

Ch. 27

25. $2d \sin \theta = m\lambda$

$$m=1$$

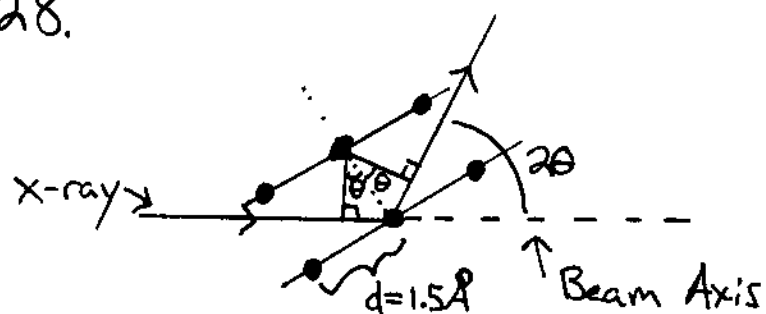
$$\theta = 25^\circ$$

$$\lambda = .090 \times 10^{-9} \text{ m}$$

$$d = \frac{m\lambda}{2 \sin \theta} = \frac{(1)(.090 \times 10^{-9})}{2(\sin 25^\circ)} = 1.065 \times 10^{-10} \text{ m}$$

or $1.065 \text{ \AA}$

28.



See fig. 27.8 c)
on pg. 1083

$$30^\circ \text{ off beam axis} \Rightarrow 2\theta = 30^\circ$$

$$\theta = 15^\circ$$

$$1^{\text{st}} \text{ order peak} \Rightarrow m=1$$

$$\therefore \lambda = 2d \sin \theta$$

$$= 2(1.5 \text{ \AA}) \sin 15^\circ = .78 \text{ \AA}$$

36. $2d \sin \theta = m\lambda$ $d = .500 \text{ \AA}$ $\theta = 30^\circ$

$$\lambda = \frac{2d \sin \theta}{m}$$

$$= \frac{2(.500 \text{ \AA}) \sin 30^\circ}{m}$$

$$= \frac{2(.500 \text{ \AA}) \left(\frac{1}{2}\right)}{m} \Rightarrow \lambda = \frac{.500 \text{ \AA}}{m} \quad m=1, 2, 3, \dots$$

38. As the α -particle moves closer to the gold nucleus, the kinetic energy of the α -particle is being converted into potential energy between it and the nucleus and consequently slowing down. When all its initial kinetic energy becomes stored as potential energy, the α -particle will not move any closer to the nucleus.

so closest when $K.E. = P.E.$

$$K.E. = 8.00 \times 10^{-13} \text{ J}$$

$$P.E. = \frac{k_e (Ze)(2e)}{r} \quad Z_{\text{gold}} = 79$$

$$\Rightarrow 8.00 \times 10^{-13} \text{ J} = \frac{k_e (79e)(2e)}{r}$$

$$k_e = \frac{1}{4\pi\epsilon_0}$$

$$\epsilon_0 = 8.85 \times 10^{-12}$$

$$r = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{158e^2}{8.00 \times 10^{-13}}$$

$$= \frac{158(1.6 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})(8.00 \times 10^{-13})}$$

$$= 4.55 \times 10^{-14} \text{ m}$$

$$\text{or } 45.5 \text{ fm}$$

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$$43. \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

Series limit as $n \rightarrow \infty$

$$\frac{1}{\lambda} = R \left[\frac{1}{4} - \frac{1}{\infty} \right] \rightarrow 0$$

$$\frac{1}{\lambda} = \frac{R}{4} \Rightarrow \lambda = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 3.646 \times 10^{-7} \text{ m} \text{ or } 364.6 \text{ nm}$$

$$44. \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right] \quad \begin{array}{l} \text{Longer wavelength} = \text{Lower energy} \\ \text{Since } n=3, 4, 5, \dots \end{array}$$

