1a. "For every action (applied force) there is an equal and opposite reaction"

When 2 objects A and B collide, at any instant

A B — force of A on B $\vec{F}_{AB}$

$\overleftarrow{F_{BA}}$ $\overrightarrow{F_{AB}}$ $= -$ force of B on A $\vec{F}_{BA}$

Therefore, integrating over the time of the collision t,

the impulse (change in momentum)

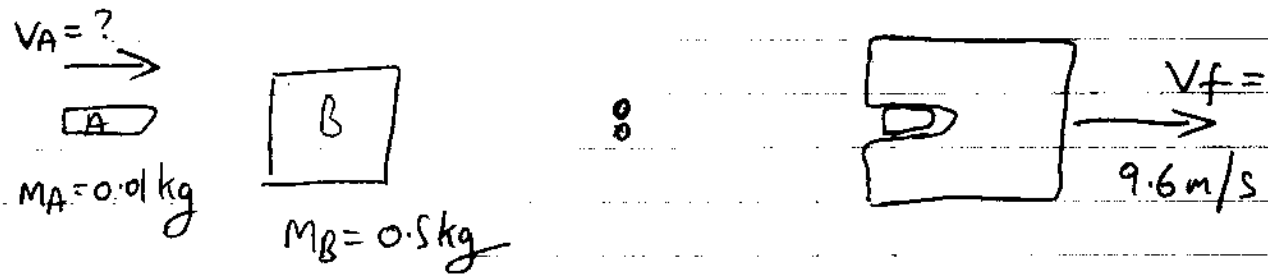
$$\Delta(M_A V_A) = \int F_{BA}\,dt = -\int F_{AB}\,dt = -\Delta(M_B V_B)$$

So for a system with no <u>external</u> forces

$$\Delta(M_A V_A) + \Delta(M_B V_B) = 0 \quad \text{i.e. } \Delta(p_A + p_B) = 0$$

so momentum is conserved.

b)



Final momentum $p_f = (M_A + M_B)V_f = (0.5 + 0.01) \times 9.6 = 4.896$ kg m/s

$= $ initial momentum $p_i = M_A V_A$

$\Rightarrow$ bullet speed $V_A = \dfrac{p_f}{M_A} = \dfrac{4.896}{0.01} = \underline{489.6 \text{ m/s}}$

c)

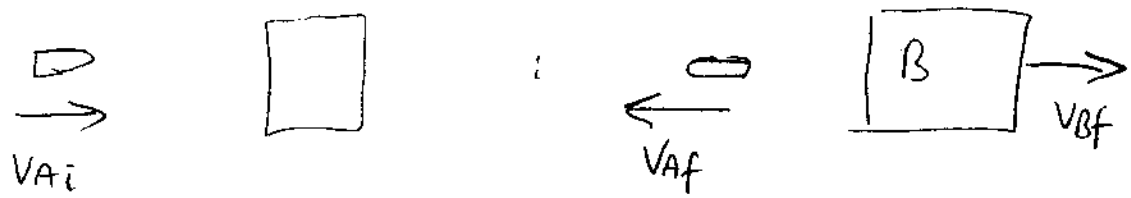
Initial momentum $p_i = M_A V_A \Rightarrow$ initial K.E. $= \frac{1}{2} M_A V_A^2 = \dfrac{p_i^2}{2M_A}$

Final " $p_f = p_i \Rightarrow$ final KE $= \dfrac{p_i^2}{2(M_A + M_B)}$

c) contd: So change in K.E. $\Delta KE = \frac{p_i^2}{2}\left(\frac{1}{m_A} - \frac{1}{m_A+m_B}\right)$

$$= \frac{4.896^2}{2}\left(\frac{1}{0.01} - \frac{1}{0.51}\right) = 1175.04\,J$$

d) For a rubber bullet, elastic collision $\Delta KE = 0$



In this case $m_A << m_B$, so bullet (m_A) rebounds

with $V_{Af} \approx -V_{Ai}$ (as if it had hit a wall)

$\therefore$ Impulse on A $\Delta(m_A v_A) \approx m_A V_{Ai} - (-m_A V_{Ai}) \approx 2 m_A V_{Ai}$

$$= -\text{impulse on } B = m_B V_{Bf}$$

Compare to inelastic case where A is almost brought to rest
i.e. Impulse $\Delta(m_A v_A) \approx m_A V_{Ai}$, only ½ of elastic case.

So, the block remains undamaged (no KE converted to work), but receives a bigger impulse ("kick") from the rubber bullet vs. the steel bullet.

