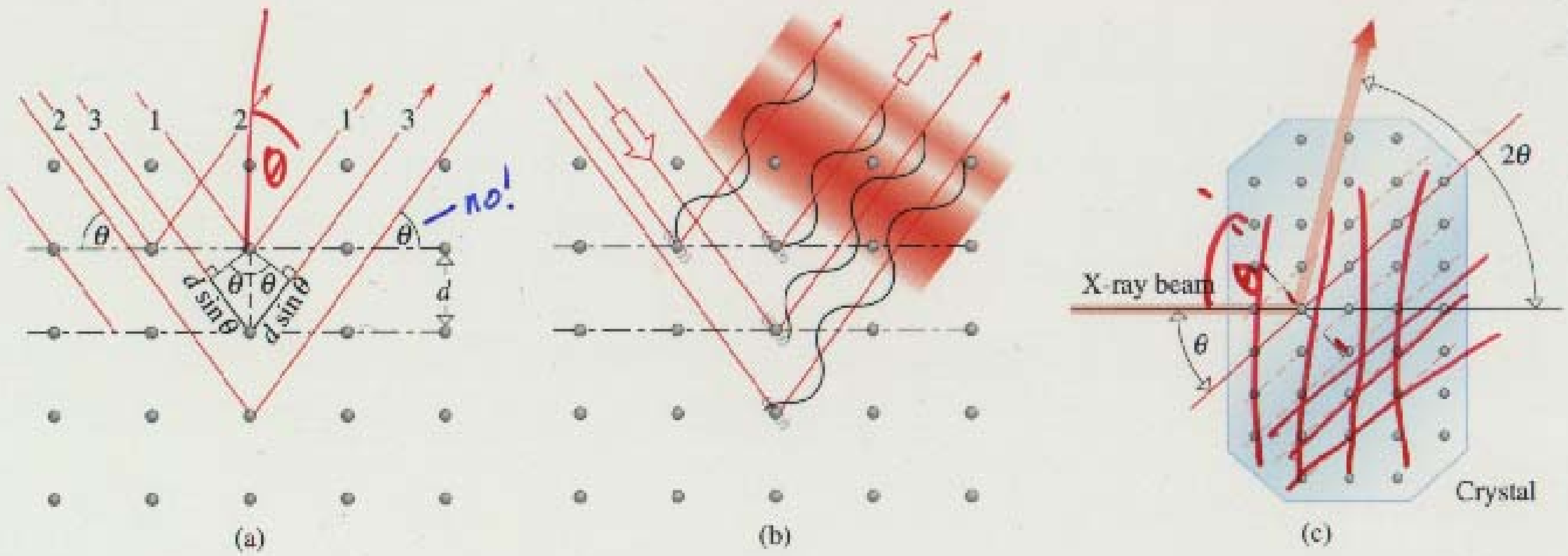
$\rightarrow$ "bright spot" in X-rays at $\theta = \cos^{-1}\left(\frac{m\lambda}{2d}\right)$

For other θ, X-rays from many planes all arrive slightly out-of-phase $\rightarrow$ cancel out (destructive)

$\therefore$ Bragg diffraction pattern depends on crystal spacings (+ structure, e.g. graphite vs. diamond)

Figure 27.8

Scattering of X-rays from the planes of a crystal



Bragg Diffraction: Applications

1. X-ray crystallography. Use known λ (e.g. X-ray line from atomic target). Measure θ

$$\rightarrow \quad d = \frac{m\lambda}{2\cos\theta}$$

. Used to find structure of DNA

2. Known crystal (d) can be used to isolate specific λ from X-ray continuum (e.g. bremsstrahlung)

In both cases, only works for

$$|\cos\theta| = \frac{|m|\lambda}{2d} \leq 1$$

i.e.

$$\lambda \leq 2d/m$$

(For EM waves with $\lambda >> d$, crystal just acts like a mirror-smooth surface)

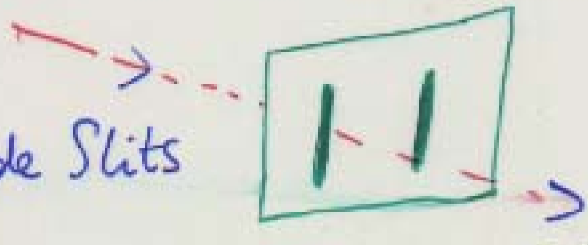
Wave - Particle Duality of Light

Example: Photoelectric effect, spectral lines, x-ray production → EM energy created/destroyed in "photon" packets .....

.... but still show <u>wave</u> properties (λ, interference, diffraction)

How does this work???

Young's Double Slits



<u>Particle:</u> passes through one slit or other ⇒ sharp shadow

Wave: non-local, must pass through <u>both</u> slits

⇒ interference pattern.

Photon ≃ "Wave Packet"

can be localized in space

Bohr: "Evidence obtained under different experimental conditions ..... <u>cannot</u> be comprehended within a single picture."

Wave / Particle Duality .... of Matter ? (ch. 29)

1923 : de Broglie used Special Relativity ($E = mc^2$) to propose that particles have associated wavelength :

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Particles move along "pilot waves" in space , depending on momentum $p = mv$

1925 : Davisson + Germer : bombarded Ni crystal with e^- beam.
- observed a Bragg diffraction pattern !

e.g. for e^- with accel voltage 100V, ⇒ energy = 0.1 keV

$\frac{1}{2}mv^2 = eV_a$ ⇒ speed $v = \sqrt{\frac{2eV_a}{m}}$, momentum $p = mv$

⇒ wavelength of electron in beam $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_a}}$

i.e. $$\lambda = \frac{6.6 \times 10^{-34}\ Js}{\sqrt{2 \times 9.1 \times 10^{-31} kg \times 1.6 \times 10^{-19} C \times 100 V}} = 0.117\ nm$$

$= J/C$ ~ X-ray wavelength
~ crystal spacing .

∴ This is a real effect. Also done with beams of neutrons, H atoms, He atoms ..... Page 1139, fig 29.3

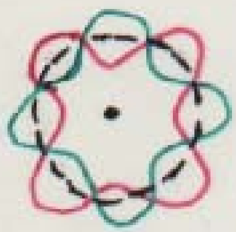
① de-Broglie Wavelength explains Bohr atom "Stationary States"

1. Bohr's Postulate: e^- in atom will not radiate if angular momentum $mvr = \frac{nh}{2\pi} = n\hbar$ ← "h-bar"

2. Using $\lambda = \frac{h}{mv}$, eliminate (mv) ⇒ $\frac{h}{\lambda} r = \frac{nh}{2\pi}$

   i.e. $n\lambda = 2\pi r$

∴ "Allowed" orbits can fit an integer # of wavelengths around the H nucleus :



e.g. $n=4$ orbit (Hecht fig. 29.1)

⇒ a "standing wave" with no flow of EM energy

② Electron-microscope : uses low energy e^- beam instead of X-rays. Can probe features on surfaces ~ λ

i.e. $\lesssim 0.1\,nm$ ⇒ can "see" individual atoms!

Note: accel voltage for e^- ~ 100V (c.f. 10kV for X-rays)
So can use e^- beam for "delicate" objects