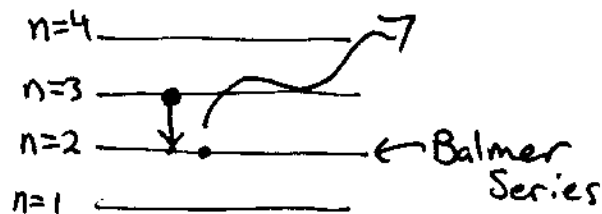
$$\frac{1}{\lambda} = R \left[\frac{1}{4} - \frac{1}{3^2} \right]$$

$$= R \left[\frac{9-4}{36} \right]$$

$$= 1.097 \times 10^7 \left(\frac{5}{36} \right)$$

$$\Rightarrow \lambda = \frac{1}{1.097 \times 10^7} \left(\frac{36}{5} \right) = 6.563 \times 10^{-7} \text{ m or } 656.3 \text{ nm}$$

Lowest energy is from the $n=3$ to $n=2$ transition



$$51. \lambda_{\min} = 390 \text{ nm}$$

$$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

$$\frac{1}{R \lambda_{\min}} = \frac{1}{4} - \frac{1}{n^2}$$

$$\frac{1}{n^2} = \frac{1}{4} - \frac{1}{R \lambda_{\min}}$$

$$51. (\text{cont.}) \quad \frac{1}{n^2} = \frac{R\lambda_{\min} - 4}{4R\lambda_{\min}}$$

$$n^2 = \frac{4R\lambda_{\min}}{R\lambda_{\min} - 4}$$

$$\begin{aligned} n &= \sqrt{\frac{4R\lambda_{\min}}{R\lambda_{\min} - 4}} = \sqrt{\frac{4(1.097 \times 10^7)(390 \times 10^{-9})}{R\lambda_{\min} - 4}} \\ &= \sqrt{\frac{17.1132}{4.2783 - 4}} \\ &= \sqrt{61.49} = 7.8 \end{aligned}$$

highest integer = 7

$\therefore \lambda_{\min} @ n=7$

$$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{4} - \frac{1}{7^2} \right] = R \left(\frac{49 - 4}{196} \right)$$

$$\Rightarrow \lambda_{\min} = \frac{1}{R} \left(\frac{196}{45} \right)$$

$$= 3.97 \times 10^7 \text{ m}$$

$$\text{or } 397 \text{ nm}$$

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$$1. \quad P = \epsilon \sigma A T^4 \quad \sigma = 5.67 \times 10^{-8} \quad \epsilon = .97$$

$$I = \frac{P}{A} = \epsilon \sigma T^4 \quad T_{\text{body}} = 306 \text{ K}$$

$$T_{\text{room}} = 293 \text{ K}$$

The skin is emitting radiation according to its temp. but it is also absorbing radiation from the room according to the room's temp.

∴ Net power radiated off the skin

= Radiation emitted by body @ 306 K

— Radiation absorbed by body @ 293 K

$$I = \frac{P}{A} = (.97)(5.67 \times 10^{-8})(306)^4 - (.97)(5.67 \times 10^{-8})(293)^4$$

$$= (.97)(5.67 \times 10^{-8})(306^4 - 293^4)$$

$$= 77 \frac{\text{W}}{\text{m}^2}$$

$$P = I A = (77 \frac{\text{W}}{\text{m}^2})(1.4 \text{ m}^2) = 108 \text{ W} \quad \text{Total energy lost per second}$$

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3. $T = 306 \text{ K}$

$$\lambda_{\text{peak}} T = .002898$$

$$\lambda_p = \frac{.002898}{306} = 9.47 \times 10^{-6} \text{ m or } 9.47 \mu\text{m}$$

7. $T = 1000 \text{ K}$

$$\lambda_p = \frac{.002898}{T} = \frac{.002898}{1000} = 2.898 \times 10^{-6} \text{ m or } 2.898 \mu\text{m}$$

