Questions to Ponder....

- A rock tied to a piece of string is whirled around in a circle.
  - How are its {speed, rotation radius} related to the rotation period (or frequency)?
  - How does the string tension depend on : speed, radius, mass,...?
- How fast can a car drive around a curve without skidding? How does a "banked" road surface help?
- If I put some water in a cup, can I get it to stay in even when the cup is upside-down?

Homework:

Ch5: 3, 11, 13, 17, 18, 21, 24

Ch10: 1, 5, 10, 61, 63, 67, 80
83, 85, 105, 112, 122.

Week 4 Reading Quiz (ch. 5, ch 10)

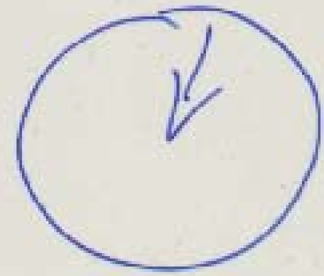
1. An object moves along a circular path at constant speed. Its acceleration is:

a) Zero

b) Towards the center

c) Away from the center

d) Tangential to the path



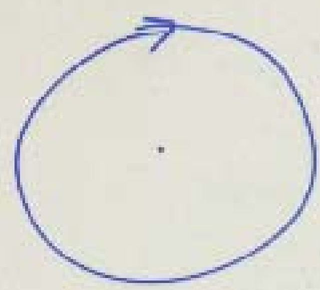
2. An athlete runs at 4 m/s around a circular track, and takes 300s to complete one circuit. What is the radius of the circle?

a) 1200 m

b) $1200 \times \pi$ m

c) $600 \times \pi$ m

d) $\frac{600}{\pi}$ m.



$c = vt = 4 \times 300$
$= 1200\,m$

$= 2\pi r$

$r = \frac{1200}{2\pi}$
$= \frac{600}{\pi}$

3. Simple Harmonic Motion is a consequence of a restoring force pushing against an object when it is displaced. This force must be proportional to :

a) displacement

b) (displacement)^2

c) mass of the object

d) speed of the object

4. For a simple pendulum, the Period T depends on:

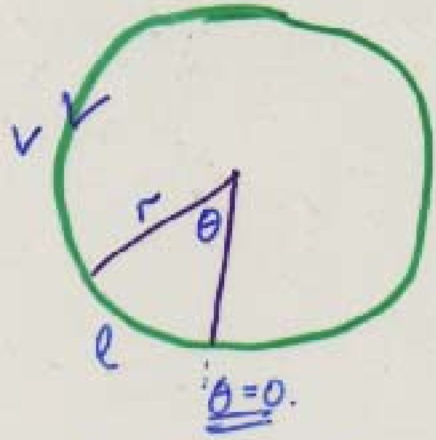
a) length of the string L

b) accel. due to gravity g

c) both <u>a</u> and <u>b</u> only.

d) both <u>a</u> and <u>b</u> AND the mass on the end of the string.

Circular Motion : Speed, Angular Speed, Period, Frequency



circumference $c = 2\pi r$

Measure θ in RADIANS

i.e. $\theta = \dfrac{\text{arc length}}{\text{radius}}$

(so for full circumference (360°) $\theta = \dfrac{2\pi r}{r} = 2\pi$ radians)

If object travels at speed v

Period T for 1 rotation $= \dfrac{\text{dist}}{\text{speed}} = \dfrac{2\pi r}{v}$

Angle swept out in time $t = \dfrac{\text{arc length}}{\text{radius}} = \dfrac{(vt)^{=l}}{r}$

Define angular speed $\omega = \dfrac{d\theta}{dt}$ (rad/s).

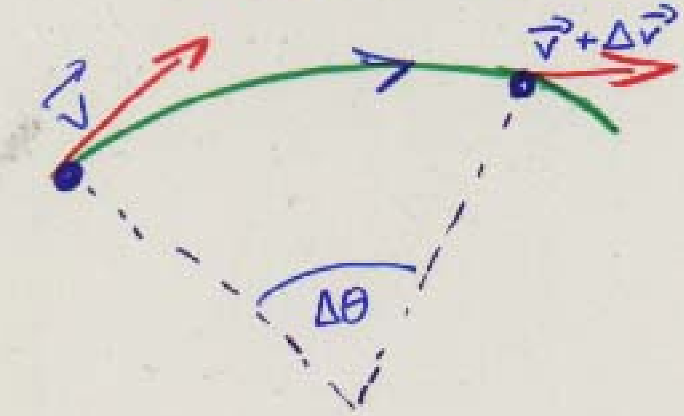
Then $\omega = \dfrac{v}{r}$ from above, or $\boxed{v = r\omega}$ [units!]

