Origins of Quantum Theory (Hecht ch. 28, 29)

Thermal Radiation

~1850 Wedgewood : noticed all objects in furnace glow same color (depends on temperature, not composition)

Kirchoff : imagined an assembly of objects (different materials, colors) in thermal eq.m. in a closed cavity. Concluded:

- Poor absorbers of EM energy are also poor radiators. e.g. reflective surfaces: white, silver
(⇒ white-painted houses, igloos, thermal blanket)

- Best absorbers also best radiators of EM energy
e.g. black or dark surfaces (matte, not shiny)
(⇒ solar panels, asphalt blacktop vs. concrete roads)

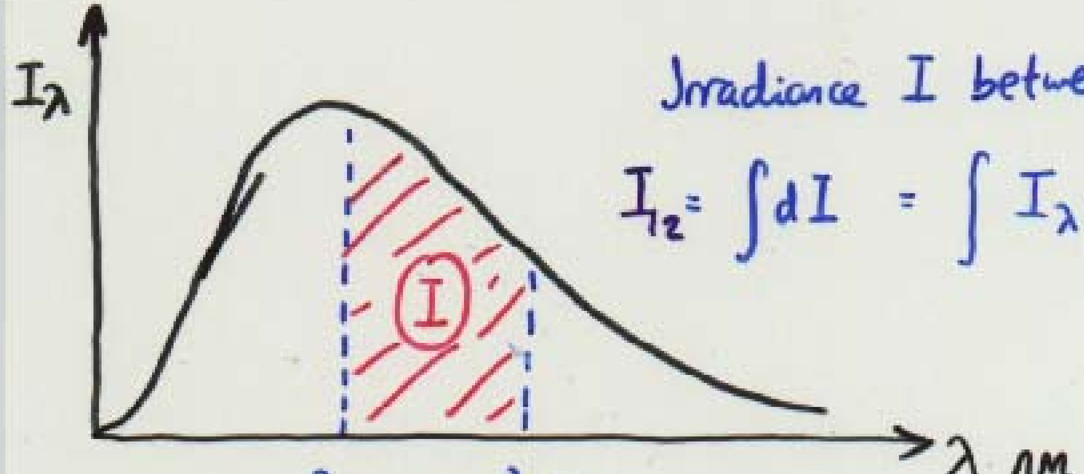
Define : "Black body" = Perfect Absorber

- absorbs all incident EM radiation
- therefore, must radiate to stay in thermal eq^m.
- spectrum ("color") depends only on T.

  c.f. objects in furnace : coals, pots, pokers all same color.

Can measure and plot spectral intensity of BB:

$I_\lambda = \frac{dI}{d\lambda}$ [W/m²/nm]: "irradiance per unit wavelength"



Irradiance I between λ_1, λ_2

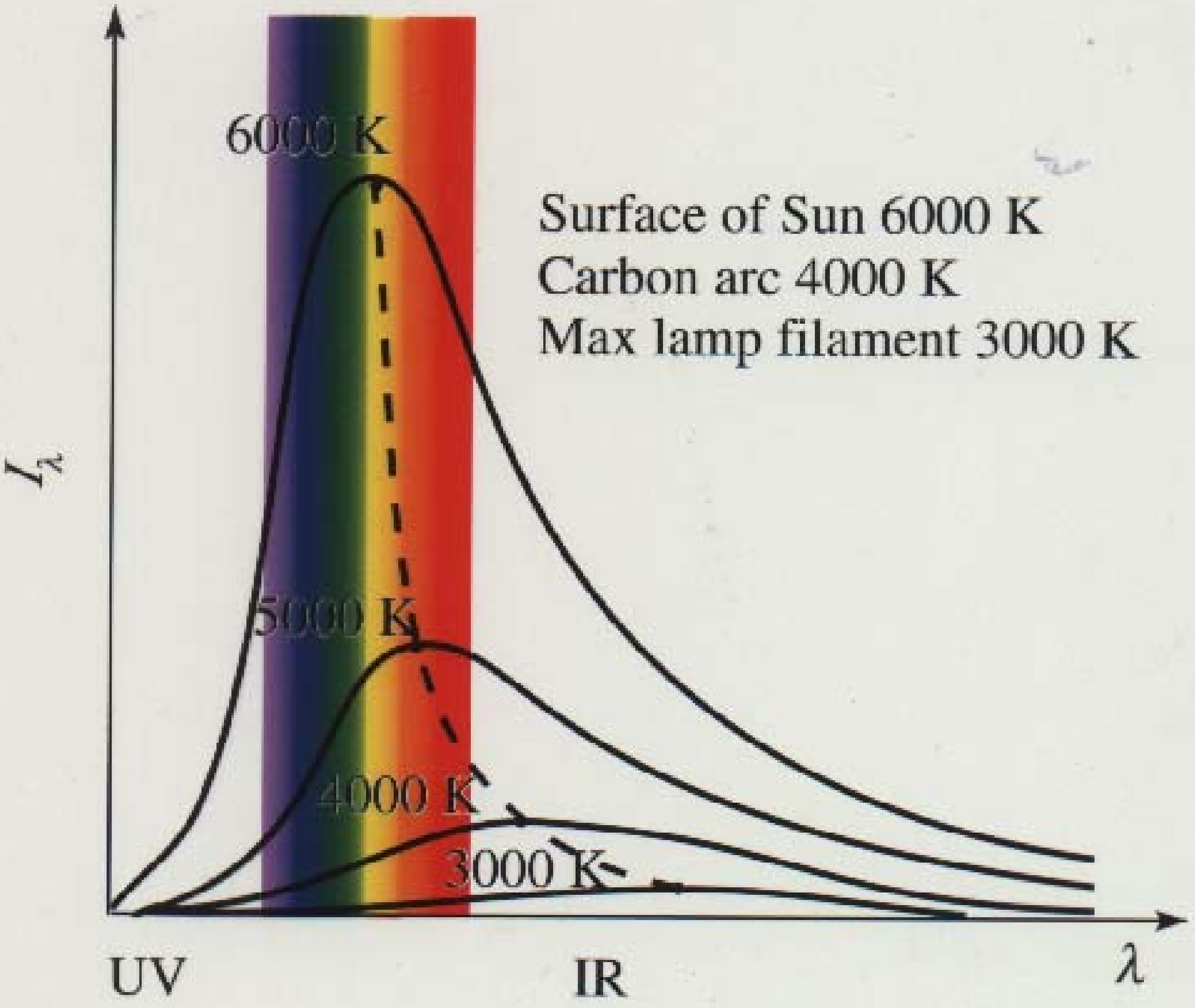
$I_{12} = \int dI = \int I_\lambda \, d\lambda$

Total power [W] of BB = surface area × irradiance

i.e. $P = A \int_0^\infty I_\lambda \, d\lambda$

Copyright © 2000 by Brooks/Cole
A division of Thomson Learning

Figure 28.3
Blackbody radiation curves



3

Examples of Blackbody Radiation

Many objects ≈ BB over some range of λ

e.g. human body reflects visible light, but absorbs + re-emits IR radiation

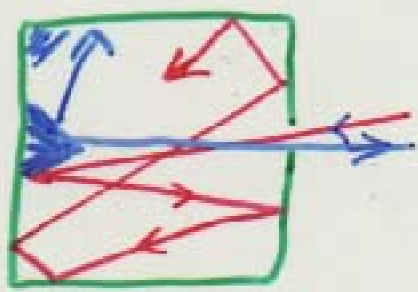
I_λ depends on T ⇒ thermal imaging.

Atmospheres of stars : opaque, dense gas absorbs + re-emits EM energy from interior

∴ color of star ⇒ temperature. (Sun: 6000K)

Light bulb filaments: glow due to high T

Small hole in thermal cavity ≈ perfect BB !



EM radiation reflected until absorbed.

- no incident light onto hole reflected back out

∴ perfect absorber

⇒ also perfect emitter !

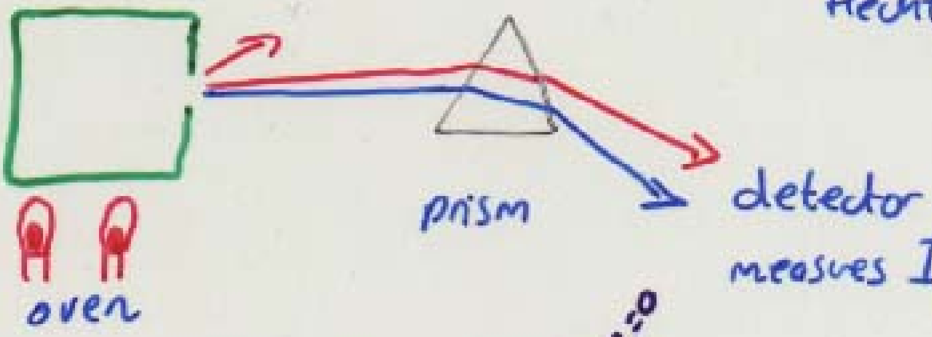
e.g. pupil of eye, empty soda can, ... 300K

Universe ≈ 2.7 K.

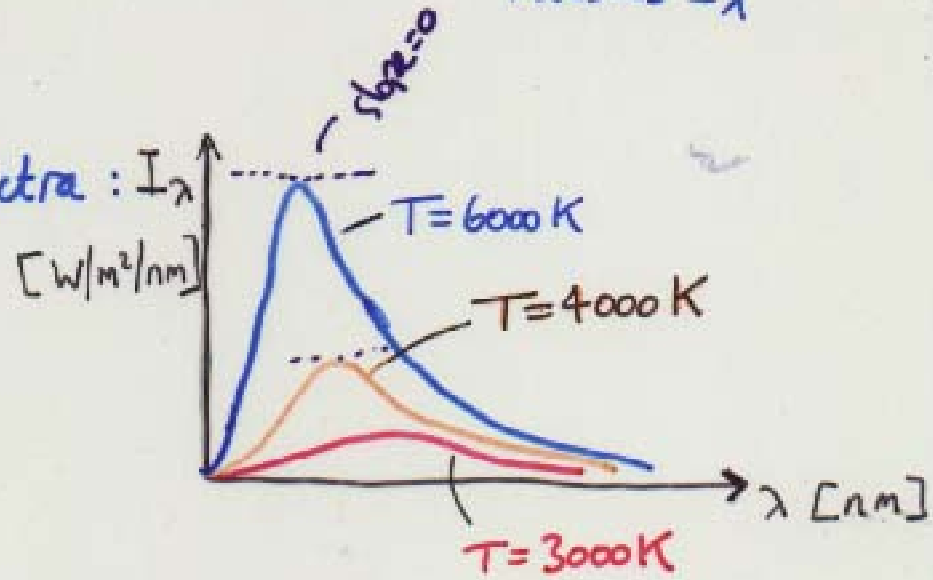
4.

Measuring I_λ for a Blackbody (Berlin ca. 1900)

Hecht fig. 30.1



Resulting Spectra:



Planck (1900) found : $I_\lambda = \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{e^{hc/\lambda kT}-1}\right]$

where $h \equiv$ Planck's constant $= 6.6\times10^{-34}$ Js.

From this formula:

1. Peak wavelength λ_{max} where $\frac{dI_\lambda}{d\lambda} = 0$

   $\Rightarrow$ Wien's Law : $\lambda_{max}.T = 3\times10^{-3}$ m. K

2. Total Power $P =$ area $\times \int_0^\infty I_\lambda . d\lambda$

   $\Rightarrow P = A\sigma T^4$: Stefan's Law

   ↑ Stefan's constant.

Examples:

Wien's Law: $\lambda_{max} T = 3 \times 10^{-3}\ m.K$

So as $T \uparrow$, wavelength of peak $I_\lambda \downarrow$

e.g. Colors of stars: measure I_λ, find many $\approx$ blackbodies, eg. Sun, $T = 6000\ K$

$\Rightarrow \lambda_{max} = 500nm$ (yellow)

c.f. "white dwarf" stars, $T = 10\,000K$ ($\Rightarrow \lambda_{max}$ in UV)

human body, room etc. $T = 300K \Rightarrow \lambda_{max} = 10\mu m$ (IR).

Stefan's Law: $P = \sigma A T^4$

e.g. to double a light bulb temp. from $1500K \Rightarrow 3000K$ requires power increase by factor $2^4 = 16$!

For distant stars, measure λ_{max} in spectrum $\Rightarrow T$.

Then, if we know distance d, and measure $I = \int I_\lambda d\lambda$

$$\Rightarrow P = (4\pi d^2) I = \sigma A T^4 = \sigma A \left(\frac{3 \times 10^{-3}}{\lambda_{max}}\right)^4$$

$I = \frac{P}{4\pi r^2}$

$\Rightarrow$ area A of star surface $\Rightarrow$ radius of star! (r)

$A = 4\pi r^2$

Planck's Explanation of I_λ for BB

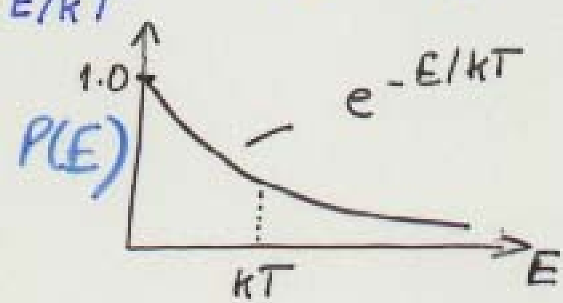
1. Classical Physics $\Rightarrow$ wrong answer !

Consider EM waves in cavity of side L [m]:

all waves have $\lambda = \frac{L}{2n}$

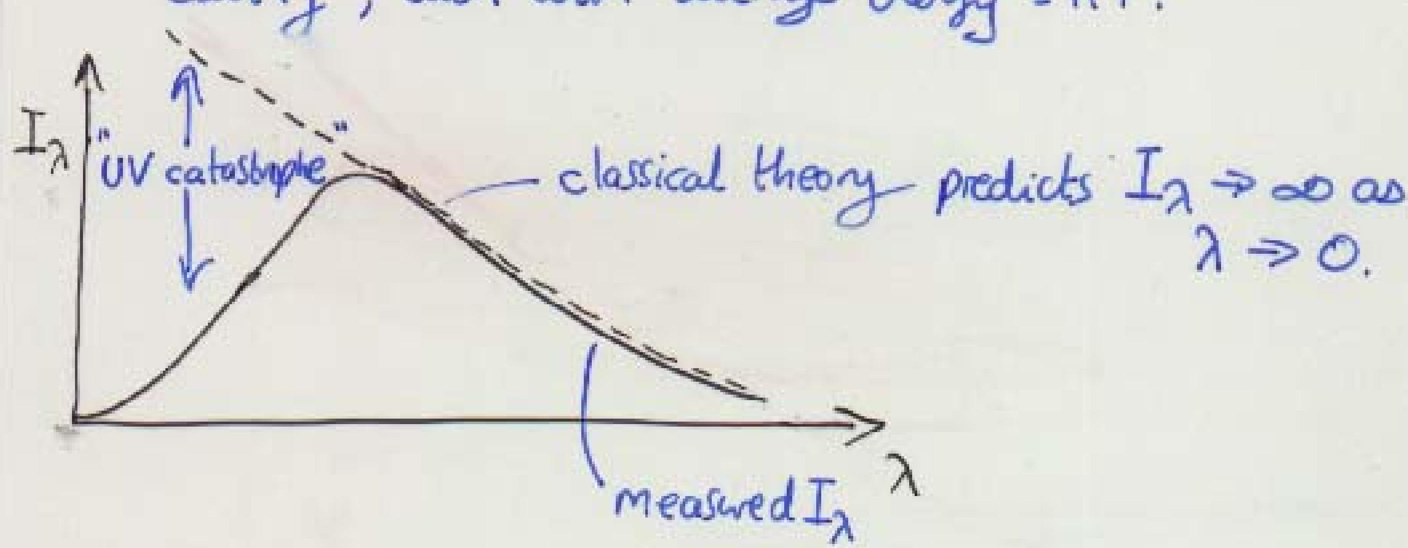
$n = 1, 2, 3, \ldots\ldots$

Boltzmann $\Rightarrow$ probability of wave having energy E is $P(E) \propto e^{-E/kT}$



