Questions

Q: Which stays airborne longer: a bullet dropped from shoulder-height, or a bullet fired horizontally from rifle at 700 m/s ?

Q: An athlete {throws javelin / dives off cliff}. How does initial speed and direction affect (i) time of flight, (ii) distance traveled, (iii) angle of impact ?

Q: Do we need to escape Earth's gravity to be (or feel) "weightless" ?

Acceleration due to Gravity - Free Fall

(Physics at last!) Galileo found by experiment

"All bodies fall at the same acceleration"

(neglecting air friction)

as angle of plane ↑

accel a ↑, but a = constant

for given angle

- measured v, t to find accel. down slope

On earth, all bodies fall at constant

$a = (-)g = (-)\ 9.81 m/s^2$

- objects pick up speed quickly!

e.g. 0 to 26.7 m/s (60 mph) in $t = \frac{26.7-0}{9.81} = 2.7s$

covering a height $h = \frac{1}{2}gt^2 = \frac{v^2}{2g} = 36.25\ m$

If initial speed $v_0 \neq 0$ (upwards or downwards)

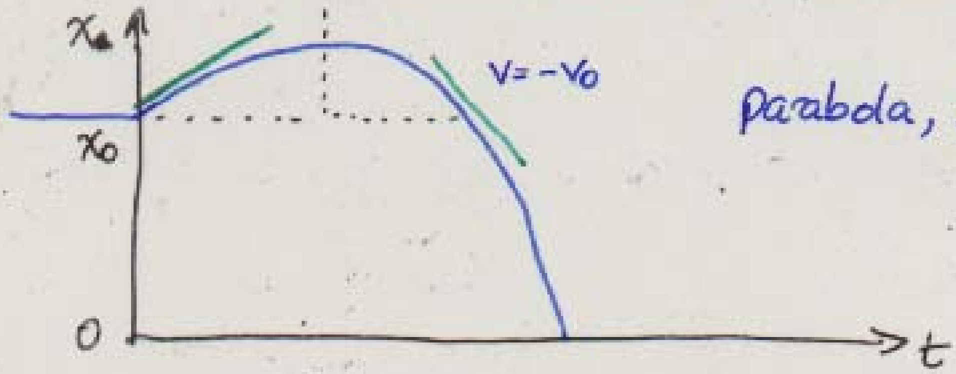
- just add or subtract from final speed

i.e. $v = v_0 - gt$: acceleration is the same

↓

change in speed $(v - v_0)$ during flight is the same for same t

Free-fall motion of diver under constant "g":



parabola, symmetric about peak x

x_0 = initial height
v_0 = initial speed

Diver's height is a max. when $\frac{dx}{dt} = 0 = v = v_0 - gt$

i.e. $t\,(\max) = v_0/g$

Max. height given by $v^2 = v_0^2 - 2g(x - x_0)_{max} = 0$

i.e. $x\,(\max) = x_0 + \frac{v_0^2}{2g}$

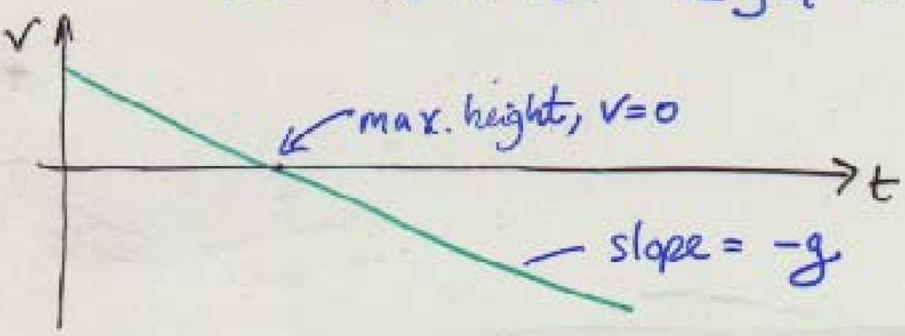
When diver hits pool ($x = 0$):

Impact speed $v_i^2 = v_0^2 - 2g(-x_0) = \underline{v_0^2 + 2g x_0}$

Time of flight $t_i = \frac{(v_i - v_0)}{a} = \frac{(v_i - v_0)}{-g}$

or given by solution of quadratic eqn:

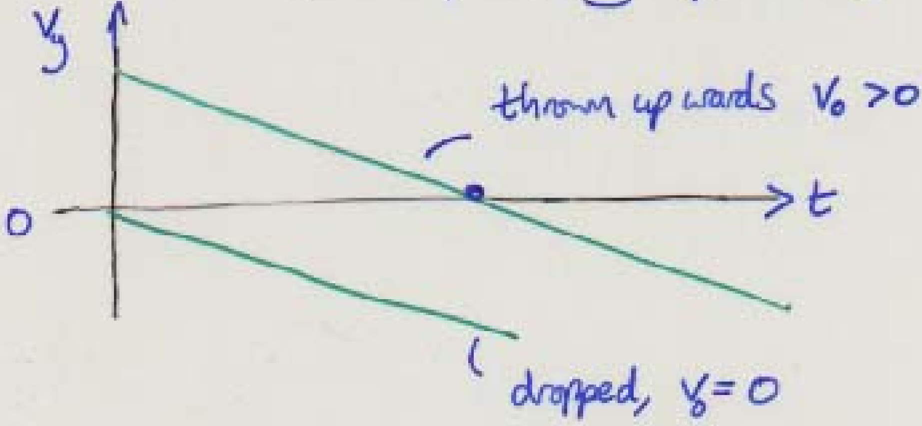
$x = x_0 + v_0 t - \frac{1}{2} g t_i^2 = 0$



Free-Fall in 2D : Projectiles

Projectile : thrown with initial velocity, moves under influence of gravity only : follows a trajectory in (x, y)

"Free-fall" = accelerating downwards under gravity
(even if moving upwards)



$v = v_0 - gt$ (y_0)
↓ slope = $-g$

Note: For now, neglect air friction : choose low speeds, dense objects (high mass, small size)

At any point along 2D trajectory, can separate object's $\vec{v}$ into components:

$\vec{v} = \vec{v}_x + \vec{v}_y$

choose $\vec{g}$ to be ∥ to (−y) axis

Then: $\vec{v} = v_x \vec{i} + v_y \vec{j}$ where $v_x = \frac{dx}{dt}$ etc.

with $\frac{d^2x}{dt^2} = 0$, $\frac{d^2y}{dt^2} = -g$ since $\vec{g} = -g\vec{j}$

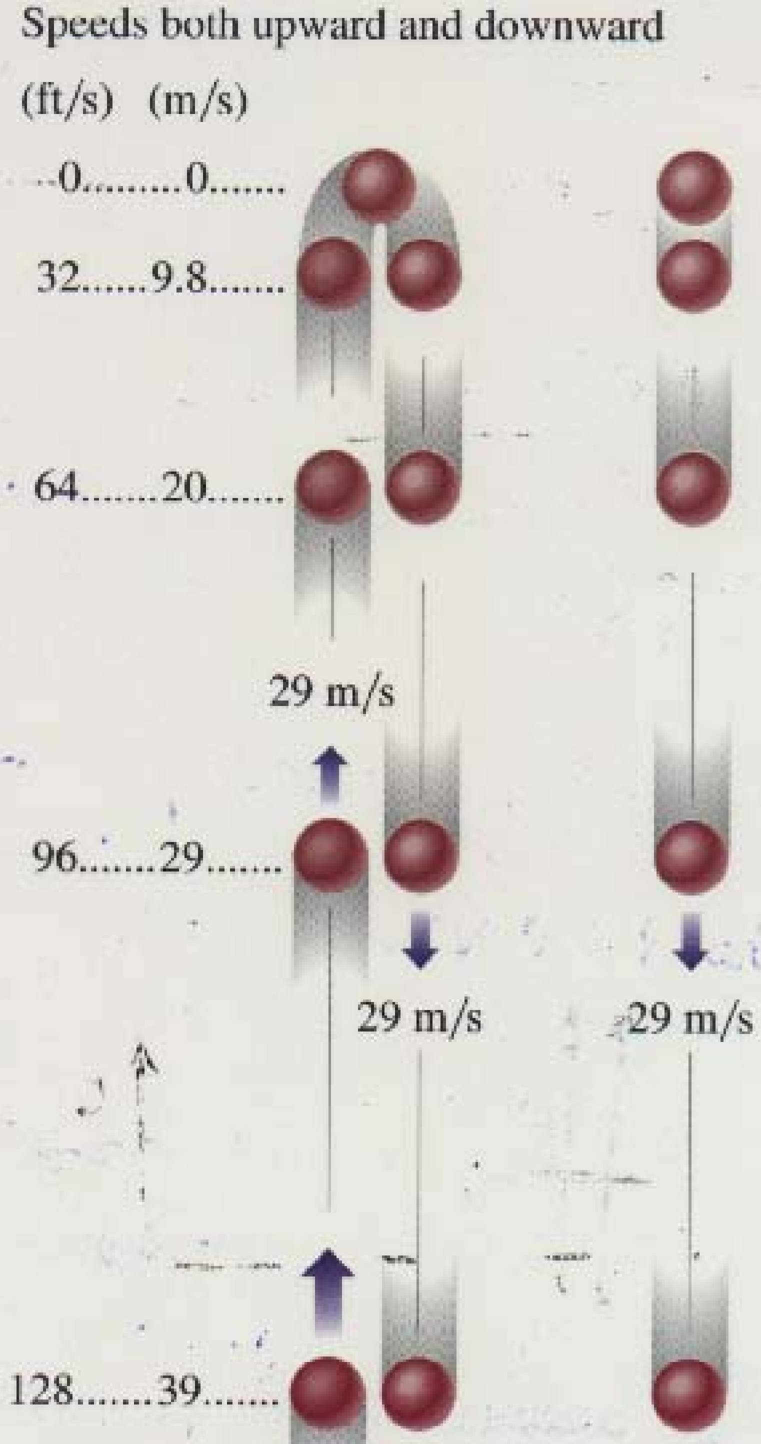
i.e. v_x = constant, $v_y = v_{y0} - gt$

