Ch. 29

1. $p = \frac{h}{\lambda}$

$\vec{p} = m\vec{v} = (60\,kg)(2.0\,\frac{m}{s}) = 120\,kg\,\frac{m}{s}$

Now, from $\vec{F} = m\vec{a}$, $1\,N = kg\,\frac{m}{s^2}$

$\Rightarrow 120\,kg\,\frac{m}{s}\left(\frac{s}{s}\right) = 120\,N\cdot s$

and $1\,J = 1\,N\cdot m$

$\Rightarrow 120\,N\cdot s\left(\frac{m}{m}\right) = 120\,\frac{J\cdot s}{m}$

Although you don't have to show explicitely how units come about, it's still definitely good to know. Particularly since the units of the answers do not intuitively flow from the units in the problem.

$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\,J\cdot s}{120\,\frac{J\cdot s}{m}} = 5.5\times10^{-36}\,m$

5. $K.E. = 6.068\times10^{-21}\,J$

$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}mv^2\left(\frac{m}{m}\right) = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$

$\Rightarrow p = \sqrt{(2m)(K.E.)}$ $\quad m_n = 1.675\times10^{-27}\,kg$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{(2m)(K.E.)}} = \frac{6.626\times10^{-34}\,J\cdot s}{\sqrt{2(1.675\times10^{-27}\,kg)(6.068\times10^{-21}\,J)}}$

$= \frac{6.626\times10^{-34}\,J\cdot s}{4.509\times10^{-24}\,kg^{1/2}\,J^{1/2}}$

5. cont.

$$\lambda = 1.470\times10^{-10}\ \frac{J\cdot s}{kg^{1/2}\,J^{1/2}}$$

Again, it's a good idea to check that these odd units work out.

$$J = N\cdot m = kg\frac{m}{s^2}\cdot m = kg\frac{m^2}{s^2}$$

$$\therefore\ \frac{J\cdot s}{kg^{1/2}J^{1/2}} = \frac{kg\frac{m^2}{s^2}\cdot s}{kg^{1/2}\cdot kg^{1/2}\frac{m}{s}} = \frac{kg\frac{m^2}{s}}{kg\frac{m}{s}} = m$$

$$\lambda = 1.470\times10^{-10}\ m \quad \text{or} \quad 1.470\,\text{Å}$$

7. $\lambda = 0.10\times10^{-9}\,m$

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ J\cdot s}{0.10\times10^{-9}\,m} = 6.626\times10^{-24}\ \frac{J\cdot s}{m}$$

Method #1

$$K.E. = \frac{p^2}{2m} \qquad M_e = 9.109\times10^{-31}\,kg$$

$$= \frac{\left(6.626\times10^{-24}\ \frac{J\cdot s}{m}\right)^2}{2(9.109\times10^{-34}\,kg)}$$

$$= \frac{4.390\times10^{-47}\ \frac{J^2\cdot s^2}{m^2}}{1.822\times10^{-30}\,kg}$$

$$= 2.410\times10^{-17}\ \frac{J^2\cdot s^2}{kg\ m^2}$$

7. cont.

$$1\,J^2 = (N\cdot m)^2 = \left(kg\cdot\frac{m}{s^2}\cdot m\right)^2 = \frac{kg^2 m^4}{s^4}$$

$$\rightarrow \frac{J^2\cdot s^2}{kg\,m^2} = \frac{\left(\frac{kg^2 m^4}{s^4}\right)s^2}{kg\,m^2} = \frac{kg^2 m^4}{kg\,m^2 s^2} = kg\frac{m^2}{s^2} = N\cdot m = J$$

$$\therefore 2.410\times10^{-17}\frac{J^2\cdot s^2}{kg\,m^2} = 2.410\times10^{-17}J$$

The voltage provides the potential energy which gets converted to the specified kinetic energy.

$$\therefore P.E. = K.E.$$

$$P.E. = qV = K.E. = 2.410\times10^{-17}J$$

$$q = e = 1.602\times10^{-19}C$$

$$\rightarrow V = \frac{K.E.}{q} = \frac{2.410\times10^{-17}J}{1.602\times10^{-19}C} = 150.4\,V$$

