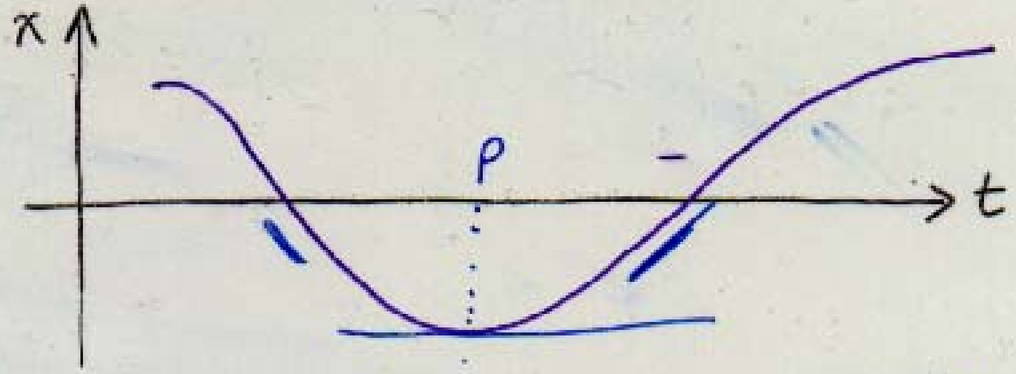
Reading Quiz #1 : Chapter 3, 4.1- 4.8

1. 

In the distance - time graph shown, the (speed, acceleration) at point P are :

a) negative, positive

b) zero, negative

c) zero, positive

d) positive, zero.

2. Atop the wall of a fort, a defender fires an arrow downwards at the attackers. Just after leaving the bow, the arrow's acceleration is

a) downwards, equal to g

b) downwards, greater than g

c) zero

d) downwards, but depends on its speed.

* (Neglect air friction) *

RQ1 cont/d.

3. The SI unit of force, the Newton, has units.

a) kg m s

b) kg m/s

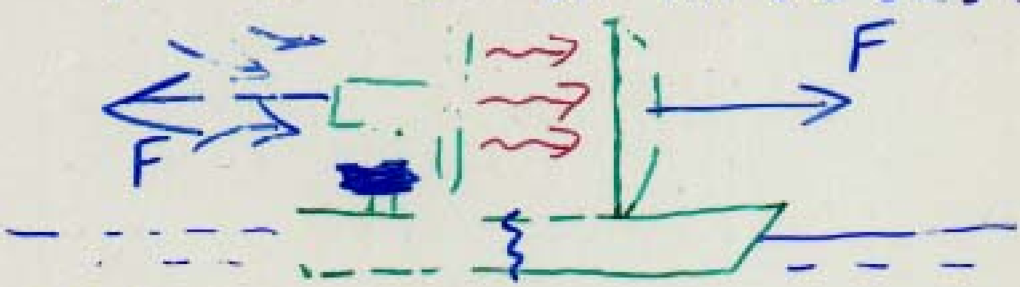
c) kg m/s^2

d) kg m/.

(Hint: Newton's 2nd law)

$F = m a$

4. In order to win a sailboat race, a clever boat owner mounts a fan on his boat to blow air into the sails:



At the start of the race, the fan is switched on. The sailboat then:

a) is blown to the right, as intended

b) moves to the left, since the fan acts like a propeller

c) moves right or left, depending on fan force vs. friction in the water

d) does not move at all.

(Hint: Newton's 3rd law!)

Week 2 HW

Ch 3: 11, 13, 15, 32, 58, 68-70, 78, 81, 94, 103, 111, 115, 121

Ch 4: 1, 3, 7, 8, 9, 13, 22, 26, 27, 39, 49, 65, 66, 67, 87, 96, 102, 106, 113, 118, 127, 134, 137.

.... still trying to get a classroom for Bart's Wednesday Problem Section.

Questions

- How does a car's stopping distance increase with speed?