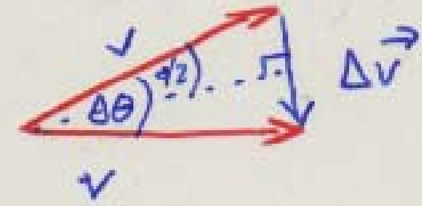
$m/s = m \times s^{-1}$.

Period $\boxed{T = \dfrac{2\pi}{\omega}}$ [s]

Cyclic Frequency (# of revs/s) $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$ [s^{-1} or Hz]

Circular Motion: Centripetal Acceleration





Watch object over small time Δt as it moves through $\Delta\theta$

$\vec{v}$ changes direction by $\Delta\theta$ but $|\vec{v}|$ constant

Change $\quad \Delta\vec{v} = \vec{v}(t+\Delta t) - \vec{v}(t)$

From geometry: $|\Delta v| = 2v\sin\frac{\Delta\theta}{2}$, also $\Delta\vec{v}$ points towards center

BUT as $\Delta\theta$ is small, $\sin\Delta\theta \approx \Delta\theta \Rightarrow \Delta v = 2v\frac{\theta}{2} = v\Delta\theta$

$\therefore$ accel. $a = \frac{\Delta v}{\Delta t} = v\frac{\Delta\theta}{\Delta t} = v\omega$

Since $\omega = v/r$

$$\boxed{\text{Centripetal accel. } a_c = \frac{v^2}{r}}$$

directed towards center.

Check: As $r \to \infty$, circle $\to$ st. line with $a_c = 0$ (Newton I)

If $v = 0$, $a_c = 0$ (no motion, Newton I)

Otherwise, a centripetal force $F_c = m a_c$ must act to keep circular motion.

Curvilinear Motion (Ch. 5)

So far: Newton's laws with $\vec{a}$ = constant (or 0)

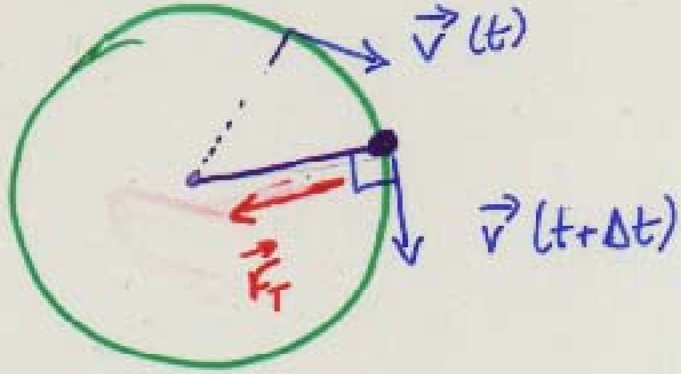
e.g. $\vec{a} = \vec{g}$, $\vec{a} = \frac{\vec{F} - \vec{F}_F}{m}$, etc.

Resulting motion of object can still be a curve $y(x)$

e.g. parabolic trajectories, $\vec{g}$ constant.

<u>BUT</u> $\vec{a}, \vec{F}$ often change direction as well as magnitude

e.g. Whirl stone on string around head at constant speed

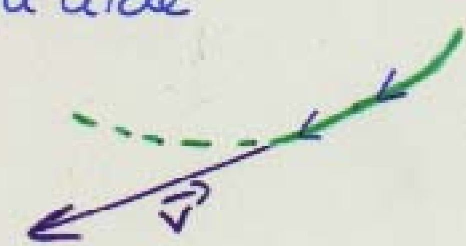


- $\vec{v}$ tangential to path $\Rightarrow$ must be $\perp$ to radius (string)
- $|\vec{v}|$ constant, but direction changes $\Rightarrow$ <u>must be a FORCE</u>

