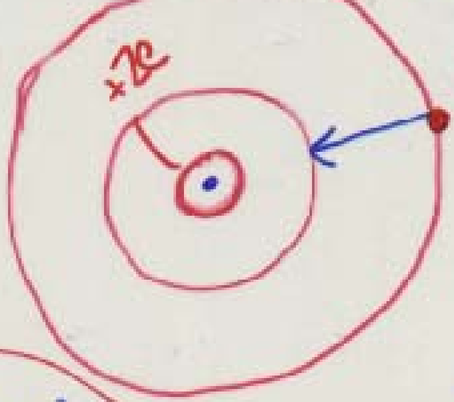
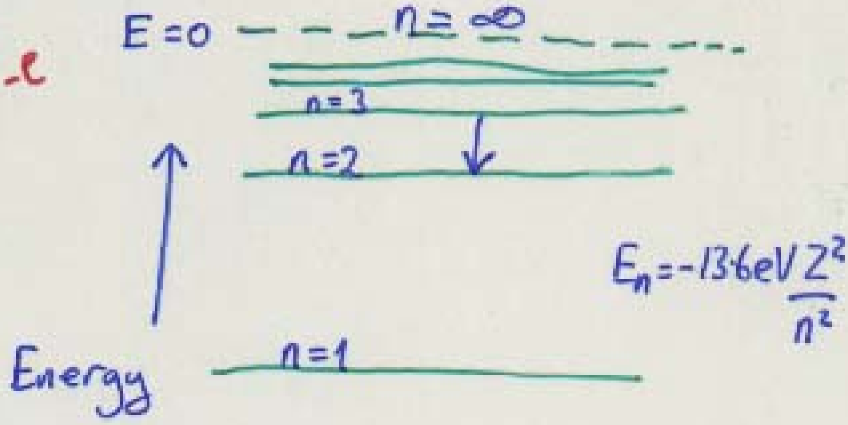
Quantized Energy Levels : Atomic Spectra.



$$r_n = a_0 \frac{n^2}{Z^2}$$



For allowed orbital radius r_n, $E = \overset{-\frac{1}{2}U}{K} + U = +U/2 = \dfrac{-Ze^2}{8\pi\varepsilon_0 r_n}$

(Note : $U < 0$ so $E < 0$: <u>bound</u> orbit)

With $r_n = \dfrac{a_0 n^2}{Z}$ $\Rightarrow$ $E_n = \dfrac{-Z^2}{n^2}\left(\dfrac{e^2}{8\pi\varepsilon_0 a_0}\right)$

$$\therefore \quad E_n = -13.6\text{eV}.\,\frac{Z^2}{n^2}$$

Transition between n_1, n_2 results in a photon of energy

$$E_{12} = \frac{hc}{\lambda_{12}} = -13.6\text{eV}.\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda_{12}} = \left(-\frac{13.6\text{eV}}{hc}\right) Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

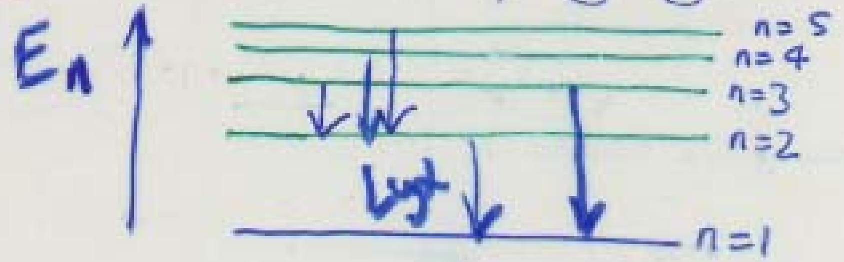
Rydberg constant $= 1.1 \times 10^{-7}\,\text{m}^{-1}$

Spectral Series : Hydrogen

$$\frac{1}{\lambda_{12}} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

When n_1 fixed, we get a series of lines close in λ by varying n_2

e.g. Balmer series of hydrogen : $n_1 = 2$



$H\alpha$ line ($n = 3 \to 2$) : $\lambda_{32} = 656.3$ nm red/pink
$H\beta$ ($n = 4-2$) : $\lambda_{42} = 486.1$ nm turquoise
$H\gamma$ ($n = 5-2$)

We also have: Paschen series ($n_1 = 3$) - infrared
Lyman " ($n_1 = 1$) - ultraviolet.