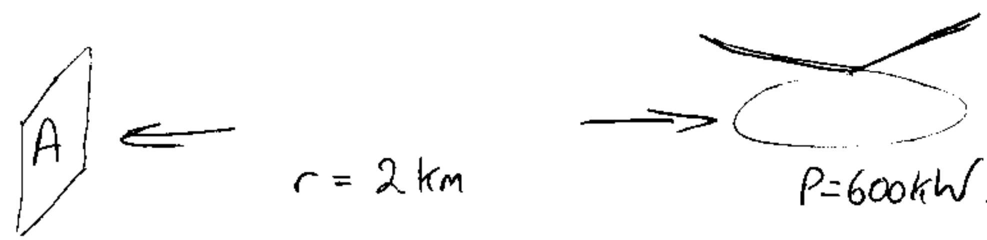
In fact the block's final speed is roughly given by

$$m_B V_{Bf} \approx 2 m_A V_{Ai} \Rightarrow V_{Bf} \approx 2\frac{m_A}{m_B} V_{Ai}$$

with $V_{Ai} = 489.6\,m/s$, $\Rightarrow V_{Bf} \approx \frac{2 \times 0.01}{0.5} V_{Ai} \approx 19.5\,m/s$,

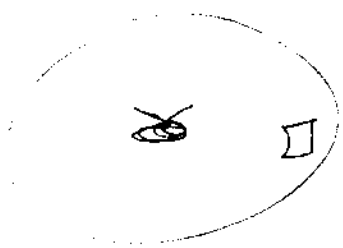
~ double the speed of part (a).

2.



a) Wavelength $\lambda = \frac{v}{f} = \frac{330\,m/s}{15\,Hz} = 22\,m$.

b) If Power $P = 600kW$ radiated over sphere



area A intercepts a fraction $\frac{A}{4\pi r^2}$ of power, so

power intercepted $= \frac{P.A}{4\pi r^2}$

For $P = 600\times10^3 W$, $A = 2.5m^2$, $r = 2\times10^3 m \Rightarrow \frac{600\times10^3 \times 2.5}{4\pi\,(2\times10^3)^2}$

$= 29.8\times10^{-3} W \approx 30mW$

c) For vibrating window on "putty springs", $\omega^2 = k/m$

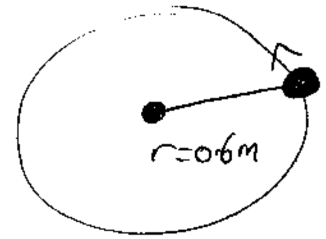
i) $\Rightarrow$ spring constant $k = m\omega^2 = m\,(2\pi f)^2$

$= 6kg \times 4\pi^2\,(15^2) = 53.30\ N/m$

ii) Restoring force $F = kx$, maximum when x = amplitude A

$\Rightarrow F_{max} = kA = 5330kN \times 2mm = \underline{106.6\,N}$

3. Top view:



a) Speed = $\dfrac{\text{Circumference}}{\text{time}} = \dfrac{2\pi r}{T} = 2\pi r f = 2\pi \times 0.6 \times 4$
$= 15.08\,m/s$
$(15.1\,m/s)$.

b) tensile force F_T provides centripetal force $F_T = F_c = \dfrac{mv^2}{r}$
$= 0.3kg \times \dfrac{(15.08)^2}{0.6} = 113.7\,N$.

c) Side view:

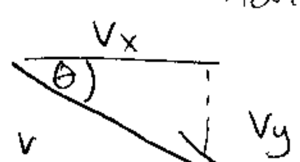
initial height y_0
$= 3.25\,m$
initial speed.
$v = 15.08\,m/s$
in x-direction.

i) Time of flight t_f given by $y = y_0 + \underbrace{V_{y0}t_f}_{=0} - \frac{1}{2}g t_f^2 = 0$

i.e. $t_f = \sqrt{\dfrac{2y_0}{g}} = \sqrt{\dfrac{2 \times 3.25}{10}} = 0.806s$

ii) Therefore range $= V_x t_f = 15.08\,m/s \times 0.806s = 12.16\,m$

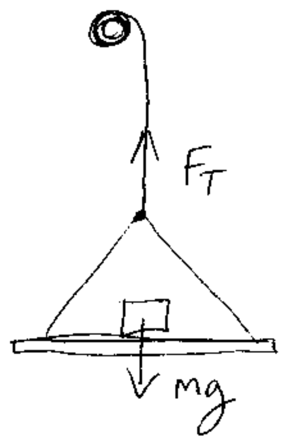
d) At $t = t_f$, vertical speed $V_y = 0 - g t_f = -10 \times 0.806 = 8.06\,m/s$
horiz: " $V_x = 15.08\,m/s$



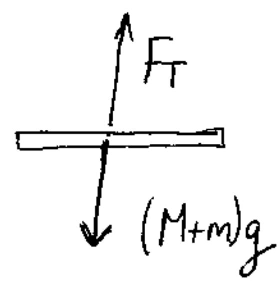
$\Rightarrow$ speed $v = \sqrt{V_x^2 + V_y^2} = 17.1\,m/s$

angle $= \tan^{-1}\left(\dfrac{V_y}{V_x}\right) = 28.1°$

4.