– motion in x direction unaffected by gravity!

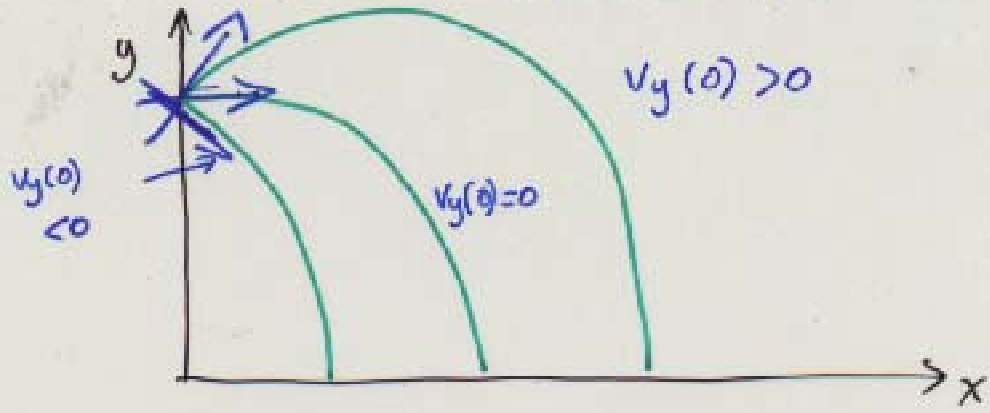
Copyright © 2000 by Brooks/Cole
A division of Thomson Learning

Figure 3.11

Peak altitude of a moving ball



Parabolic paths



Shape of parabola (incl. max height, max. range) depends on magnitude and direction of $\vec{v}_0$

Max. height is when $v_y = \frac{dy}{dt} = v_{y0} - g t_{max} = 0$

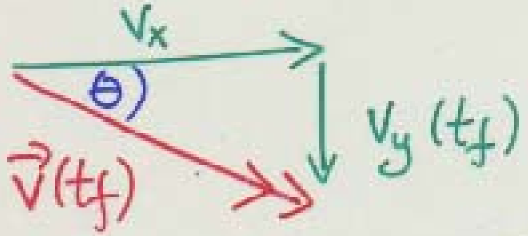
Time of flight t_f given by $y(t_f) = y_0 - v_{y0} t_f - \frac{1}{2} g t_f^2 = 0$

Can then find distance traveled (range) $x(t_f) = x_0 + v_x t_f$

Speed of impact, angle of impact found by adding components:

$$v_y(t_f) = v_{y0} - g t_f$$

$$v_x(t_f) = v_x(0) \; (= v_0 \cos\theta)$$



Ex 3.16, 3.17

Speed $v(t_f) = \sqrt{v_x^2 + v_y^2(t_f)}$

Angle $\theta = \tan^{-1} \frac{v_y(t_f)}{v_x(t_f)}$

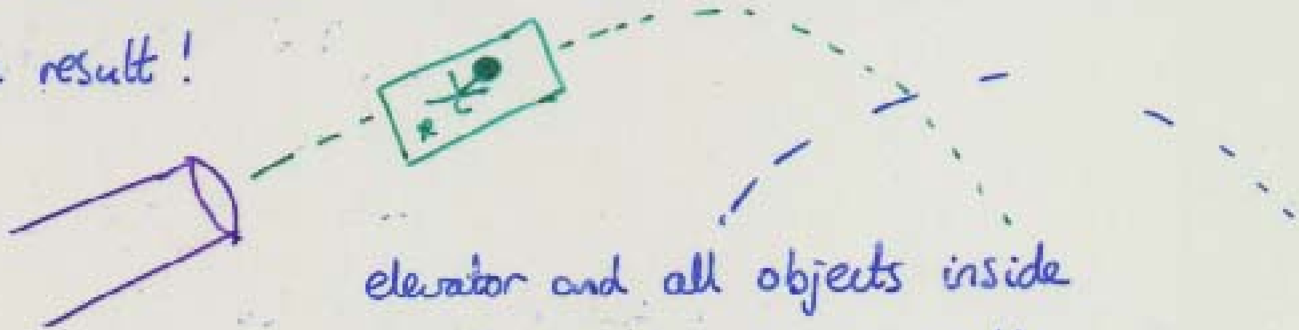
Free-fall and "Weightlessness"

Since all freely-falling objects accelerate at g

.... in falling elevator, no net accel. of objects (keys, internal organs) relative to each other.

So a being born in a falling elevator (or space-ship) will not know that gravity exists (if no windows)

We can also launch elevator upwards and still have same result!



elevator and all objects inside follow a free-fall parabolic path

- Used by NASA for astronaut training (and "Apollo 13" movie)
  - ~~Boeing 747~~ C130 "Vomit Comet" flies in a parabolic climb/dive

    ⇒ few minutes of "weightlessness"

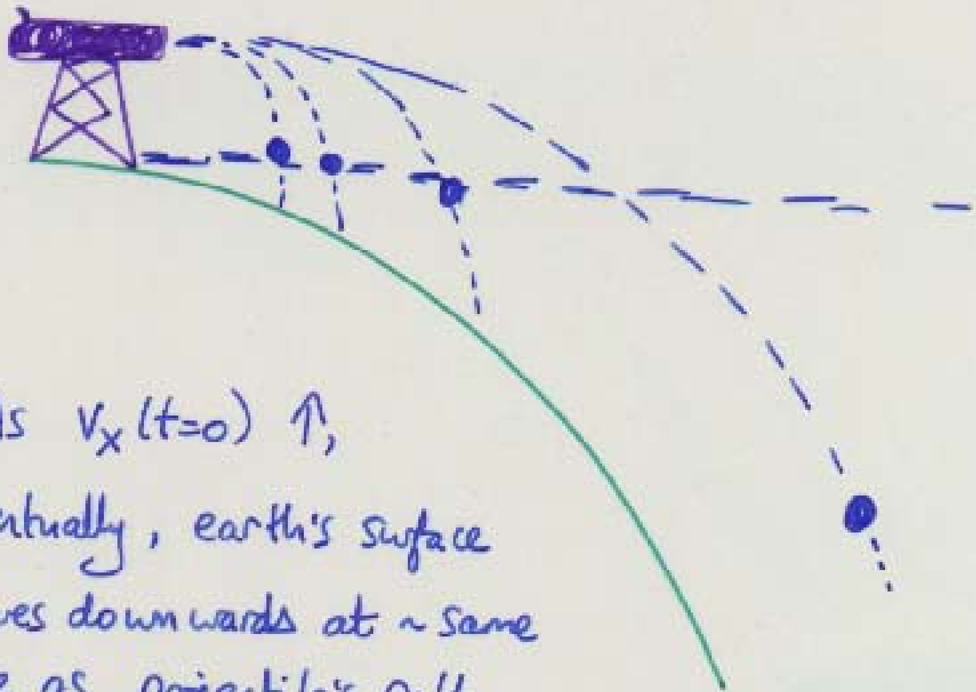
"Free-fall" ride at Magic Mountain

<u>But</u> height of ride $y_0 = \frac{1}{2} g t_f^2$

\- expensive!

Newton's Cannon - Weightlessness in Orbit

Newton theorized: build a powerful cannon on tall mountain. Fire horizontally, increasing initial speed:



As $v_x(t=0) \uparrow$,
eventually, earth's surface curves downwards at ~ same rate as projectile's path

→ projectile now in orbit!

∴ Shuttle crew in orbit are also in free-fall.
Gravity is still strong close to earth but no <u>net</u> acceleration inside shuttle → "weightlessness".