Method #2

$$p = \frac{h}{\lambda} = \frac{4.136\times10^{-15}eV\cdot s}{0.10\times10^{-9}m} = 4.136\times10^{-5}\frac{eV\cdot s}{m}$$

$$K.E. = \frac{p^2}{2m_e}$$

$$E_e = m_e c^2 \quad m_e = 9.109\times10^{-31}kg$$

$$= (9.109\times10^{-31}kg)\left(9.00\times10^{16}\frac{m^2}{s^2}\right)$$

$$= 8.198\times10^{-14}J\left(\frac{1eV}{1.602\times10^{-19}J}\right)$$

$$= 0.511\times10^{6}eV$$

This is a particularly useful value to know, it shows up everywhere

$m_e \sim \frac{1}{2}MeV$

$= .511\,MeV$

7. cont. $E_e = 0.511 \times 10^6 eV \Longrightarrow m_e = \frac{.511 \times 10^6 eV}{c^2}$

$$\therefore K.E. = \frac{p^2}{2m_e} = \frac{p^2}{2\frac{E}{c^2}} = \frac{(pc)^2}{2E_e}$$

$$= \frac{\left(4.136 \times 10^{-5} \frac{eV \cdot s}{m}\right)^2 \left(3.00 \times 10^8 \frac{m}{s}\right)^2}{2(0.511 \times 10^6 eV)}$$

$$= \frac{1.54 \times 10^8 \frac{eV^{\cancel{2}} \cdot \cancel{s^2}}{\cancel{m^2}} \cdot \frac{\cancel{m^2}}{\cancel{s^2}}}{1.022 \times 10^6 \cancel{eV}}$$

$$= 150.7\, eV$$

$P.E. = eV = 150.7 eV$

$$V = \frac{K.E.}{e} = \frac{150.7 eV}{e} = 150.7 V$$

I prefer this method, with Energy in (eV) and mass in $\left(\frac{E}{c^2}\right)$. It is easier and faster with these type (quantum) problems to work in these units; particularly the $m = \frac{E}{c^2}$ substitution which I will use on most of these problems.

9. $K.E. = 20\,eV$

$$= \frac{1}{2} m_e v^2 = \frac{m_e^2 v^2}{2m_e} = \frac{p^2}{2m_e} = 20\,eV$$

$$\Rightarrow p = \sqrt{20\,eV(2m_e)}$$

$$m_e = \frac{.511\times10^6\,eV}{c^2} \qquad p = \sqrt{20\,eV\left(2\cdot 0.511\times10^6\,eV \over (3.00\times10^8\,\frac{m}{s})^2\right)}$$

$$\lambda = \frac{h}{p} = \frac{4.136\times10^{-15}\,eV\cdot s}{\sqrt{(20\,eV)(2)\left(\frac{.511\times10^6\,eV}{(3.00\times10^8\,\frac{m}{s})^2}\right)}}$$

$$= \frac{4.136\times10^{-15}\,eV\cdot s}{1.507\times10^{-5}\,\frac{eV\cdot s}{m}} = 2.744\times10^{-10}\,m$$

$$= 2.744\,\text{Å}$$

if you worked in Joules you had to convert the K.E.

$$\rightarrow p = \sqrt{20\,eV(2m_e)} \qquad m_e = 9.109\times10^{-31}\,kg$$

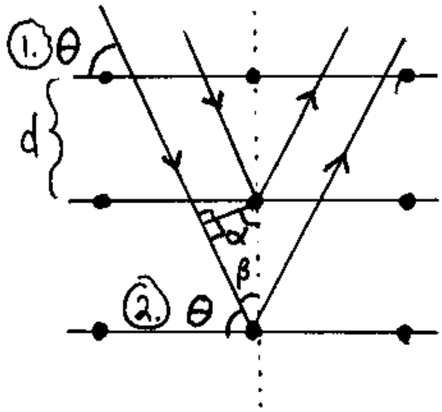
$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\,J\cdot s}{\sqrt{20\,eV\left(\frac{1.602\times10^{-19}\,J}{1\,eV}\right)(2)(9.109\times10^{-31}\,kg)}}$$