8. Total Energy = Power $\times$ time $t = 5 \text{ hrs}$

$$P = \sigma A T^4 \quad \text{Energy} = Pt = 500 \text{ J @ } T_0 = 293 \text{ K}$$

$$= X \quad \text{@ } T_1 = 1273 \text{ K}$$

$$Pt = \sigma A T_0^4 t = 500 \text{ J} \Rightarrow t = \frac{500 \text{ J}}{\sigma A T_0^4}$$

$$\sigma A T_1^4 t = X$$

$$t = \frac{X}{\sigma A T_1^4}$$

$$\therefore \frac{500 \text{ J}}{\sigma A T_0^4} = \frac{X}{\sigma A T_1^4}$$

$$\rightarrow \left(\frac{T_1^4}{T_0^4} \right) (500 \text{ J}) = X$$

$$\left(\frac{1273^4}{293^4} \right) (500 \text{ J}) = 178,000 \text{ J}$$

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9. Total Energy = $Pt = 180 \text{ J}$

$$t = 8 \text{ hr} \left(\frac{3600 \text{ sec}}{1 \text{ hr}} \right) = 28,800 \text{ sec}$$

$$P = \sigma AT^4 \quad T = 293 \text{ K}$$

$$\Rightarrow \sigma AT^4 t = 180 \text{ J}$$

$$A = \frac{180}{\sigma T^4 t} = \frac{180}{(5.67 \times 10^{-8})(293)^4(28,800)} \\ = 1.5 \times 10^{-5} \text{ m}^2$$

19. Energy per photon $\Rightarrow E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{550 \times 10^{-9}} \\ = 3.614 \times 10^{-19} \text{ J/photon}$

Total photon ~~Power~~ = (energy per photon) $\left(\frac{\# \text{ of photons}}{\text{sec}} \right)$
 $= (3.614 \times 10^{-19} \frac{\text{J}}{\text{photon}}) \left(2 \times 10^{13} \frac{\text{photons}}{\text{sec}} \right)$
 $P = 7.228 \times 10^{-6} \frac{\text{J}}{\text{s}} \text{ or } \text{W}$

$$A = 1 \text{ cm}^2 \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^2 = 1 \times 10^{-4} \text{ m}^2$$

$$I = \frac{P}{A} = \frac{7.228 \times 10^{-6} \text{ W}}{1 \times 10^{-4} \text{ m}^2} = 0.0723 \frac{\text{W}}{\text{m}^2}$$

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$$21. E_{in} = E_{out}$$

$$E_{in} = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}) (3 \times 10^8)}{650 \times 10^{-9}} = 3.058 \times 10^{-19} \text{ J}$$

$$E_{out} = \cancel{K.E.}_{\text{electron}} + \phi \text{ (stopping potential)}$$

0 (problem states that k.e. is negligible)

$$E_{out} = \phi$$

$$\Rightarrow 3.058 \times 10^{-19} \text{ J} = \phi$$

All this energy got absorbed by the metal and converted into mechanical energy and heat.

$$E_{\phi} = hf = 3.058 \times 10^{-19} \text{ J}$$

$$f = \frac{3.058 \times 10^{-19}}{6.626 \times 10^{-34}} = 4.62 \times 10^{14} \text{ Hz}$$

Threshold frequency because it is the highest frequency, and highest energy, photon that the metal can completely absorb before it has to get rid of energy by spitting out an electron.

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25. The electron's kinetic energy gets converted to potential energy as it moves towards the charged plate. As the potential is increased the electrons arrive at slower and slower velocities until the potential is increased to a point where electrons don't arrive at all. At that instant, all the kinetic energy of the electron gets stored as electrostatic potential energy with no energy to continue motion towards the plate. Since the electrons don't jump the gap the current goes to zero, as in this problem.

$$\therefore \text{all K.E.} \rightarrow \text{P.E.} \Rightarrow \text{K.E.} = \text{P.E.}$$

$$\text{Electrostatic potential energy for a charge} = qV \quad \begin{array}{l} \text{Voltage or} \\ \text{potential at that} \\ \text{point in space} \end{array}$$

$$q = e^-$$

$$V = 1.25 \text{ V}$$

$$\text{K.E.} = \frac{1}{2} m_e v^2$$

$$\Rightarrow \frac{1}{2} m_e v^2 = eV$$

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.6 \times 10^{-19})(1.25)}{9.11 \times 10^{-31}}}$$

$$= 6.63 \times 10^5 \frac{\text{m}}{\text{s}}$$

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28. First we need to solve for the stopping potential, ϕ , which will give us the minimum energy, and thus frequency, required to kick out an electron.

