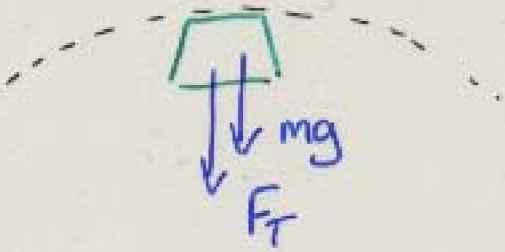
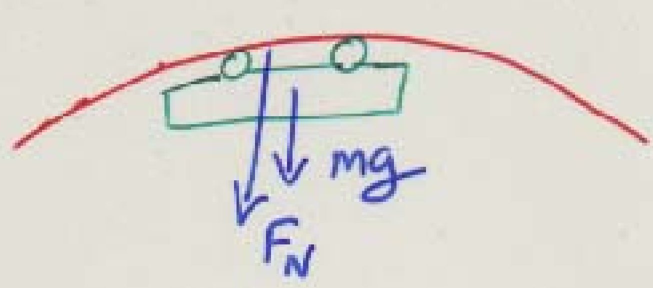
Vetical Rotations under Gravity

Bucket on String



Coaster on track



At top of circle, net inward fore must $= F_c = mv^2/r$

$$F_c = \frac{mv^2}{r} = F_T + mg$$

$$\frac{mv^2}{r} = F_N + mg$$

$\Rightarrow$ tension $F_T = \frac{mv^2}{r} - mg$

Normal Fore = eff. weight $F_N = \frac{mv^2}{r} - mg$

For taut string $F_T \geq 0$

To stay on the track $F_N \geq 0$

$$\Rightarrow \frac{mv^2}{r} - mg \geq 0$$

$$\text{or} \quad v^2 \geq gr \qquad v \geq \sqrt{gr}$$

- otherwise (v too small), object starts to free-fall.

Note: At bottom of circle,

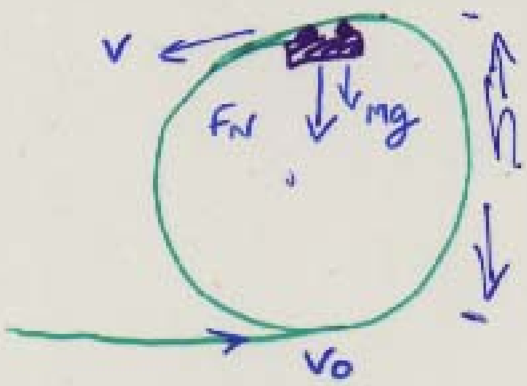
$$F_c = \frac{mv^2}{r} = \begin{Bmatrix} F_N \\ F_T \end{Bmatrix} - mg \Rightarrow \begin{Bmatrix} \text{eff. weight} \\ \text{tension} \end{Bmatrix} = m\left(g + \frac{v^2}{r}\right)$$

$>$ actual weight mg.

Motion in a Vertical Circle: Example

A roller-coaster enters a vertical loop of height $h = 16\,m$ un-powered. What speed must it have in order to

(a) make it around the loop (b) keep passengers from falling out?



(a) to climb to height $h = 2r = 16m$

KE at bottom $\geq \Delta PE$ at top

$$\tfrac{1}{2} m v_0^2 \geq mgh = 2mgr$$

$$\Rightarrow \underline{v_0^2 \geq 4gr} \quad (v_0 > 18\,m/s)$$

(b) At top of loop, must keep moving for $F_N > 0$

$$F_c = \frac{mv^2}{r} = F_N + mg \quad \Rightarrow \quad F_N = \frac{mv^2}{r} - mg \geq 0$$ or everyone falls out!