$$= \frac{6.626\times10^{-34}\,J\cdot s}{2.416\times10^{-24}\,J^{1/2}\cdot kg^{1/2}} \qquad \frac{J\cdot s}{J^{1/2}kg^{1/2}} = \frac{kg\frac{m^2}{s^2}\cdot s}{kg^{1/2}\frac{m}{s}\cdot k^{1/2}} = \frac{kg\frac{m^2}{s}}{kg\frac{m}{s}} = m$$

$$= 2.742\times10^{-10}\,m$$

or $2.742\,\text{Å}$

17.



Proof for θ:

① This θ is the definition of the angle that the incoming wave/particle makes with the atomic plane.

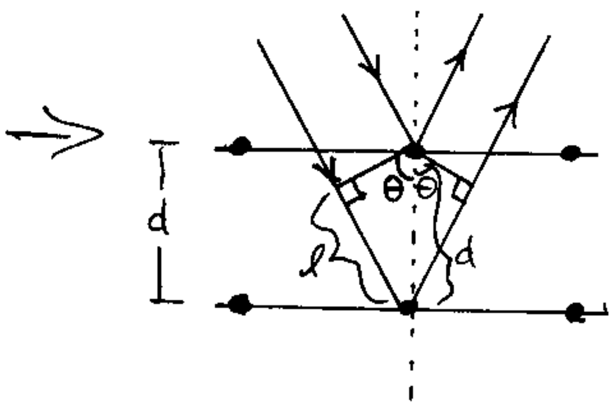
② This θ is equal to ① by laws of two parallel lines cut by a transverse.

Now: $\alpha + \beta + 90° = 180°$

$\alpha + \beta = 90°$

But: $\theta_{②} + \beta = 90°$

$\therefore \theta = \alpha$



$\sin\theta = \frac{\ell}{d} \quad \ell = d\sin\theta$

Total pathlength $= 2\ell$

$= 2d\sin\theta$

Constructive interference when Total pathlength $= m\lambda \quad m = 1, 2, 3, \ldots$

$\Rightarrow 2d\sin\theta = m\lambda$

1^{st} order $\therefore m = 1 \qquad \theta = 65° \qquad d = 0.091 \times 10^{-9}\,m$

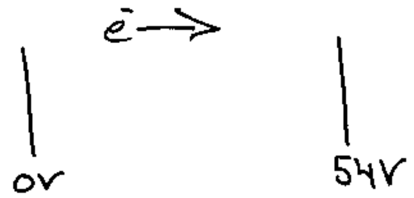
$\Rightarrow \lambda = 2d\sin(65°) = 2(0.091 \times 10^{-9}\,m)(.9063)$

$= 1.649 \times 10^{-10}\,m$

$= 1.65\,Å$

$= 1.7\,Å$ (only 2 sig figs)

17. cont.



$$P.E. = qV = (e)(54V) = 54eV$$

$$P.E. \rightarrow K.E. \Rightarrow 54eV = \frac{p^2}{2m_e} \qquad m_e = \frac{0.511 \times 10^6 eV}{c^2}$$

$$p = \sqrt{54eV(2m_e)} = \frac{\sqrt{(54eV)(2)(0.511 \times 10^6 eV)}}{c}$$

$$= \frac{\sqrt{5.52 \times 10^7 eV^2}}{3 \times 10^8 \frac{m}{s}}$$

$$= 2.48 \times 10^{-5} \frac{eV \cdot s}{m}$$

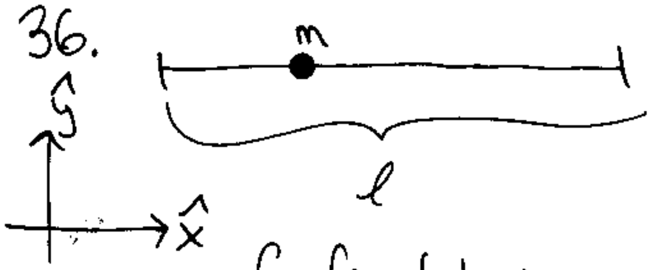
$$p = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p} = \frac{4.136 \times 10^{-15} eV \cdot s}{2.48 \times 10^{-5} \frac{eV \cdot s}{m}} = 1.67 \times 10^{-10} m$$

$$= 1.7 \text{Å}$$

$1.65 \text{Å} \approx 1.67 \text{Å}$, but 54V is 2 sig figs $\rightarrow$ $1.7 \text{Å} = 1.7 \text{Å}$