For elevator mass M



Net upwards force $F_T - (M+m)g$

$= M \times a$ (Newton II)

i.e. $Ma = F_T - (M+m)g$ (1)

For load, mass m:

net force upwards:

$ma = F_N - mg$ (2) with same acceleration as in (1)

a) At constant speed or stationary, $a = 0$

$\Rightarrow$ i) $F_T = (M+m)g = (10+20)\times 10 = \underline{300N}$

ii) eff. weight = normal force $F_N = mg = 20 \times 10 = \underline{200N}$.

b) i) With $a = -2\,m/s^2$, from (1): $F_T = (M+m)g + Ma$

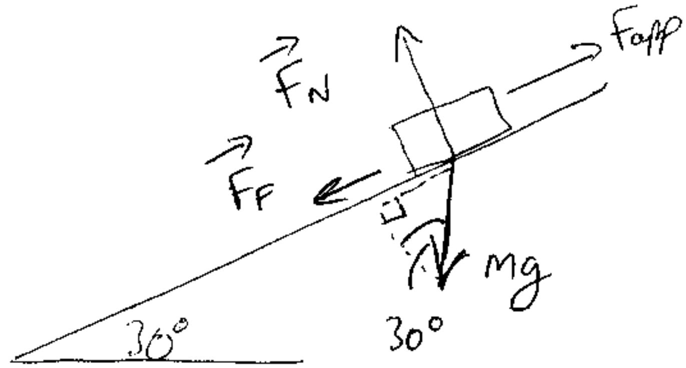
$= 300N - 10 \times 2 = \underline{280N}$

ii) From (2), $F_N = m(g+a) = 20 \times (10-2) = \underline{160N}$

c) Work done = $\Delta PE = (M+12m)gh = (10+240)\times 10 \times 6 = 15kJ$

$\Rightarrow$ required power $= \frac{\text{Work}}{\text{time}} = \frac{15000J}{10s} = 1500W$ or $\underline{1.5kW}$

5.

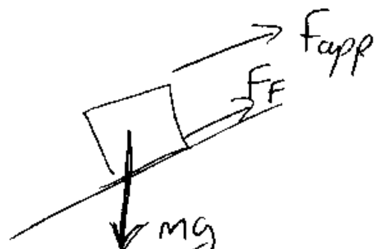


a) Max friction force $F_F \le \mu F_N$

equating forces ⊥ to slope, $F_N = mg\cos\theta$

$\Rightarrow F_F \le \mu mg\cos\theta = 0.1 \times 2000\text{N} \times \cos 30° = \underline{173.2\text{N}}$

b)
i)



To prevent sliding downwards
(F_F acts uphill)

equate ∥ forces, $F_{app} + F_F = mg\sin\theta$

$\Rightarrow \quad F_{app} = mg\sin\theta - \mu mg\cos\theta = mg(\sin 30° - \mu\cos 30°)$
$= \underline{826.8\text{N}}$

ii) To pull uphill, minimum $F_{app} = mg\sin\theta + F_F$
$= mg(\sin 30 + \mu\cos 30) = 1000\text{N} + 173.2\text{N}$
$= \underline{1173.2\text{N}}$.

c) If $F_{app} = 1500\text{N}$, along slope of length $\ell = \frac{h}{\sin\theta} = 24\text{m}$
i) $\Rightarrow$ Work done $W = 1500 \times 24 = 36000\text{J}$

ii) Work done $= \Delta PE + \Delta KE +$ (work lost to friction)
$W = mgh + \frac{1}{2}mv^2 + F_F \cdot \ell = 36000$

$\Rightarrow \frac{1}{2}mv^2 = 36000 - mgh - F_F\ell$
$= 36000 - 2000 \times 12 - 173.2 \times 24 = 7843.2\text{J}$

$\Rightarrow v = \sqrt{\frac{2 \times 7843.2}{m}} = \underline{8.85\text{ m/s}}$