$$\Rightarrow mv^2 \geq mgr \text{ at top}$$

$$\text{or KE at top}: \tfrac{1}{2} mv^2 \geq \tfrac{1}{2} mgr$$

$$\text{KE at top} = \text{KE at bottom} - PE = \tfrac{1}{2} m v_0^2 - 2mgr$$

$$\Rightarrow \tfrac{1}{2} m v_0^2 - 2mgr \geq \tfrac{1}{2} mgr$$

$$\underline{\text{OR}} \quad \underline{v_0^2 \geq 5gr} \quad (v_0 > 20\,m/s)$$

Also: At bottom, eff. weight $F_N = mg + \frac{mv_0^2}{r}$

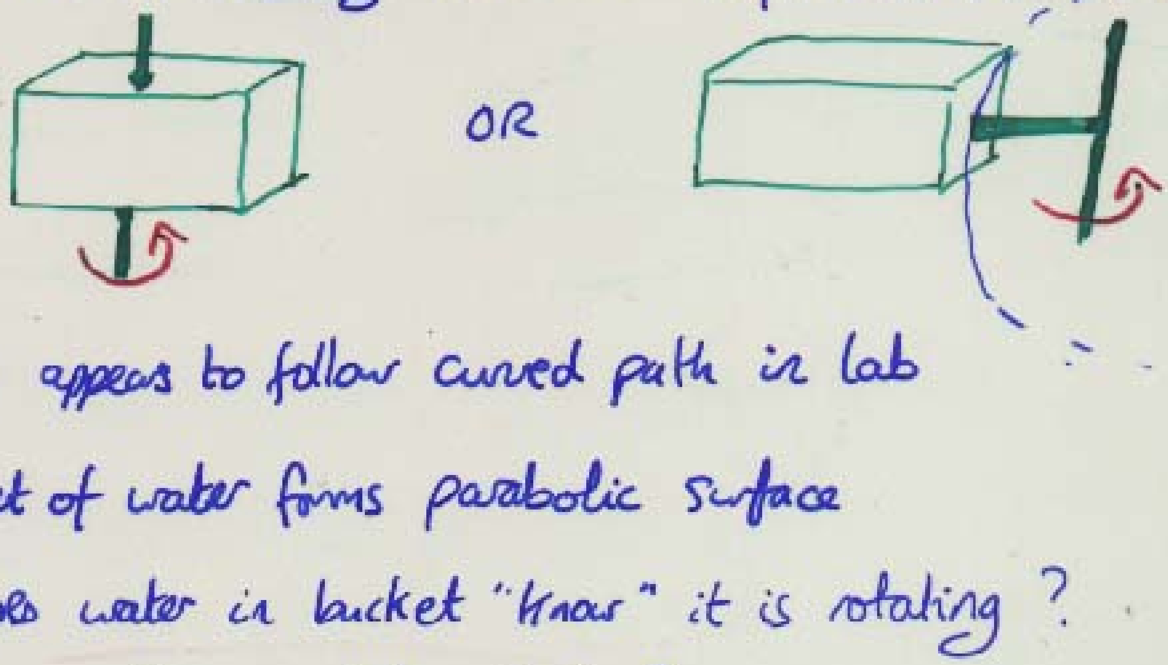
$v_0 = 20\,m/s \Rightarrow F_N = mg + 50m = \underline{\underline{6mg}}$!

Rotation and Inertial Reference Frames

- Cannot perform experiment in lab to tell if lab is
  - stationary
  - moving with $\vec{v}$ = constant
  - falling freely under gravity (all objects have same accel.)

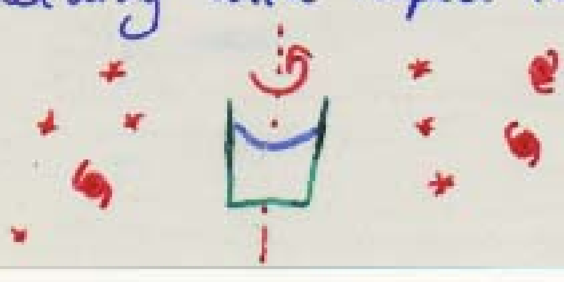
e.g. ball with no forces acting obeys Newton I
(uniform motion, straight line)

<u>BUT</u> in a <u>rotating</u> lab ..... (c.f. lab #1 turntable)



- ball appears to follow curved path in lab
- bucket of water forms parabolic surface

- How does water in bucket "know" it is rotating?
- Rotating with respect to what?



(Mach: distant stars + galaxies define "rotational standard of rest")