$\therefore$ Electrons with momentum of $2.48 \times 10^{-5} \frac{eV \cdot s}{m}$ and corresponding de Broglie wavelength of 1.7 Å show the same interference pattern as EM radiation of wavelength 1.7 Å.

36.



$m = 100g = 0.100kg$

$\ell = 1.00m$

Confined to x-axis.

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad p_x = mv_x \qquad \Delta x = \ell = 1.00m$$

$$\Delta p_x = m\Delta v_x$$

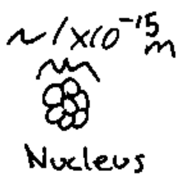
$$\Delta x (m\Delta v_x) \geq \frac{\hbar}{2}$$

$$\Delta v_x \geq \frac{\hbar}{2m\Delta x} = \frac{1.055\times10^{-34} J\cdot s}{2(1m)(.100kg)}$$

$$= 5.28\times10^{-34} \qquad \frac{kg\frac{m^2}{s^2}\cdot s}{m\,kg} = \frac{\frac{m^2}{s}}{m} = \frac{m}{s}$$

$$5.28\times10^{-34}\frac{m}{s}$$

$$\left(5.28\times10^{-34}\frac{m}{s}\right)\left(\frac{3600s}{1hr}\right)\left(\frac{24hr}{1day}\right)\left(\frac{365.25days}{1year}\right) = 1.67\times10^{-26}\,m/year$$



The nuclei of most atoms are over 9 orders of magnitude larger than this distance.

41. $\Delta E \Delta t \geq \frac{\hbar}{2}$ $\quad \Delta t = 0.10\,\mu s = 1.0\times10^{-7}\,s$

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{6.582\times10^{-16}\,eV\cdot s}{2(1.0\times10^{-7}\,s)} = 3.29\times10^{-9}\,eV$$

or $5.28\times10^{-28}\,J$

46. $\Delta x \Delta p \geq \frac{\hbar}{2}$ $\quad \Delta p = m\Delta v$

$m = 10\,g = 0.010\,kg$

$\Delta x = 1.00\,nm = 1.00\times10^{-9}\,m$

$$\rightarrow \Delta p \geq \frac{\hbar}{2\Delta x}$$

$$m\Delta v \geq \frac{\hbar}{2\Delta x} \rightarrow \Delta v \geq \frac{\hbar}{2m\Delta x} = \frac{1.055\times10^{-34}\,J\cdot s}{2(.010\,kg)(1.00\times10^{-9}\,m)}$$

$$= 5.275\times10^{-24}\,\frac{m}{s}$$

$$\frac{1\,nm}{year} = \frac{1\times10^{-9}\,m}{1\,year}\left(\frac{1\,year}{365.25\,days}\right)\left(\frac{1\,day}{24\,hr}\right)\left(\frac{1\,hr}{3600\,s}\right)$$

$$= 3.17\times10^{-17}\,\frac{m}{s} > 5.275\times10^{-24}\,\frac{m}{s}$$

$\therefore$ Uncertainty Principle is not violated because $\vec{v} > \frac{\hbar}{2m\Delta x}$