- Ionization energy for $n = 1 \to \infty$ = $13.6\,eV \cdot Z^2$

  For hydrogen ($Z = 1$), absorb photon with $\lambda < 0.912$ nm. $\frac{hc}{\lambda} = 13.6\,eV$

Other single-electron atoms have similar series

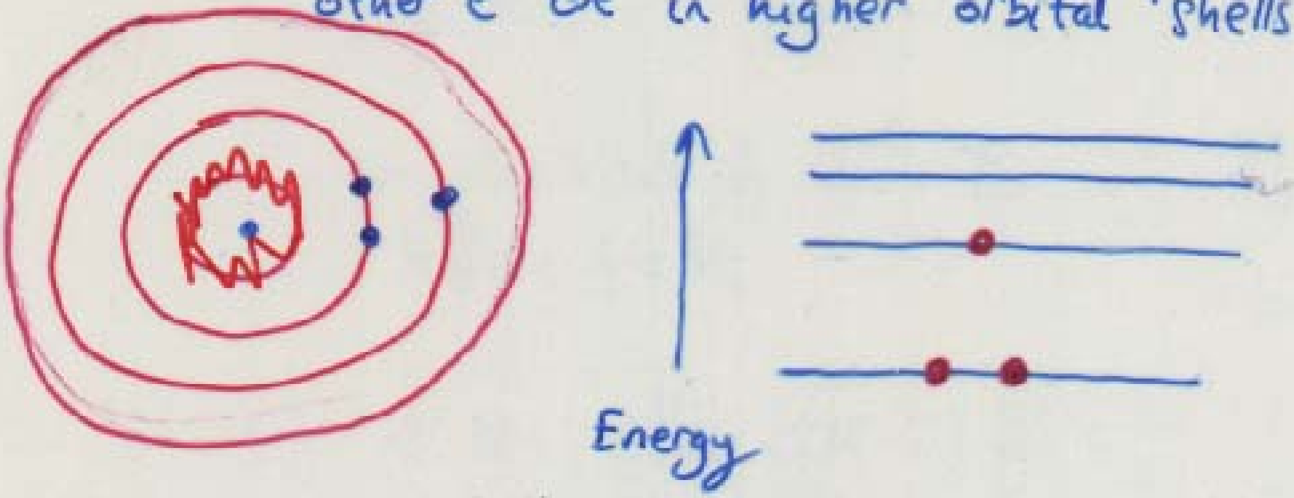
e.g. H, He^+, Li^{2+}, ...
$Z =$ 1, 2, 3

(use "reduced" nuclear charge $Z' < Z$)

as well as single "outer" electron atoms, e.g. Na

Multi-electron atoms : Pauli Exclusion Principle

→ only 2 e^- can occupy $n = 1$ state
other e^- are in higher orbital "shells", $n \geq 2$



e.g. Lithium $Z = 3$.

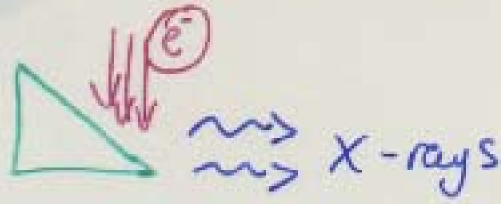
Heavier atoms (higher atomic # Z) fill shells according to the Pauli E.P.

→ chemistry !