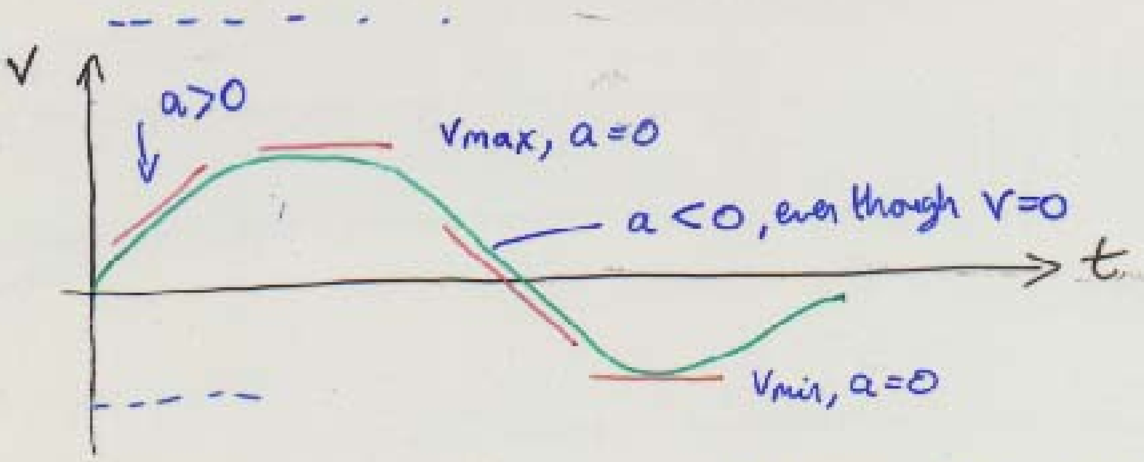
- If I throw a ball vertically upwards, how does its (1) time of flight (2) height reached ..., depend on the speed of launch?

- In a window-less, soundproof lab, can we easily tell if the lab is
  - stationary or moving at constant $\vec{v}$? No
  - accelerating ? Yes
  - rotating ? Yes.

10 Acceleration : Rate of Change of Speed

When motion is not uniform, i.e. $v(t) \neq$ constant

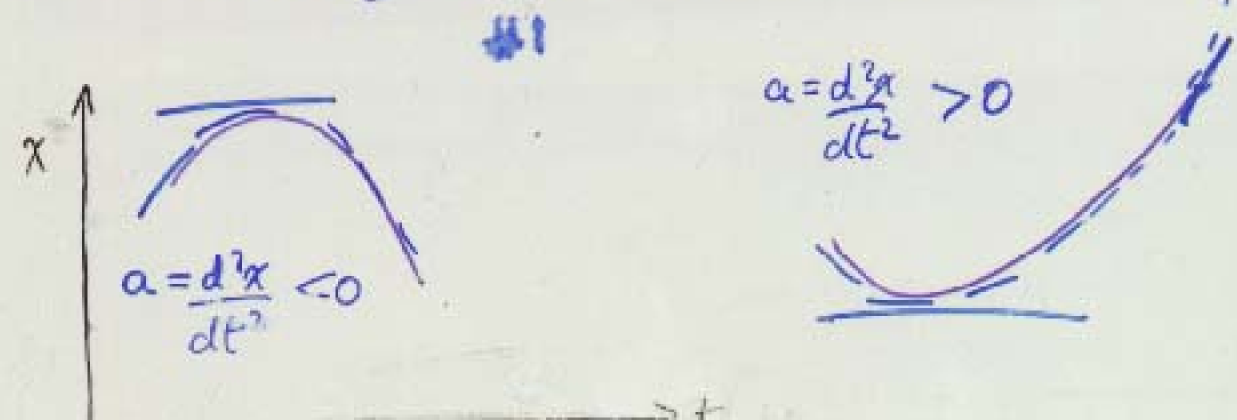
(test : equal time intervals $\not\Rightarrow$ equal distances).