<u>Tensile force $\vec{F}_T$</u> keeps motion in a circle

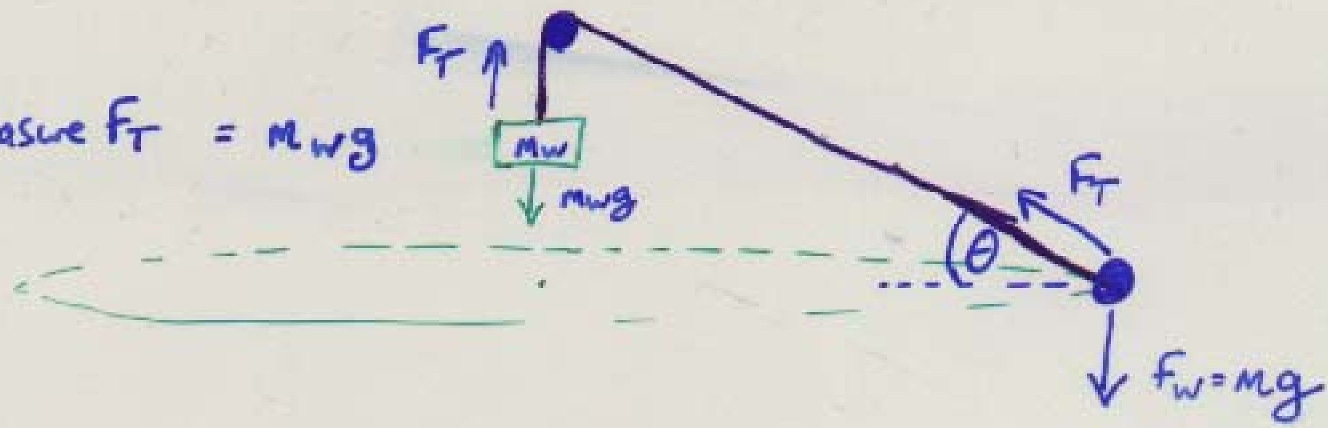
Break string $\rightarrow F_T = 0 \rightarrow$

(Newton I)



Tension: Mass on a String in Circular Motion

measure $F_T = m_w g$



For mass m on string at θ to horizontal,

tensile force F_T provides F_c. Measure F_T at other end by balancing weights ($F_T = m_w g$)

For rotating mass m:

Vert: $F_T \sin\theta = mg$ (1) so constant $F_T \Rightarrow \theta = \text{constant}$

Horiz: $F_T \cos\theta = F_c = \dfrac{mv^2}{r}$ (2)

$\frac{(1)}{(2)}$ $\Rightarrow$ angle of string $\tan\theta = \dfrac{gr}{v^2}$: so as $r\uparrow$, $v \uparrow$ (θ fixed)

Hard to measure v directly, but period $T = \dfrac{2\pi r}{v}$

$$\Rightarrow \tan\theta = gr\frac{T^2}{4\pi^2 r^2} = \frac{gT^2}{4\pi^2 r} = \text{constant}$$

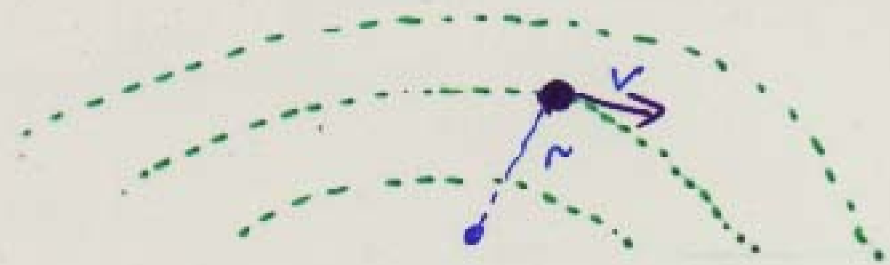
i.e. $T^2 \propto r$ for lab.