$$E_{in} = \frac{hc}{\lambda} \quad \lambda = 250 \times 10^{-9} \text{ m}$$
$$= \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{250 \times 10^{-9}} = 7.95 \times 10^{-19} \text{ J}$$

$$E_{out} = \text{K.E.} + \phi \quad \text{K.E.} = 1 \times 10^{-19} \text{ J}$$
$$= \frac{1}{2}mv^2 + \phi$$

$$E_{in} = E_{out}$$
$$7.95 \times 10^{-19} \text{ J} = 1 \times 10^{-19} \text{ J} + \phi$$
$$\phi = (7.95 - 1) \times 10^{-19} \text{ J}$$
$$= 6.95 \times 10^{-19} \text{ J}$$

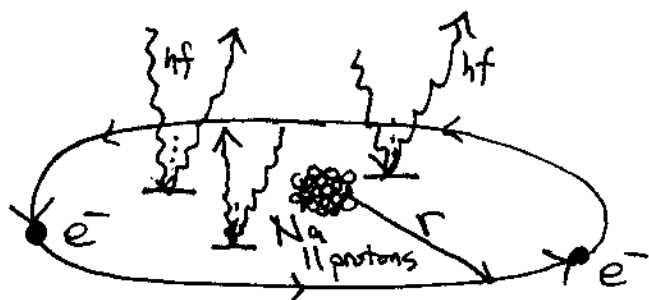
Lowest frequency photon is the lowest energy photon, and the lowest E_{in} occurs when $\text{K.E.} = 0$

$$\rightarrow E_{in} = \phi$$

$$hf = 6.95 \times 10^{-19} \text{ J}$$

$$f = \frac{6.95 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{ J}\cdot\text{s}} = 1.05 \times 10^{15} \text{ Hz}$$

31.

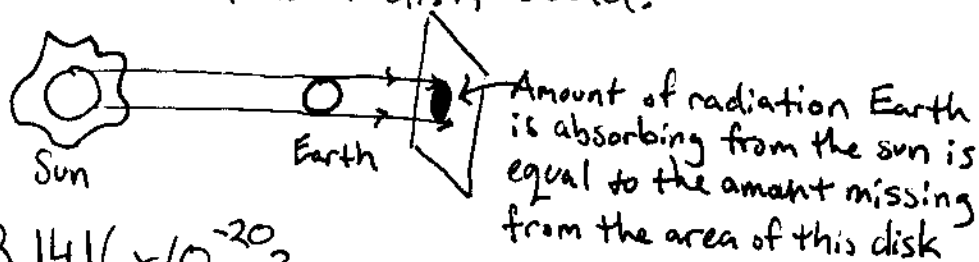


$$\text{Na: } \phi_{\text{Na}} = 2.28 \text{ eV (pg. 1113)}$$

$$Z_{\text{Na}} = 11$$

$$r = .10 \text{ nm}$$

The effective area the intensity is distributed over is $A = \pi r^2$. Just as the sun's intensity incident on Earth is computed with the area of the shadow the Earth makes against a plane, so is it here; even though the area of the hemisphere is $\frac{1}{2}A_{\text{sphere}} = 2\pi r^2$ it only absorbs the same amount of radiation as a disk would.



$$A = \pi (.1 \times 10^{-9})^2 = 3.1416 \times 10^{-20} \text{ m}^2$$

$$I_{\text{aver}} = 10 \frac{\text{W}}{\text{m}^2}$$

The electron needs to absorb at least as much energy as ~~the~~ stopping potential before it will ~~escape~~ ~~the metal~~.
escape the metal.

$$P_e = IA = (10 \frac{\text{W}}{\text{m}^2}) (3.1416 \times 10^{-20} \text{ m}^2) = 3.1416 \times 10^{-19} \text{ W}$$

$$3.1416 \times 10^{-19} \frac{\text{J}}{\text{s}} \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) = 1.96 \frac{\text{eV}}{\text{sec}}$$

31 (cont.)

