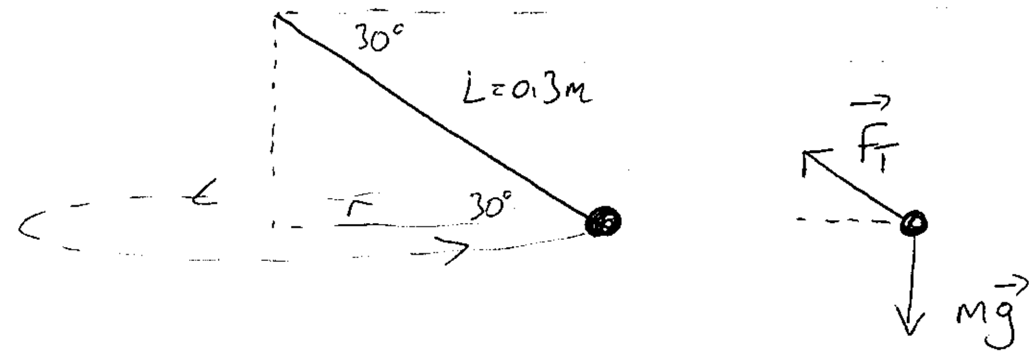
Quiz 4 Solutions

1.



a) Equate vertical forces: $mg = F_T \sin 30° = \frac{1}{2} F_T$

$$\Rightarrow F_T = 2mg = 2 \times 0.01\,\text{kg} \times 10\,\text{m/s}^2 = 0.2\,\text{N}$$

b) i) Rotation radius $r = L \cos 30° = 0.3 \times \frac{\sqrt{3}}{2} = 0.2598\,\text{m}$

ii) Equating horizontal forces $F_c = \frac{mv^2}{r} = F_T \cos\theta$

Using ~~mg~~ $F_T = 2mg$ from (a)

$$mv^2/r = 2mg\cos\theta \Rightarrow v^2 = gr\cos\theta = gL\cos^2\theta$$

i.e. $v = \cos 30° \sqrt{gL} = 1.5\,\text{m/s}$

iii) So period $T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi r}{v} = 1.088\,\text{s}.$

c) If F_T now increases to 0.4N

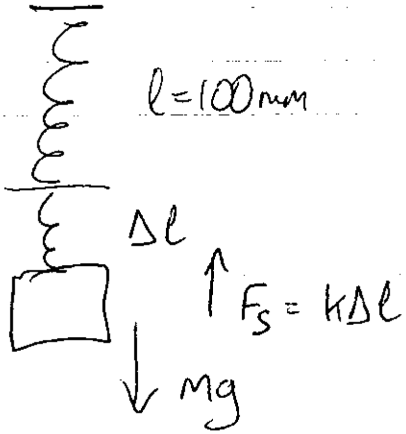
from $F_T \sin\theta = mg$, $\sin\theta = \frac{mg}{F_T} = \frac{0.01 \times 10}{0.4} = 1/4$

i) so $\theta = 14.48°$

ii) Use $v^2 = gr\cos\theta = gL\cos^2\theta$ with $\theta = 14.48°$

$$\Rightarrow v = \cos 14.48 \sqrt{gL} = 1.677\,\text{m/s}$$

2a)



At equilibrium, forces on the mass

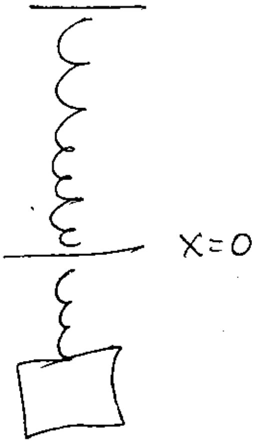
$$mg = k\Delta l$$

$$\Rightarrow \Delta l = \frac{mg}{k} = \frac{10 \times 0.2\,kg}{25}$$

$$= 0.08\,m \text{ or } 80\,mm$$

So new length $= \underset{l}{100\,mm} + \underset{\Delta l}{80\,mm} = \underline{180\,mm}$

b)



Use $\omega^2 = k/m$

$$\Rightarrow \omega = 2\pi f = \sqrt{k/m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25}{0.2}}$$

$$= 1.78\ Hz$$

c) Max. accel $= \pm\,\omega^2 A$ with amplitude $A = 0.02\,m$ (20

$$= \frac{k}{m}A = \frac{25}{0.2} \times 0.02 = 2.5\ m/s^2$$

d) Max. KE $\frac{1}{2}mV_{max}^2$ = Max P.E. $= \frac{1}{2}kA^2$

$$\Rightarrow V_{max} = \sqrt{\frac{k}{m}} \cdot A = \omega A$$

$$= \sqrt{\frac{25}{0.2}} \times 0.02 = 0.2236\ m/s$$

3.

$x = A \sin \omega t$

a) For $\omega^2 = \frac{g}{L}$, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = 1s$

$\Rightarrow$ Length required $L = \frac{gT^2}{4\pi^2} = \frac{10 \times 1^2}{4\pi^2} = 0.25$

b) For this length, rearranging: $g_M = \frac{4\pi^2 L}{T_M^2}$ with $T_M = 2.4$

$g_M = \frac{4\pi^2 \times 0.253}{2.46^2} = 1.65\ m/s^2$

c) Speed $v = \frac{dx}{dt} = \omega A \cos \omega t$. Initial speed $v(0) = \omega A$
$= 0.2\ m/s$

$\Rightarrow$ amplitude $A = \frac{v(0)}{\omega} = v(0)\sqrt{\frac{L}{g}}$

i) On earth ($g = 10\ m/s^2$), $A_E = 0.2\sqrt{\frac{0.253}{10}} = 0.0318m$

ii) On Mars, $g = 1.65 m/s^2$, $A = 0.2\sqrt{\frac{0.253}{1.65}} = 0.0783m$

- lager swing on Mars, since bob must swing higher to