Elasticity + Hooke's Law

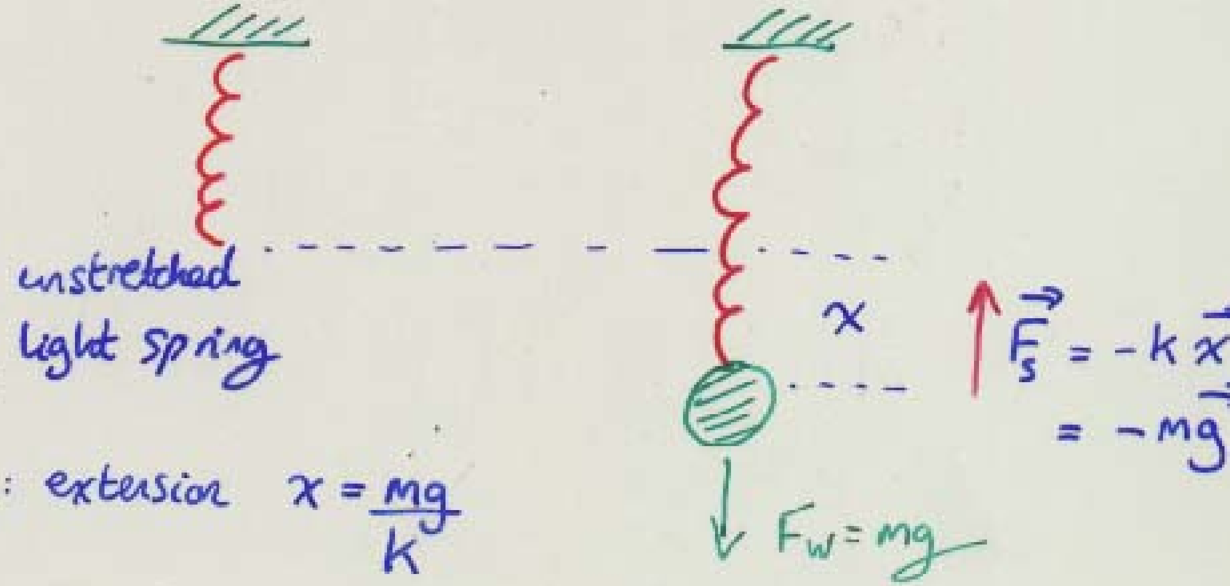
Robert Hooke (1676) : Springs, some wires etc. deform temporarily under applied force

Hooke's law : spring exerts equal and opp. RESTORING FORCE $\propto$ displacement.

i.e. $$\vec{F}_s = -k\vec{x}$$

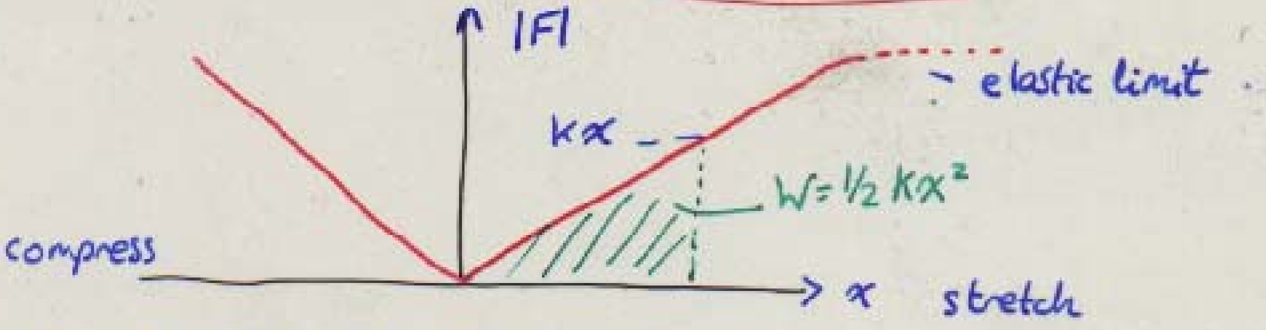
k: Spring constant [N/m] : = "stiffness" of spring

e.g. Spring balance measures weight:



Calibrate: extension $x = \dfrac{mg}{k}$

Hooke's Law cont/d. $|F| = k|x|$:



Most springs, wires obey Hooke's law up to an elastic limit
(increase F → permanent deformation, breakage)

Work and Potential Energy

To stretch/compress spring by displacement x

$$W = \int F.dx = \int_0^x kx.dx = \tfrac{1}{2}kx^2$$

(= area of Δ ½ base × height)
½ x × kx

- Work is "stored" as Potential Energy
- Compression or stretching → same result for springs
  (rope, wire, rubber bands only store P.E. by stretching)
- P.E. can be converted back into work or KE.

(rubber band : loses some work to heat , don't get it all back).

Example: Pinball machine with $k = 32\,N/m$ spring, $m = 0.02\,kg$ pinball. Compress spring by $x = 0.04\,m$

$v = 0$, $KE = 0$

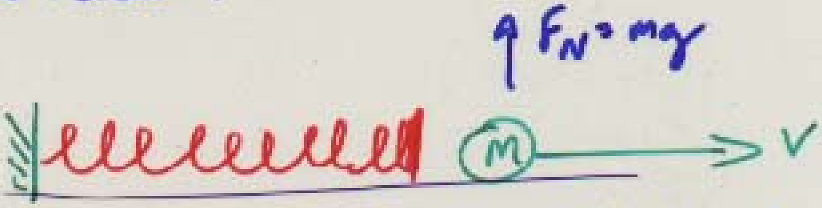
$x = -0.04\,m$, $PE = \frac{1}{2}kx^2 = 25.6\,mJ$

Force $F = kx = 1.28\,N$

$\Rightarrow$ initial accel $a = F/m = 64.0\,m/s^2$

$\approx 6.4g$!