$$E_{\text{Total absorbed}} = P t = (1.96 \frac{\text{eV}}{\text{s}}) t$$

$$\phi = 2.28 \text{ eV} \quad P t = \phi$$

$$t = \frac{\phi}{P} = \frac{2.28 \text{ eV}}{1.96 \frac{\text{eV}}{\text{s}}} = 1.16 \text{ sec}$$

However, if $I = 1 \frac{\mu\text{W}}{\text{m}^2}$

$$\begin{aligned} \text{then } t &= \frac{\phi}{P} = \frac{\phi}{I A} = \frac{2.28 \text{ eV}}{(1 \times 10^{-6} \frac{\text{W}}{\text{m}^2}) \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) (3.1416 \times 10^{-20} \text{ m}^2)} \\ &= 1.16 \times 10^7 \text{ sec} \\ &= 134 \text{ days} \end{aligned}$$

∴ The classical electrodynamic interpretation of this problem is obviously incorrect.

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$$32. E = hf_{\max} = \frac{hc}{\lambda_{\text{shortest}}} = eV \quad V = 30 \times 10^3 \text{ V}$$

$$\begin{aligned} \Rightarrow \lambda &= \frac{hc}{eV} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \frac{\text{m}}{\text{s}})}{(1.6 \times 10^{-19} \text{ C})(30 \times 10^3 \text{ J/C})} \\ &= \frac{1.988 \times 10^{-25} \text{ J}\cdot\text{m}}{4.8 \times 10^{-15} \text{ J}} \\ &= 4.14 \times 10^{-11} \text{ m} \quad \text{or } 0.414 \text{ \AA} \end{aligned}$$

$$44. E_n = -\frac{13.6 \text{ eV}}{n^2} \quad \text{1st excited state} \Rightarrow n=2$$

$n=1$ for ground state

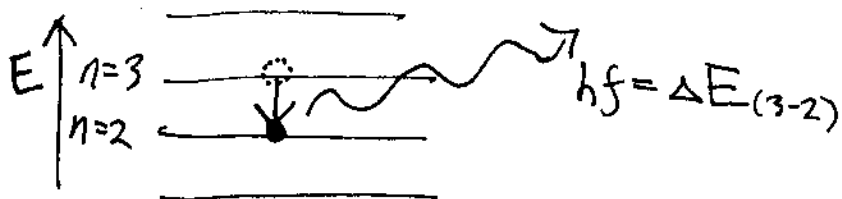
$$\begin{aligned} E_2 &= -\frac{13.6 \text{ eV}}{2^2} = -3.40 \text{ eV} \\ -3.40 \text{ eV} \left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) &= -5.44 \times 10^{-19} \text{ J} \end{aligned}$$

$$45. \text{ 2nd excited state} \Rightarrow n=3$$

$$E_3 = -\frac{13.6 \text{ eV}}{3^2} = -1.51 \text{ eV}$$

$$46. E_3 - E_2 = E_{\text{photon}}$$

$$-1.51 \text{ eV} - (-3.40 \text{ eV}) = 1.89 \text{ eV} \left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = 3.024 \times 10^{-19} \text{ J}$$



47. The difference in energy between the energy levels goes into making the photon.

$$E_{\text{photon}} = \Delta E_{(3-2)} = E_3 - E_2 = 1.89 \text{ eV}$$

$$\therefore E_{\text{photon}} = hf = \frac{hc}{\lambda} = 1.89 \text{ eV} \Rightarrow f = \frac{1.89 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{sec}} = 4.57 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{hc}{1.89} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3 \times 10^8 \frac{\text{m}}{\text{s}})}{1.89 \text{ eV}}$$

$$= 6.57 \times 10^{-7} \text{ m}$$

$$\text{or } 657 \text{ nm}$$

58. for K_α lines $f = \left(\frac{3cR}{4}\right)(Z-1)^2$

$$\text{or } \frac{1}{\lambda} = R(Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right] \quad \lambda = .0759 \text{ nm}$$

$$= R(Z-1)^2 \left(\frac{3}{4} \right)$$

$$\frac{4}{3R\lambda} = (Z-1)^2$$

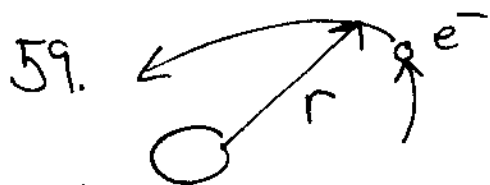
$$\sqrt{\frac{4}{3R\lambda}} = Z-1$$

$$\Rightarrow Z = \sqrt{\frac{4}{3R\lambda}} + 1 = \sqrt{\frac{4}{3(1.097 \times 10^7)(.0759 \times 10^{-9})}} + 1$$

$$= 40.02 + 1$$

$$= 41$$

$Z = 41$ protons for Niobium



Angular Momentum: $L_n = m_e v_n r_n = n \hbar$ $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J}\cdot\text{s}$

Ground State $\rightarrow n=1$

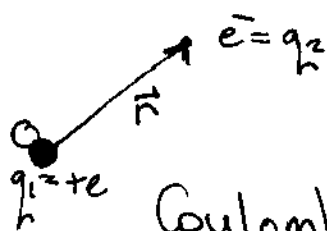
$\therefore r_n = n^2 r_1 = 1^2 r_1 = r_1 = .0529 \times 10^{-9} \text{ m}$ (Bohr Radius)

$\Rightarrow m_e v r_1 = (1) \hbar$

$$v = \frac{\hbar}{m_e r_1} = \frac{1.05 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.109 \times 10^{-31} \text{ kg})(.0529 \times 10^{-9} \text{ m})}$$

$$= 2.18 \times 10^6 \frac{\text{m}}{\text{s}}$$

60.



Ground State: $r = r_1 = .0529 \times 10^{-9} \text{ m}$

Coulomb's Law: $\vec{F} = \frac{k_o q_1 q_2}{r^2} \hat{r}$

$q_1 = \text{hydrogen nucleus (1 proton)} = +e$

$q_2 = \text{electron} = -e$

$k_o = \frac{1}{4\pi\epsilon_o} = \frac{1}{4\pi(8.85 \times 10^{-12})} = 8.99 \times 10^9$

$$\vec{F} = \frac{k_o (+e)(-e)}{r_1^2} \hat{r} = \frac{-(8.99 \times 10^9)(1.6 \times 10^{-19})^2}{(.0529 \times 10^{-9})^2} \hat{r} = \frac{-2.3 \times 10^{-28}}{2.798 \times 10^{-21}} \hat{r}$$

$$= -8.22 \times 10^{-8} \hat{r} \text{ N} \quad \text{or} \quad 8.22 \times 10^{-8} \left(\underset{\substack{\text{attractive} \\ \text{force}}}{\hat{r}} \right) \text{ N}$$

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66. Atomic # for Helium = $Z_{\text{He}} = 2$

$n = 4$ (3rd excited state)

General E_n equation for Elements other than Hydrogen:

$$\rightarrow E_n = - \frac{2\pi^2 k_e^2 e^4 m_e}{h^2} \frac{Z^2}{n^2}$$

$$E_{4\text{He}} = \frac{-2\pi^2 (8.99 \times 10^9)^2 (1.6 \times 10^{-19})^4 (9.109 \times 10^{-31})}{(6.626 \times 10^{-34})^2} \cdot \frac{2^2}{4^2}$$

$$= \frac{-9.524 \times 10^{-85}}{4.39 \times 10^{-67}} \left(\frac{1}{4}\right)$$

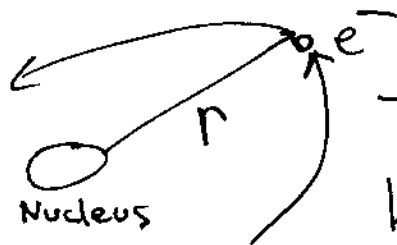
$$= -5.42 \times 10^{-19} \text{ J} \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) = \underline{-3.39 \text{ eV}} \quad E_{4\text{Helium}}$$

$$E_{2\text{Hydrogen}} = \frac{-13.6 \text{ eV}}{n^2} = \frac{13.6 \text{ eV}}{4} = \underline{-3.4 \text{ eV}}$$

1st exc. state
 $n=2$

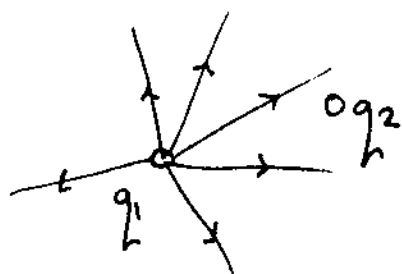
Apparently Helium's 3rd excited state has almost the same exact energy as Hydrogen's 1st excited state. Not that ~~that~~ any of it is that exciting at all...

67.



The orbiting electron has kinetic and potential energy.

Potential Energy between 2 charges is $\frac{k_0 q_1 q_2}{r}$



Potential or "voltage" from

$$q_1: V(r) = \frac{k_0 q_1}{r}$$

$$\text{Potential Energy of } q_2 = (\text{Potential at } q_2 \text{'s point in space}) (\text{Charge of } q_2)$$

$$= V(r) q_2$$

$$\text{P.E.} = \frac{k_0 q_1 q_2}{r}$$

$$\text{By Symmetry, the P.E. of } q_1 = \text{P.E. of } q_2 = \frac{k_0 q_1 q_2}{r}$$

let q_1 be the nucleus

$$q_1 = (\# \text{ of protons}) (\text{charge of proton})$$

$$= (\underbrace{Z}_{\substack{\text{Atomic} \\ \#}}) (\underbrace{1.6 \times 10^{-19} \text{ C}}_{+e})$$

let q_2 be the electron

$$q_2 = (-1.6 \times 10^{-19} \text{ C})$$

$$\quad \quad \quad \underbrace{\quad}_{-e}$$

67. (cont.)

$$\begin{aligned}\therefore P.E. &= \frac{k_o q_1 q_2}{r} = \frac{k_o (Z)(+e)(-e)}{r} \\ &= -\frac{k_o Z e^2}{r}\end{aligned}$$

$$\text{Kinetic energy} = \frac{1}{2} m_e v^2$$

$$\text{from Angular Momentum: } L_n = m_e v r = \frac{n h}{2\pi}$$

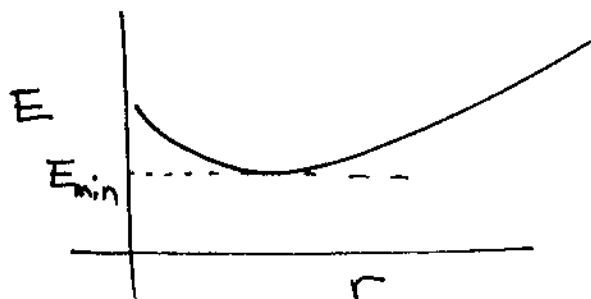
$$\Rightarrow v = \frac{n h}{2\pi m_e r}$$

$$v^2 = \frac{n^2 h^2}{4\pi^2 m_e^2 r^2}$$

$$\rightarrow \frac{1}{2} m_e v^2 = \frac{m_e}{2} \frac{n^2 h^2}{4\pi^2 m_e^2 r^2} = \frac{n^2 h^2}{8\pi^2 m_e r^2} = K.E.$$

$$E_{\text{total}} = K.E. + P.E.$$

$$E(r) = \frac{n^2 h^2}{8\pi^2 m_e r^2} - \frac{k_o Z e^2}{r} \quad \text{as required in book}$$



$$E_{\min} \text{ when } \frac{dE}{dr} = 0 \quad \& \quad \frac{d^2 E}{dr^2} > 0$$

67. (cont.)