49. $\Delta x \Delta p \geq \frac{\hbar}{2}$

$\Delta x = 0.100\,\text{nm} = 1.00\times10^{-10}\,\text{m}$

$\Delta p = m\Delta v$

$m_e = \frac{0.511\times10^{6}\,\text{eV}}{c^2}$

$$\Delta x (m\Delta v) \geq \frac{\hbar}{2}$$

$$\Delta v \geq \frac{\hbar}{2m_e\Delta x} = \frac{\hbar c^2}{2E_e\,\Delta x} = \frac{(6.582\times10^{-16}\,\text{eV}\cdot\text{s})(3.00\times10^{8}\,\frac{\text{m}}{\text{s}})^2}{2(0.511\times10^{6}\,\text{eV})(1.00\times10^{-10}\,\text{m})}$$

$$= 5.80\times10^{5}\,\frac{\text{m}}{\text{s}}$$

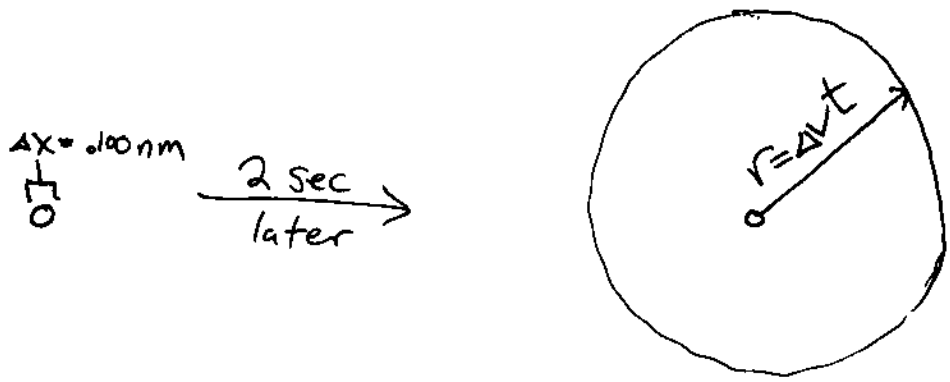
Minimum position uncertainty when Δv is minimum.

$$\Delta v_{min} = \frac{\hbar c^2}{2E_e\Delta x}$$

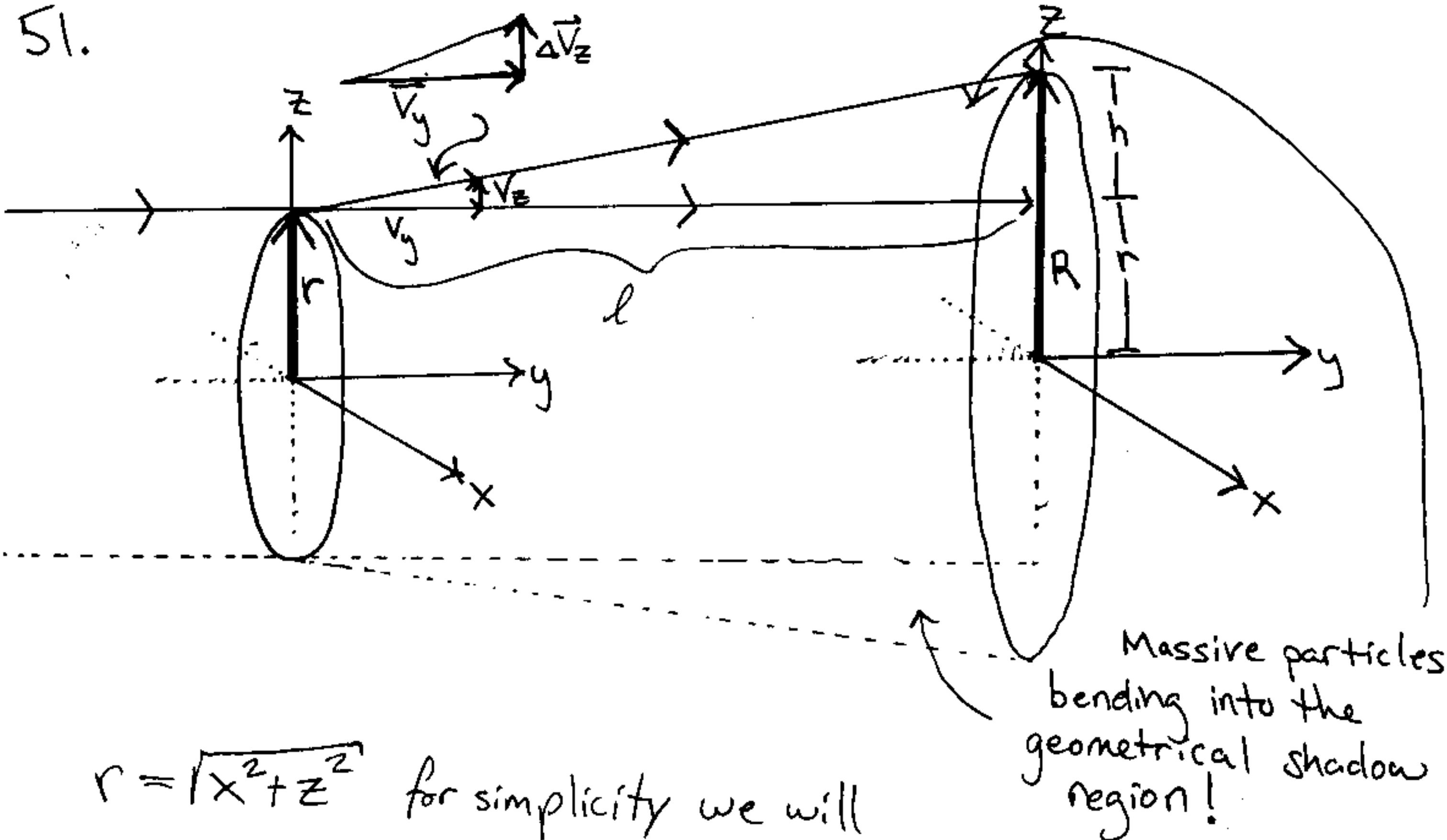
You may (or may not) find it useful, but $\hbar c = 1973\,\text{eV}\cdot\text{Å}$. It is an easy value to remember and very easy to work with. If you use it just make sure all your lengths are in angstroms