Define Acceleration $a = \dfrac{dv}{dt}$ $[m/s^2]$

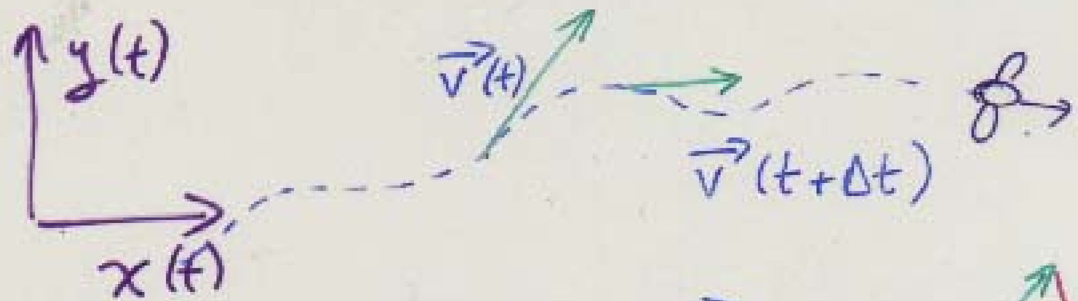
- Can be >0, <0 or $=0$ (constant v, or when v is a max. or a min.)
- does not depend on sign or size of v - only the rate of change.

Since $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$, can find "a" from curvature (change of slope) of position $x(t)$



4/2

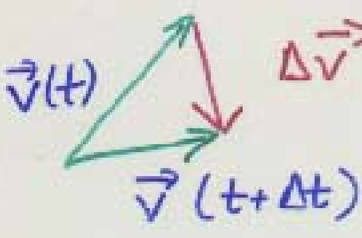
>1D Acceleration = Rate of Change of Velocity



Path of bee (top view)

$\vec{v}(t+\Delta t) = \vec{v}(t) + \Delta\vec{v}$

Change in velocity: $\Delta\vec{v}$:



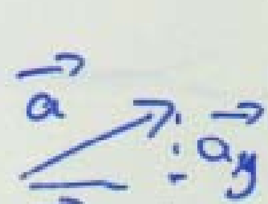
Can divide by Δt to get acceleration vector:

$$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} \rightarrow \frac{d\vec{v}}{dt} \text{ as } \Delta t \to 0$$

Notes:

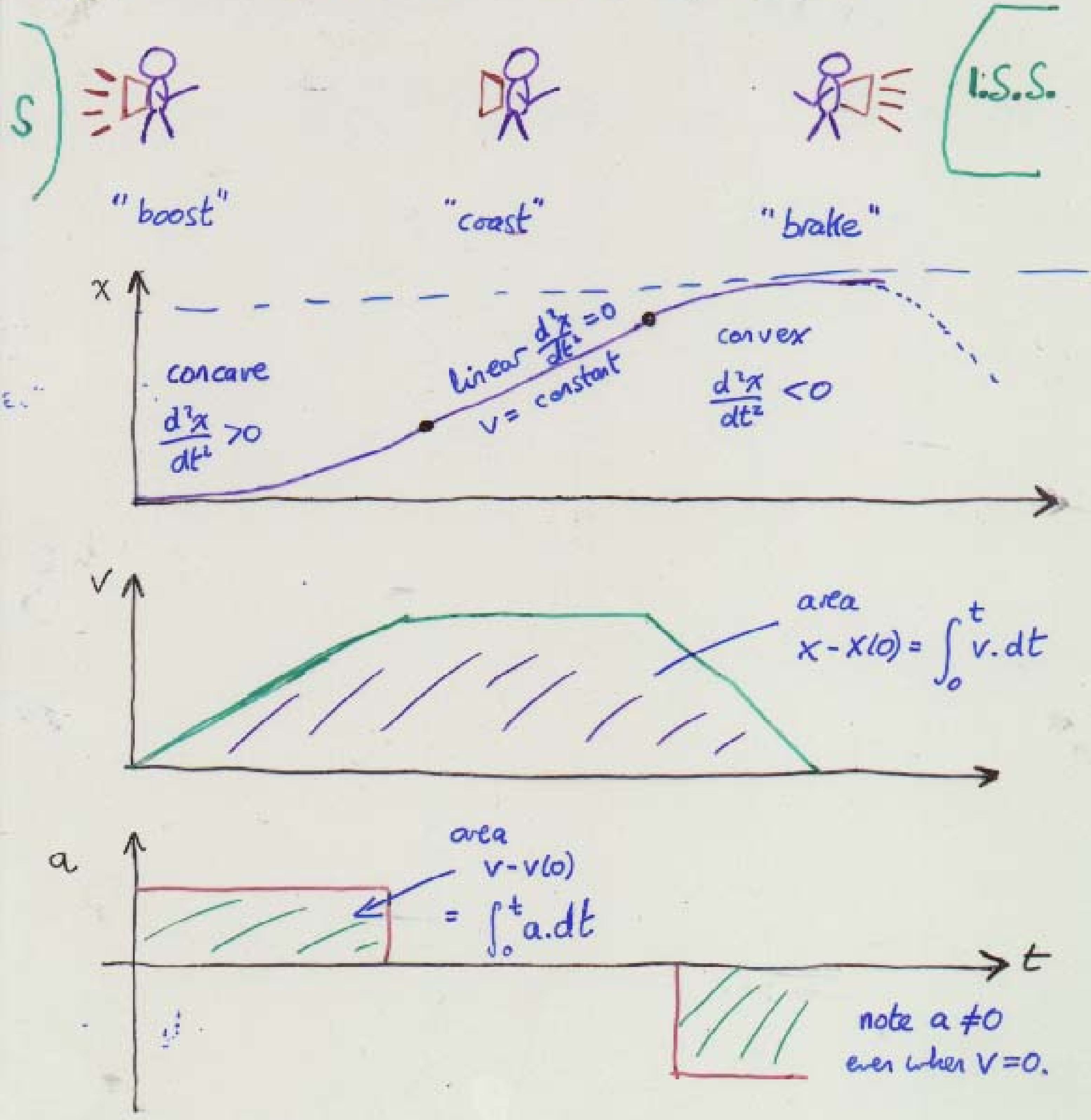
- $\vec{a}$ non-zero even if speed constant (but direction changes)
  i.e. $\vec{a}=0$ only when $\vec{v}$ = constant: speed and direction
  (c.f. circular motion, lab #1)
- Direction: $\vec{v}$ always tangential to path, but
  $\vec{a} = \frac{d\vec{v}}{dt}$ can be any direction, even ⊥ to path
- Components + Magnitude: If $\vec{v} = v_x(t)\vec{i} + v_y(t)\vec{j}$