X-ray Spectra : Production

1

1 Bremsstrahlung - beam of fast e^- smash into metal target :

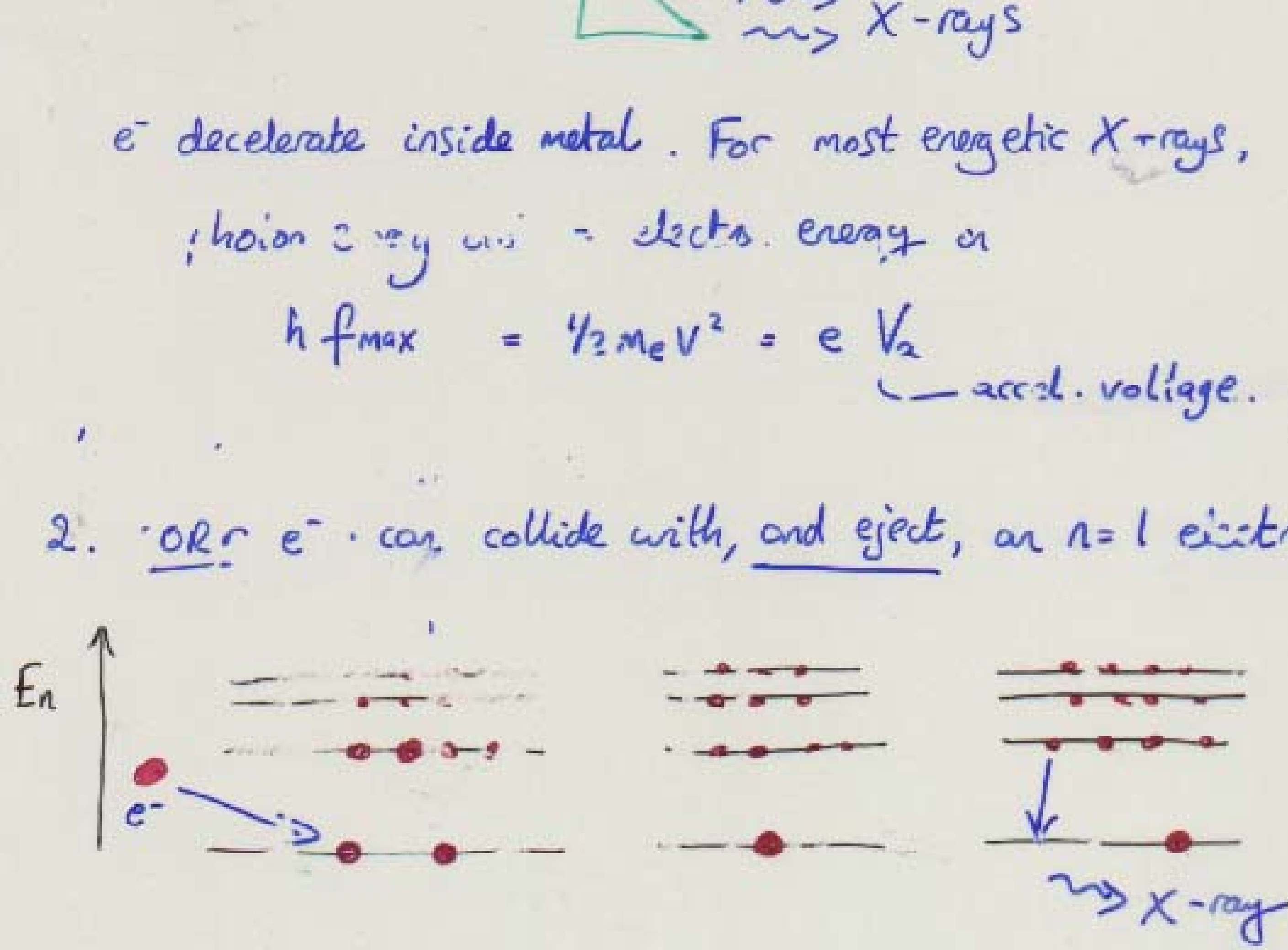


e^- decelerate inside metal. For most energetic X-rays, photon energy = electron energy or

$$h f_{max} = \tfrac{1}{2} m_e v^2 = e V_a$$

V_a — accel. voltage.

2. OR e^- can collide with, and eject, an $n=1$ electron :



then, $n=2$ e^- fills the vacancy by dropping $\rightarrow n=1$

emits photon : $\frac{hc}{\lambda} = E_2 - E_1 = 13.6\,eV (Z-1)^2 \underbrace{\left[\frac{1}{1^2} - \frac{1}{2^2}\right]}_{=3/4}$

"reduced nuclear charge" due to remaining $n=1$ electron

Example: An electron beam with $V_a = 10\,kV$

a Vanadium $(Z=23)$ target.

1. Bremsstrahlung X-rays have $hf_{max} = \dfrac{hc}{\lambda_{min}} = 10\,keV$

$$\Rightarrow \lambda_{min} = \frac{hc}{10^4 e} = \frac{(6.6\times10^{-34}\times 3\times10^{8})}{1.6\times10^{-19}}\cdot\frac{1}{10^4}$$

$$= 0.124\ nm$$

2. Strong emission line produced at

$$E_2 - E_1 = \frac{hc}{\lambda_{line}} = (Z-1)^2\cdot 13.6\,eV\left(\frac{3}{4}\right) \quad \text{with } Z=23$$

$$\Rightarrow \lambda_{line} = 0.251\ nm$$

Actual X-ray spectrum: