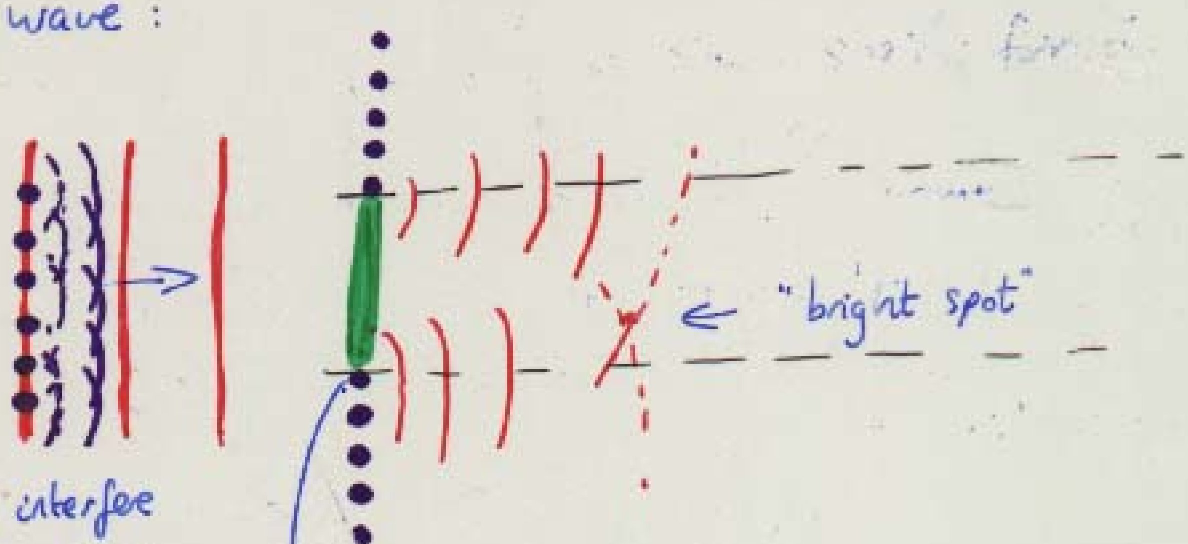
Diffraction - Light as a Wave

17th Century : Newton's "corpuscle" theory of light prevailed

"corpuscles" (with axles to explain refraction)

1817 : Fresnel showed light responds to obstructions like a wave :



wavelets interfere
⇒ advances wave front

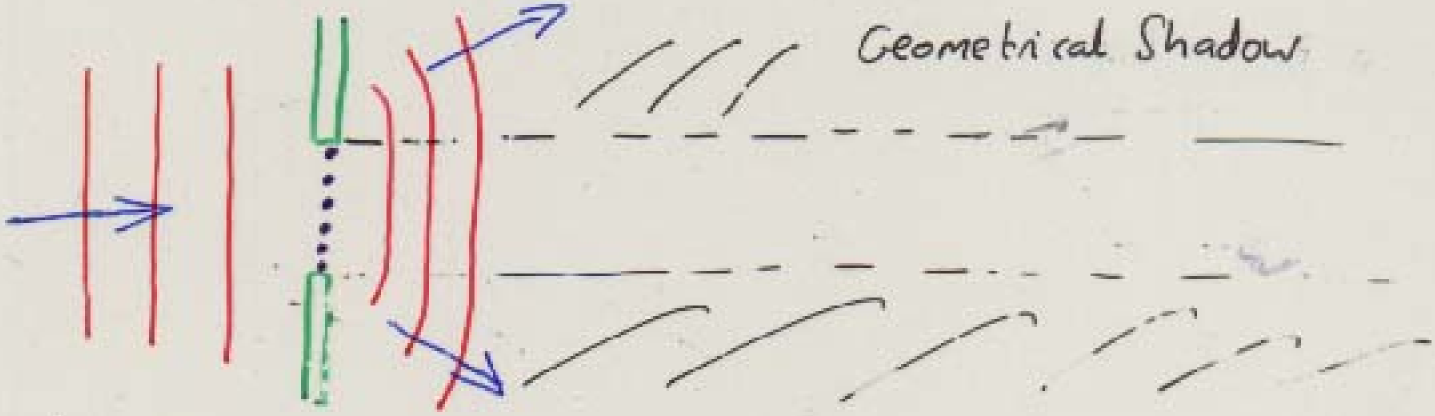
no wavelets inside disk to cancel spherical wave front

Predicted : bright spot formed "downstream" inside geometrical shadow of illuminated disk.

Confirmed !

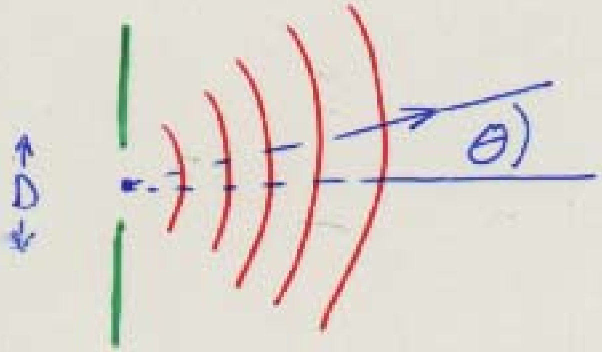
Diffraction = response ("bending") of wave due to an obstruction.

e.g. for aperture (c.f. ocean waves → harbor)



Huygens' wavelets → wavefront "bends" into geometrical shadow region : no sharp shadow formed.

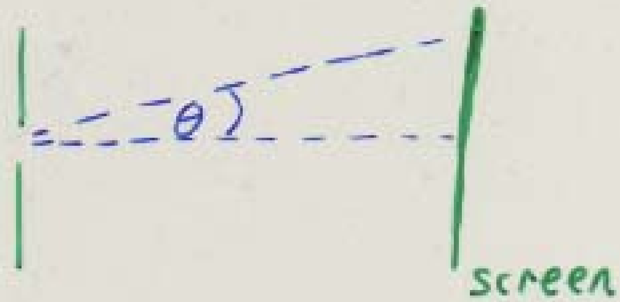
As aperture with $D \to \lambda$, fewer wavelets contribute
→ greater spreading of wavefront
(plane wave → spherical)



So pinhole does not produce sharp image on screen
In fact, as aperture $D \downarrow$, image size $\uparrow$

Diffraction Pattern of a 1-D slit

(As before)

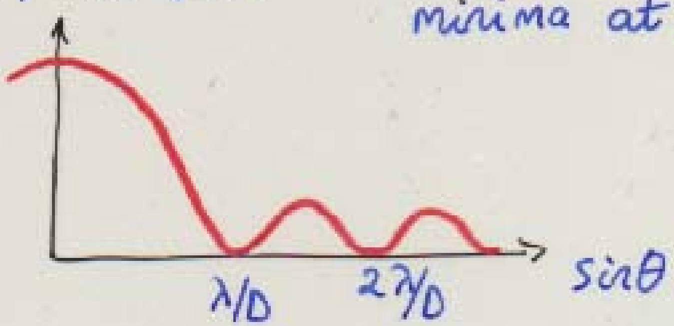


Can show that wavelets inside slit interfere constructively, destructively → non-uniform irradiance on screen (c.f. geometrical shadow of slit)

For $D \approx \lambda$, we find:

minima at $\sin\theta = \frac{m\lambda}{D}$

$m = \pm 1, \pm 2, \ldots$



with central maximum at $\theta = 0$ (opposite slit)

- Can use this pattern width to measure λ (if slit size D known) or D (for given source λ).

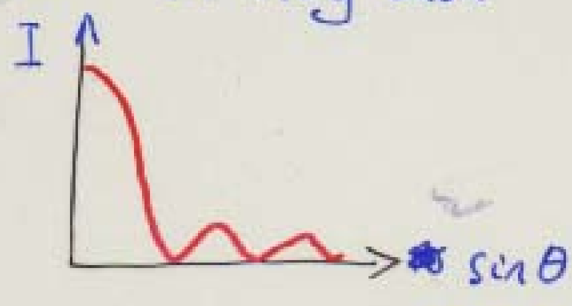
Note: If slit width $D < \lambda$, no minima possible ($\sin\theta > 1$)

⇒ pattern becomes uniform, spherical wave front

Diffraction by Circular Aperture in 2D

(opposite of Fresnel's disk obstruction)

We find : circular hole → diffraction rings with concentric maxima / minima : an Airy disk



First minimum is at $\sin\theta = 1.22\,\dfrac{\lambda}{D}$

So any aperture collecting light causes wavefronts to diffract

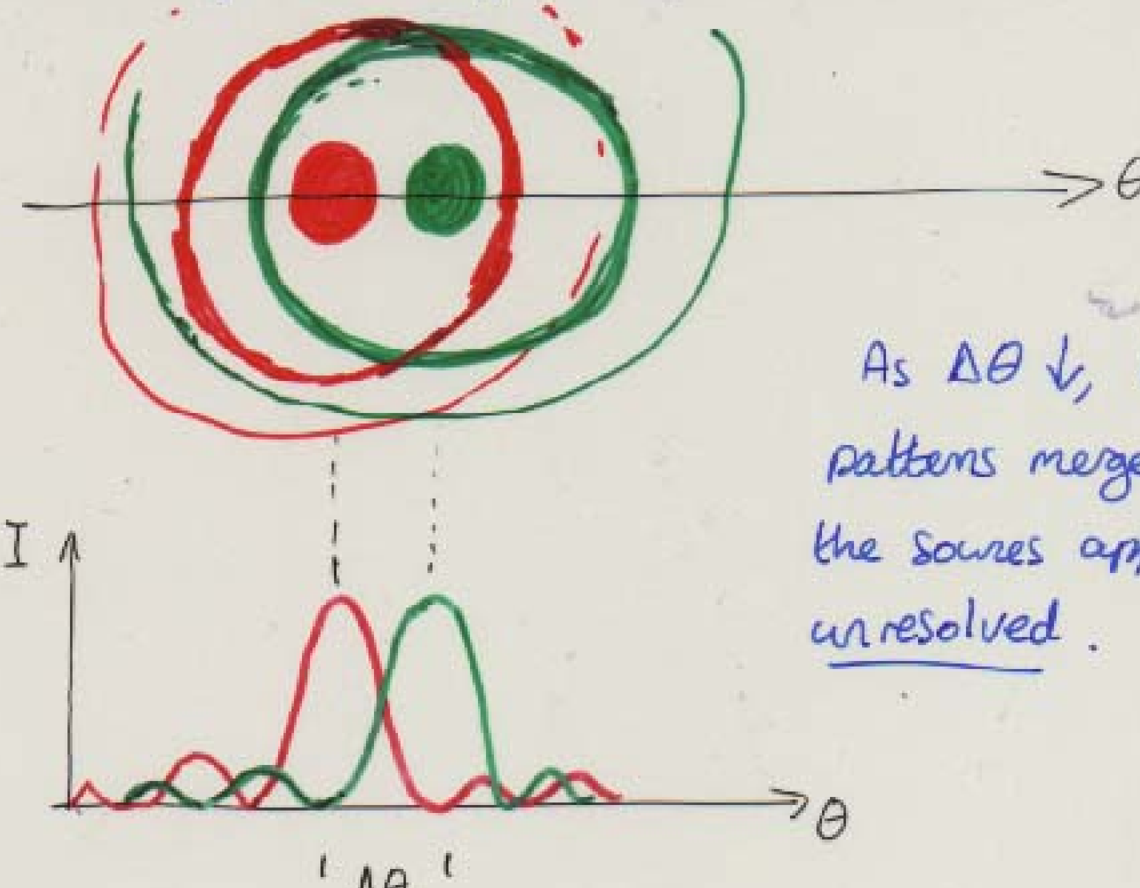
∴ Distant point source → plane wavefront incident
→ Airy disk pattern , not a point image

So no perfectly sharp images possible!

To keep pattern size small
- increase D (bigger telescope)
- decrease λ (image in violet light, not red)

Angular Resolution : Rayleigh Criterion

When 2 point sources (e.g. stars) are close together ($\Delta\theta$ small), their diffraction patterns overlap:



As $\Delta\theta \downarrow$, the patterns merge and the sources appear <u>unresolved</u>.

Rayleigh criterion: 2 sources are barely resolved if maximum of one lies on top of minimum of other

i.e. $$\Delta\theta \geq \Delta\theta_R \approx \sin\Delta\theta_R = \frac{1.22\lambda}{D}$$

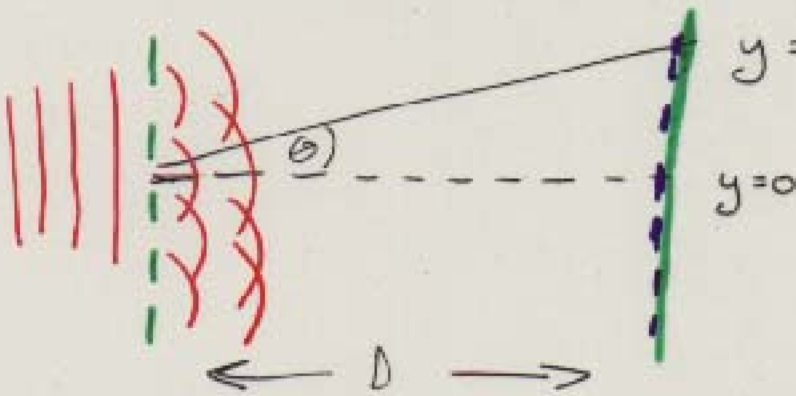
e.g. Human eye has pupil $D = 8\,mm$
- c.f. Hubble telescope objective $D = 2\,m$

$\Big\}\ \lambda_0 = 550\,nm$

- c.f. radio telescope receiving at $\lambda_0 = 2\,cm$.

Diffraction + Interference for Multiple Slits ($N > 2$)

Can produce a diffraction grating of $N > 2$ slits, separation $= a$
(c.f. Young's 2-slit experiment)

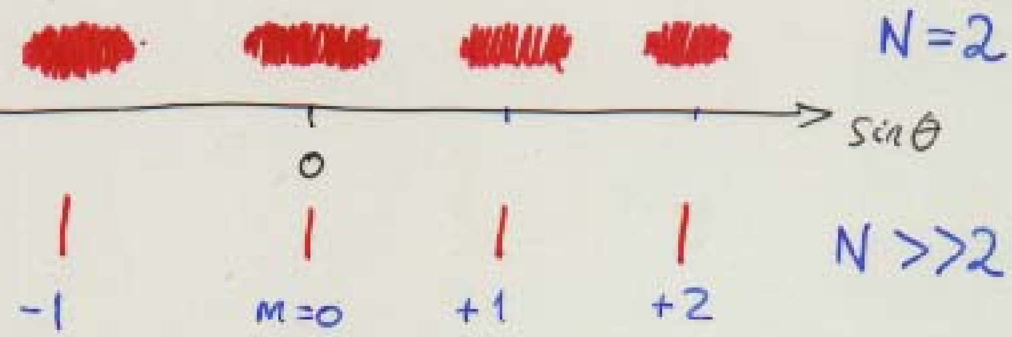


(dark)
destructive : $\sin\theta = \dfrac{(m+\frac{1}{2})\lambda}{a}$

As before, find constructive interference maxima at

bright $\sin\theta_m = \dfrac{m\lambda}{a}$ $\quad m = 0, \pm 1, \ldots\ldots$ (Young's slits)

BUT As # of slits ↑, width of fringes ↓ to produce sharp spectral lines



$m = 0$: "zeroth order", all wavelengths

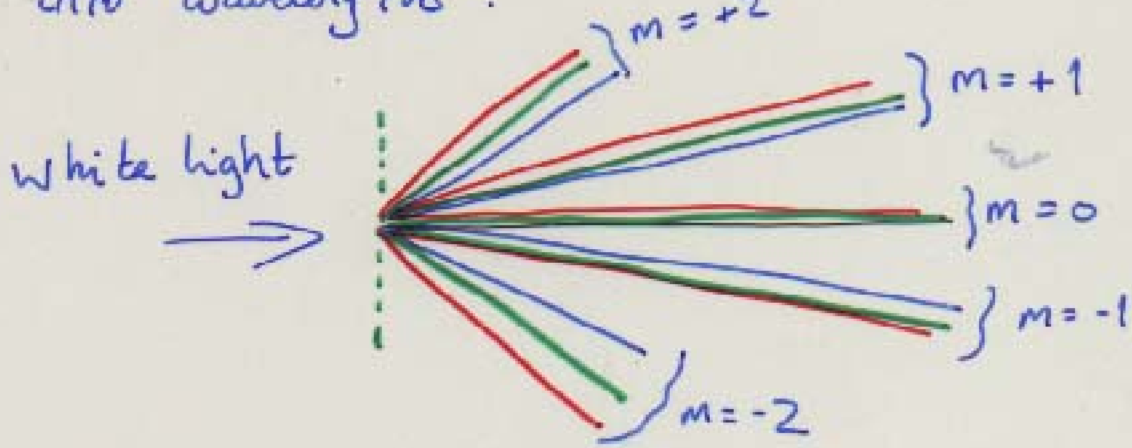
$m = \pm 1$: "first order", line appears at $\sin\theta = \pm\dfrac{\lambda}{a}$

etc.

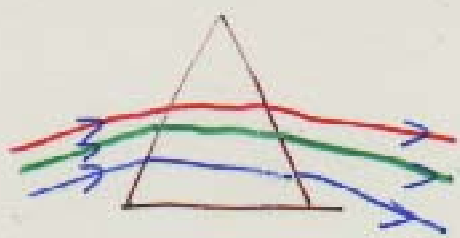
Diffraction Grating Spectroscopy

Grating Equation: $a \sin\theta_m = m\lambda$

Gratings with spacings $a \gtrsim \lambda$ can be used to disperse light into wavelengths:



(Note: prisms disperse shorter λ more, opposite to grating)



So can use a diffraction grating spectrometer to analyze light source (week 4 lab)

For 2 wavelengths close together; λ and $\lambda + \delta\lambda$ angular separation given by $a \cos\theta_m \dfrac{d\theta}{d\lambda} = m$

or $\delta\theta = \dfrac{m\,\delta\lambda}{a\cos\theta_m}$ $\therefore$ choose small a, large m for best dispersion.