$$|\vec{a}| = \sqrt{\underbrace{\left(\frac{dv_x}{dt}\right)^2}_{a_x^2} + \underbrace{\left(\frac{dv_y}{dt}\right)^2}_{a_y^2}} = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}$$



1D Position, Speed, Acceleration

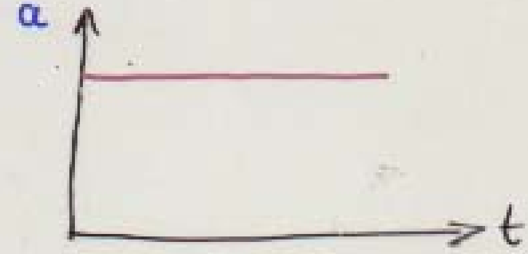
Eg. Astronaut "space walk" between Shuttle and ISS :



Constant Acceleration : Equations of Motion

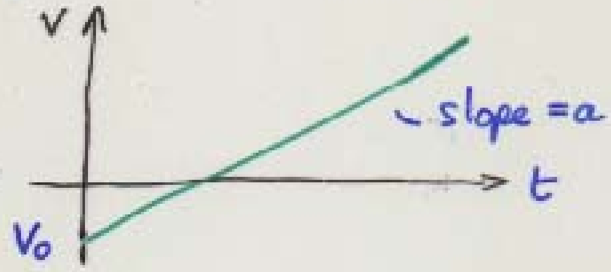
Note: If $\vec{a}$ constant, can choose direction to lie along either x or y or z axis:

If $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} =$ constant



Then speed $v = \int a.dt = v_0 + \int_0^t a.dt = \underline{v_0 + at}$

i.e. $v(t)$ is a straight line



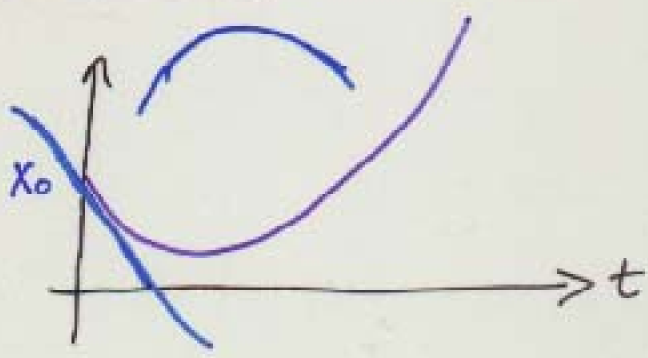
And position $x(t) = \int v.dt = x_0 + \int_0^t \underbrace{(v_0 + at)}_{v(t)}.dt$

$$\Rightarrow \quad \underline{x(t) = x_0 + v_0 t + \tfrac{1}{2}at^2}$$

$x(t)$ is a parabola

$x = x_0$ at $t = 0$

$v = v_0 = \frac{dx}{dt}$ at $t = 0$



If $a > 0$, parabola curves upwards (concave)

$a < 0$, " " downwards (convex)

(1D) Constant Acceleration: Equations of Motion cont'd:

We have $v = v_0 + at$ (1)

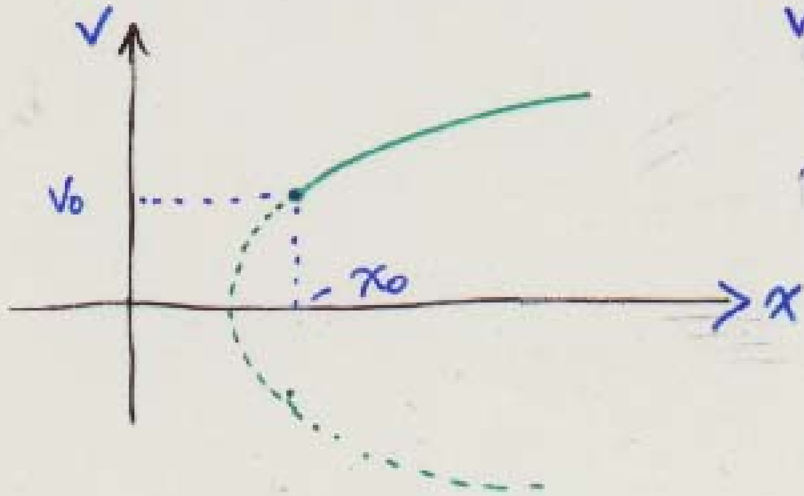
$x = x_0 + v_0 t + \frac{1}{2} a t^2$ (2) $= 0$?

$\rightarrow$ position and speed of object at any time t.

Can we find speed v as a function of position x?

Yes! From (1) $t = \frac{v - v_0}{a}$, substitute into (2)

$$\Rightarrow \quad v^2 = v_0^2 + 2a(x - x_0) \quad (3)$$



$v(x)$ is a sideways parabola

Note: Be careful with sign of $v = \pm\sqrt{v_0^2 + 2a(x - x_0)}$

Slope $\frac{dv}{dx} = \frac{\left(\frac{dv}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{a}{v}$ at any point.

Examples:

(1) Sports car accelerates from 0 to 60mph (= 96km/h or 26.67 m/s) in 5.5s. Find accel. and distance covered.

At $t=0$: given $v_0 = 0$, set $x_0 = 0$

Then at $t = 5.5s$: $a = \frac{v - v_0}{t} = \frac{26.67 - 0}{5.5} \frac{m/s}{s} = \underline{4.85\ m/s^2}$

Distance $x = \frac{1}{2}at^2$ <u>OR</u> $x = \frac{v^2}{2a} = 73.3\ m$

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(2) Same car has disc brakes that can decelerate it at $6\ m/s^2$.

What is the stopping distance at (i) $v_0 = 13.3\ m/s$ (30mph), (ii) $v_0 = 26.67\ m/s$ (60mph) ?