$\Delta x_{min} = \Delta v_{min} t \qquad t = 2.00\,\text{s}$

$$= (5.80\times10^{5}\,\tfrac{\text{m}}{\text{s}})(2.00\,\text{s}) = 1.16\times10^{6}\,\text{m}$$



51.

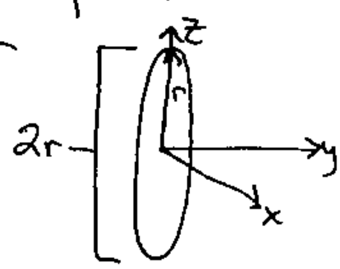


$r = \sqrt{x^2+z^2}$ for simplicity we will evaluate this problem at $x=0$

$\Longrightarrow r = z$

Total uncertainty in position along z-axis: $-r \leq z \leq r$

$\Longrightarrow \Delta z = 2r$



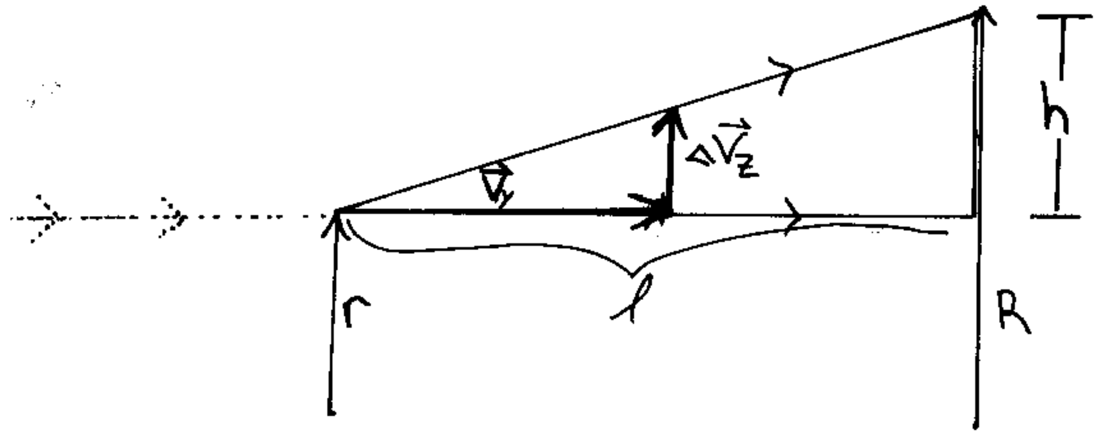
$\Delta z \Delta p_z \geq \frac{\hbar}{2}$ $\quad \Delta p_z = m\Delta V_z \quad m = 3.271\times10^{-25}\,kg$