$$E(r) = \frac{n^2 \hbar^2}{8\pi^2 m_e} r^{-2} - k_0 Z e^2 r^{-1}$$

$$\frac{dE}{dr} = \frac{n^2 \hbar^2}{8\pi^2 m_e} (-2) r^{-3} - k_0 Z e^2 (-1) r^{-2} = 0$$

$$-\frac{n^2 \hbar^2}{4\pi^2 m_e r^3} + \frac{k_0 Z e^2}{r^2} = 0$$

$$\frac{k_0 Z e^2}{r^2} = \frac{n^2 \hbar^2}{4\pi^2 m_e r^3}$$

$$k_0 Z e^2 = \frac{n^2 \hbar^2}{4\pi^2 m_e r}$$

$$r = \frac{n^2 \hbar^2}{4\pi^2 m_e (k_0 Z e^2)} = \frac{\hbar^2}{4\pi^2} \frac{n^2}{m_e k_0 Z e^2} \quad \hbar = \frac{h}{2\pi}$$

$$r_{\text{min or max}} = \frac{\hbar^2 n^2}{m_e k_0 Z e^2}$$

$$\frac{d^2 E}{dr^2} = \frac{n^2 \hbar^2}{8\pi^2 m_e} (-2)(-3) r^{-4} - k_0 Z e^2 (-1)(-2) r^{-3}$$

$$= \frac{3 n^2 \hbar^2}{4\pi^2 m_e r^4} - \frac{2 k_0 Z e^2}{r^3}$$

67. (cont.)

$$\therefore \left. \frac{d^2E}{dr^2} \right|_{r=r_{\min} \text{ or } r_{\max}} = \frac{3n^2 \hbar^2}{4\pi^2 m_e \left[\frac{\hbar^2 n^2}{4\pi^2 m_e k_0 Z e^2} \right]^4} - \frac{2k_0 Z e^2}{\left[\frac{\hbar^2 n^2}{4\pi^2 m_e k_0 Z e^2} \right]^3}$$

$$= \frac{3 \cancel{n^2} \hbar^2 (256 \overset{64}{\pi^6} \overset{6}{m_e^3} \overset{4}{Z^4} \overset{8}{e^8} \overset{4}{k_0^4})}{4 \cancel{\pi^2} m_e \hbar \overset{6}{n^6}} - \frac{2k_0 Z e^2 (64 \pi^6 m_e^3 k_0^3 Z^3 e^6)}{\hbar^6 n^6}$$

$$= \frac{192 \pi^6 m_e^3 Z^4 e^8 k_0^4}{\hbar^6 n^6} - \frac{128 \pi^6 m_e^3 Z^4 e^8 k_0^4}{\hbar^6 n^6}$$

$$= \frac{(192-128) \pi^6 m_e^3 Z^4 e^8 k_0^4}{\hbar^6 n^6}$$

$$= \frac{64 \pi^6 m_e^3 Z^4 e^8 k_0^4}{\hbar^6 n^6} > 0$$

$$\therefore r_{\min \text{ or } \max} = \frac{\hbar^2 n^2}{m_e k_0 Z e^2} = \underline{r_{\min}}$$

$$\therefore E_{\min}(r=r_{\min}) = \frac{n^2 \hbar^2}{8\pi^2 m_e \left[\frac{\hbar^2 n^2}{4\pi^2 m_e k_0 Z e^2} \right]^2} - \frac{k_0 Z e^2}{\frac{\hbar^2 n^2}{4\pi^2 m_e k_0}}$$

67(cont.)

$$= \frac{\cancel{n^3} \cancel{h^2} (16 \cancel{\pi^2} \cancel{m_e^2} \cancel{k_0^2} \cancel{Z^2} e^4)}{\cancel{8 \pi^2} \cancel{m_e} \cancel{h^2} \cancel{n^2}} - \frac{k_0 Z e^2 (4 \pi^2 m_e k_0 Z e^2)}{h^2 n^2}$$

$$= \frac{2 \pi^2 m_e k_0 Z^2 e^4}{h^2 n^2} - \frac{4 \pi^2 m_e k_0 Z^2 e^4}{h^2 n^2}$$

$$= \frac{(2-4) \pi^2 m_e k_0 Z^2 e^4}{h^2 n^2} = \frac{-2 \pi^2 m_e k_0 Z^2 e^4}{h^2 n^2}$$

$$= \frac{-4 \pi^2 m_e k_0 Z^2 e^4}{h^2 2 n^2}$$

$$E_{\min} = \frac{-m_e k_0 Z^2 e^4}{2 h^2 n^2}$$