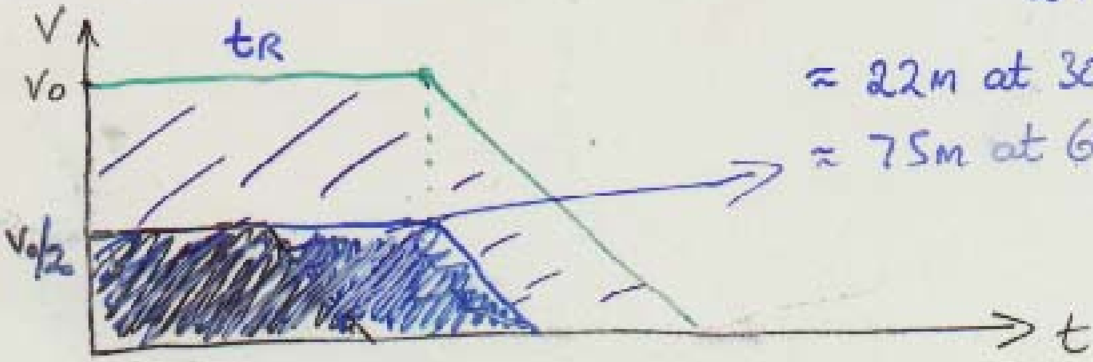
<u>Given</u> $a = -6 m/s^2$ (opposite to v), $v = 0$ (car stopped).

$\Rightarrow$ <u>"braking distance"</u> $x - x_0 = \frac{-v_0^2}{2a} \propto (\text{speed})^2$!!

$\approx$ 15m at 30mph, $\approx$ 60m at 60mph. (4x)

<u>BUT</u> must also add "<u>thinking distance</u>" $x_0 = v_0 t_R$ $\longrightarrow$ <u>reaction time</u>

$\Rightarrow$ total stopping distance $x = v_0 t_R + \frac{v_0^2}{2|a|}$



$\approx$ 22m at 30mph

$\approx$ 75m at 60mph.

Relative Motion - Inertial Frames of Reference

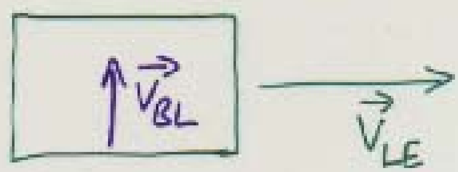
Velocity $\vec{v}$ is measured relative to some frame of reference (no "absolute standard of rest" in universe)

Can use vectors to transform motion from one frame to another.

e.g. "Lab on wheels" moves at constant $\vec{v}_{LE}$ w.r.t. earth

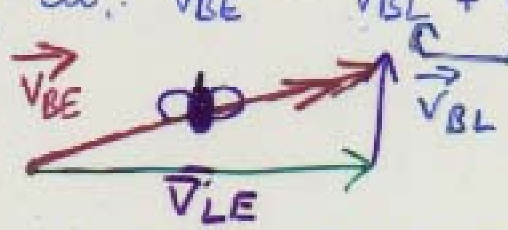
bee flies across lab at $\vec{v}_{BL}$

Top view:



We see bee fly at $\vec{v}_{BE} = \vec{v}_{BL} + \vec{v}_{LE}$

i.e.



If (lab, earth) are at relative rest or constant <u>velocity</u> then they are <u>inertial reference frames</u>

i.e. constant velocity in one frame ⇒ constant velocity in the other.

c.f. <u>rotating</u> frames in Lab #1

Test for Inertial Frames (week 2 lab)

Imagine a windowless lab

If lab moves with constant $\vec{v}$ i.e. $\frac{d\vec{v}}{dt} = 0$

(no acceleration, incl. no rotation)

then we can not tell whether we are moving or stationary

However, if "lab" accelerates $\frac{d\vec{v}_{LE}}{dt} \neq 0$ (speed and/or direction changes)

then objects in lab (incl. you!) starting with $\vec{v}_{OL}$ = constant

now have $\frac{d\vec{v}_{OL}}{dt} \neq 0$: appear to follow curved path relative to lab reference frame

e.g. In a train car, drop keys from one hand into other:

If $\vec{v}_{LE}$ = constant, keys fall into lower hand

If train accelerates/decelerates, keys "miss" lower hand

"left behind" appear to fly forwards

If train rotates (!), keys appear to move sideways.