$\Delta z\, m\Delta V_z \geq \frac{\hbar}{2}$

$$\Delta V_z \geq \frac{\hbar}{2m\Delta z} = \frac{1.055\times10^{-34}\,J\cdot s}{2(3.271\times10^{-25}\,kg)(2r)}$$

$$= \frac{8.063\times10^{-11}}{r}\,\frac{m}{s}$$

51. cont.



$$R = r + h$$

$$h = \Delta V_z t$$

$$V_y = \frac{\ell}{t} \Rightarrow t = \frac{\ell}{V_y}$$

The gold atoms are in the gas phase and, as stated in the problem, we assume that every Au atom is moving at the same speed (no distribution of speeds like a real gas). Also, since it is a parallel beam (parallel to the y-axis) means that there is no motion in the $\hat{x}$ and $\hat{z}$ directions. No motion in these directions means no kinetic energy in those directions. Therefore we assume that all the kinetic energy, more specifically all the thermal kinetic energy, is directed along the y-axis. Thermal because the only piece of information given to us from the problem is the temperature of the gas.

51. cont.

$$\Longrightarrow K.E._{(gas)} = \frac{3}{2} \underbrace{k_B}_{\text{Boltzmann Constant}} T$$

$$\frac{1}{2} m v_y^2 = \frac{3}{2} k_B T \qquad T = 1600\,K$$

$$k_B = 1.381\times10^{-23}\,\frac{J}{K}$$

$$v_y = \sqrt{\frac{3k_BT}{m}}$$

$$t = \frac{l}{v_y} = \sqrt{\frac{ml^2}{3k_BT}} = \sqrt{\frac{(3.271\times10^{-25}\,kg)(1\,m)^2}{3(1.381\times10^{-23}\,\frac{J}{K})(1600\,K)}}$$

$$= \sqrt{\frac{3.271\times10^{-25}\,kg\,m^2}{6.629\times10^{-20}\,J}}$$

$$= 2.221\times10^{-3}\,s$$

$$h = \Delta v_z t = \left(\frac{8.063\times10^{-11}}{r}\,\frac{m}{s}\right)\left(2.221\times10^{-3}\,s\right)$$

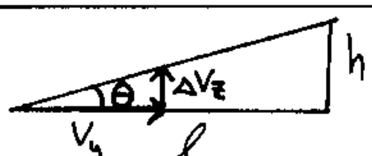
$$= \frac{1.79\times10^{-13}}{r}\,m$$

$$\therefore\ R = r + \frac{1.79\times10^{-13}}{r}\,m$$

$$\lim_{r\to0^+} \arctan\left[\frac{k}{r}\right] = 90^\circ = \theta$$

$$\lim_{r\to\infty} \arctan\left[\frac{k}{r}\right] = 0^\circ = \theta$$

It is important to note that smaller apertures (smaller r) correspond to greater bending around the edges, just as light does, $\theta_a = \frac{1.22\lambda}{D}$!



$$\tan\theta = \frac{h}{l}$$

$$h = \Delta v_z t,\quad l = v_y t \rightarrow \tan\theta = \frac{\Delta v_z t}{v_y t} = \frac{\frac{\hbar}{2m\Delta z}}{\sqrt{\frac{3k_BT}{m}}} = \frac{\hbar}{2m(2r)}\sqrt{\frac{m}{3k_BT}}$$

$$= \frac{1}{r}\cdot\underbrace{\frac{\hbar}{4\sqrt{3k_BTm}}}_{\text{Some constant, call it "k"}} \qquad \therefore\ \theta = \arctan\left[\frac{k}{r}\right]$$

The degree to which the particles "bend" or diffract around the edges of the aperature is given by θ in our original diagram showing how far into the shadow region the rays are.