$\Rightarrow$ average energy $= kT$

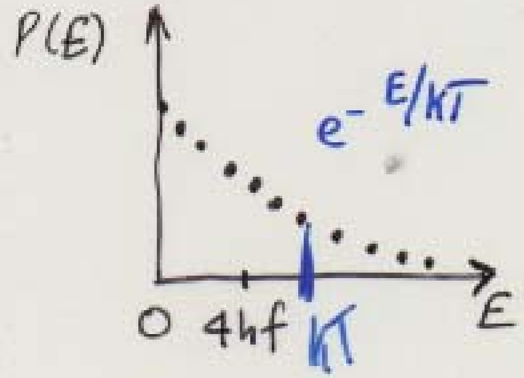
<u>But</u>, as $\lambda \downarrow$, we can fit more and more waves into cavity, each with average energy $= kT$.



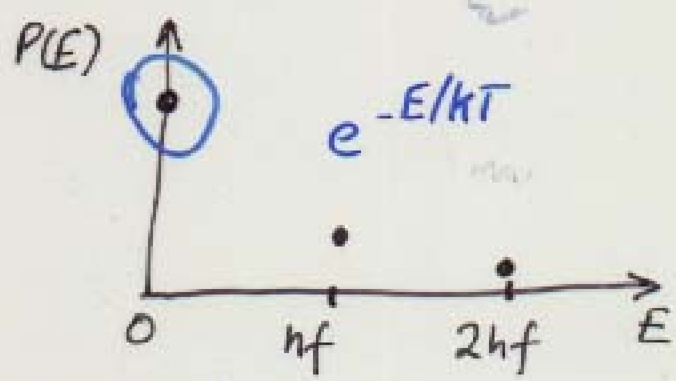
2. Planck's Solution to the "UV catastrophe"

Planck theorized that EM energy must be **quantized** such that energy of wave with wavelength λ (freq. $f = \frac{c}{\lambda}$) can only be

$$E_n = nhf \quad : \quad n = 0, 1, 2, \ldots$$



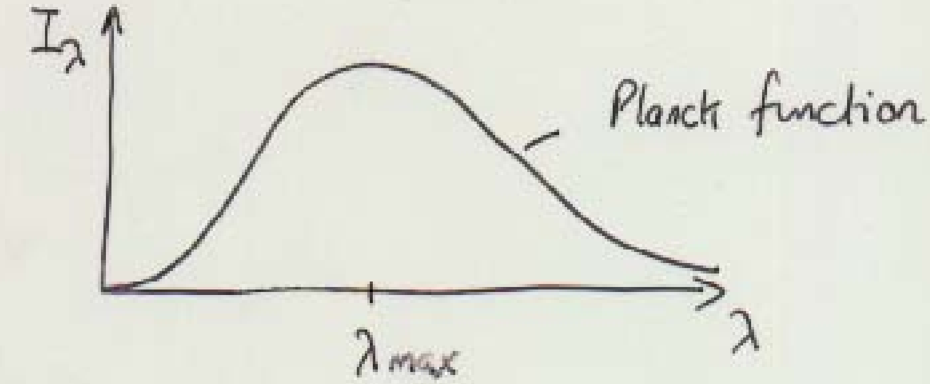
low f



high f

∴ now, low f waves have av. energy = kT as before, but high f waves have av. energy $\to 0$ (not kT) as $\lambda \to 0$.

∴ Intensity $I_\lambda \to 0$ as $\lambda \to 0$, as observed.



Bottom Line: We can explain thermal (BB) radiation only if EM waves are restricted to have energies $E_n = nhf$; $n = 0, 1, 2, \ldots$

Then they can absorb/emit EM energy in discrete multiples (quanta) of hf.

quantum of EM energy $E = hf$: photon

e.g. for $\lambda = 500nm$, how much energy in one "photon"

$$\boxed{E = hf = \frac{hc}{\lambda}} = \frac{6.6 \times 10^{-34}\,Js \times 3 \times 10^{8}\,m/s}{500 \times 10^{-9}\,m}$$

i.e. $E = 3.96 \times 10^{-19}\,J$.

$\therefore$ In a $10W = 10\,Js^{-1}$ flash light beam there are

$\sim \frac{10\,Js^{-1}}{3.96 \times 10^{-19}\,J} \simeq 2.5 \times 10^{+19}$ photons emitted per second !