Then release!



$PE = 0$, $KE = \frac{1}{2}mv^2$

After release:

KE gained = PE lost − (work against friction)

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \underbrace{\mu mg}_{F_F}\,x.$$

(neglect mass + KE of spring). If we neglect friction ($\mu = 0$)

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}kx^2 \quad \Rightarrow \quad v = x\sqrt{\frac{k}{m}} = 1.13\,m/s$$

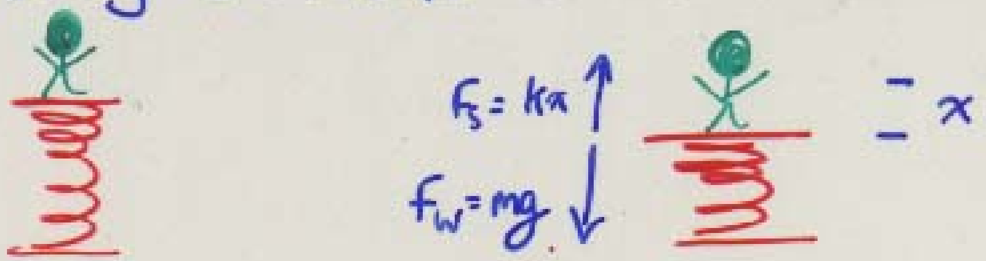
Note: accel $a = \frac{kx}{m}$ <u>not</u> constant, so good use of "energy arguments" to get final v.

(Otherwise, need to solve Newton II $m\frac{d^2x}{dt^2} = -kx$

: we shall!)

Spring P.E. vs gravity P.E - Example.

Person with $mg = 500N$ steps onto bath scale with $k = 2000\,N/m$



Scale drops to new eqm. position $F_s = F_W \Rightarrow x = \frac{mg}{k} = 0.25m$

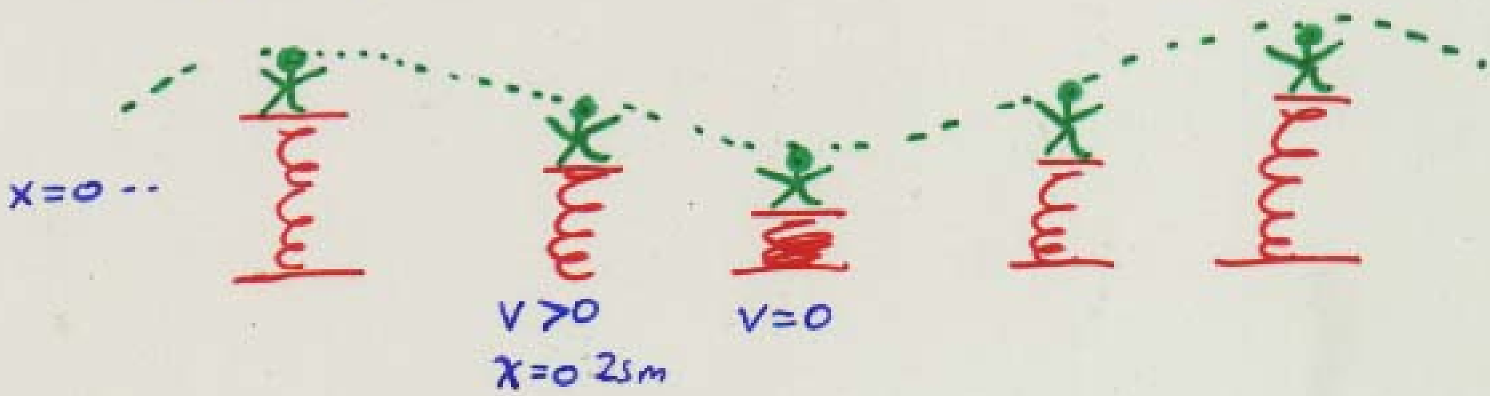
$\therefore$ grav. PE lost $= -mg\,x = 500N \times 0.25m = (-)125J$

Elastic P.E. gained $= \frac{1}{2}kx^2 = \frac{1}{2} \times 2000 \times 0.25^2 = +62.5J$

- only ½ of the lost grav P.E !

Q. What happened to the remaining 62.5 J of energy?

A: (If no friction) : MOTION!



Person vibrates up/down around eqm. position

Total energy P.E. + K.E. at equilibrium $= \frac{1}{2}kx^2 + \frac{1}{2}Mv^2$

$= 125J.$