$T \propto \sqrt{r}$

Centripetal Force : Friction

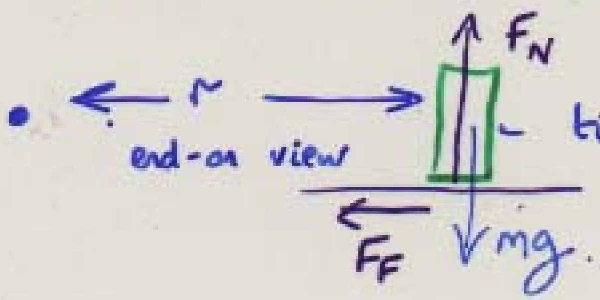
e.g. Cars race around circular track :

Top view



Force required to provide a_c is $F_c = ma_c = \frac{mv^2}{r}$
(Newton II)

On flat track, friction provides F_c (otherwise: skid)



$F_F \leq \mu F_N$

i.e. $F_F \leq \mu mg$

So for circular motion $F_c = \frac{mv^2}{r} = F_F \leq \mu mg$

$\therefore \frac{\cancel{m}v^2}{r} \leq \mu \cancel{m}g$ for friction to hold tire in circular path

$\Rightarrow v_{max} \leq \sqrt{\mu g r}$: max safe speed of curve

Note: • v_{max} indep. of mass, but $\propto \sqrt{\mu}$: "slippery when wet"

• In race, fastest lap time $T_{min} = \frac{2\pi r}{v_{max}} \geq \frac{2\pi\sqrt{r}}{\sqrt{\mu g}}$

– so inside track still wins.

Centripetal Force and "Effective Weight"

e.g. Centrifuge with $\theta \approx 0$ (horizontal)

F_N provides F_c i.e. eff. weight $F_N = \frac{mv^2}{r}$

c.f. $F_N = mg$ at rest. $\Rightarrow$ "effective gravity" $F_N/m = a_c = v^2/r$

e.g. for $r \approx 0.1$m test tube at 1500rpm

period $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{60s}{1500rpm} = 0.04s$

$\Rightarrow$ angular speed $\omega = \frac{2\pi}{T} = 157$ rad/s

$\Rightarrow a_c = \underline{\frac{v^2}{r}} = \frac{(r\omega)^2}{r} = \underline{r\omega^2} = 2467\ m/s^2$

$\approx 250 \times$ "g" !

Also, as tube rotates, ω = constant along its length

but $a_c = r\omega^2 \propto r$

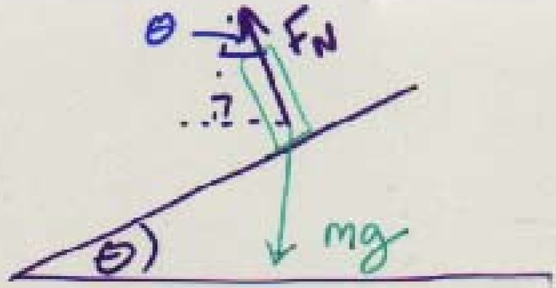
$\Rightarrow$ "effective gravity" increases with r

$\Rightarrow$ more massive objects separate to bottom of tube.

c.f. rotating space station

Banked Turns

Gravity



Instead of friction, can use $\vec{F_N}$ directly to provide $\vec{F_c}$

Equate:

Horiz: Required $F_c = \frac{mv^2}{r} = F_N \sin\theta$

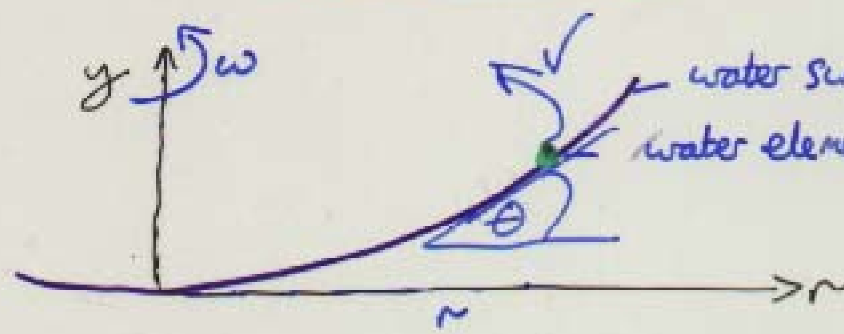
Vert: $mg = F_N \cos\theta$ $(F_N > mg)$

Divide (1) by (2) $\Rightarrow$ $\tan\theta = \frac{v^2}{rg}$: no friction required at this angle

e.g. for $r = 30\,m$, $v = 12\,m/s$, $\theta = 34.7°$

Spinning Bucket

Cross-section:

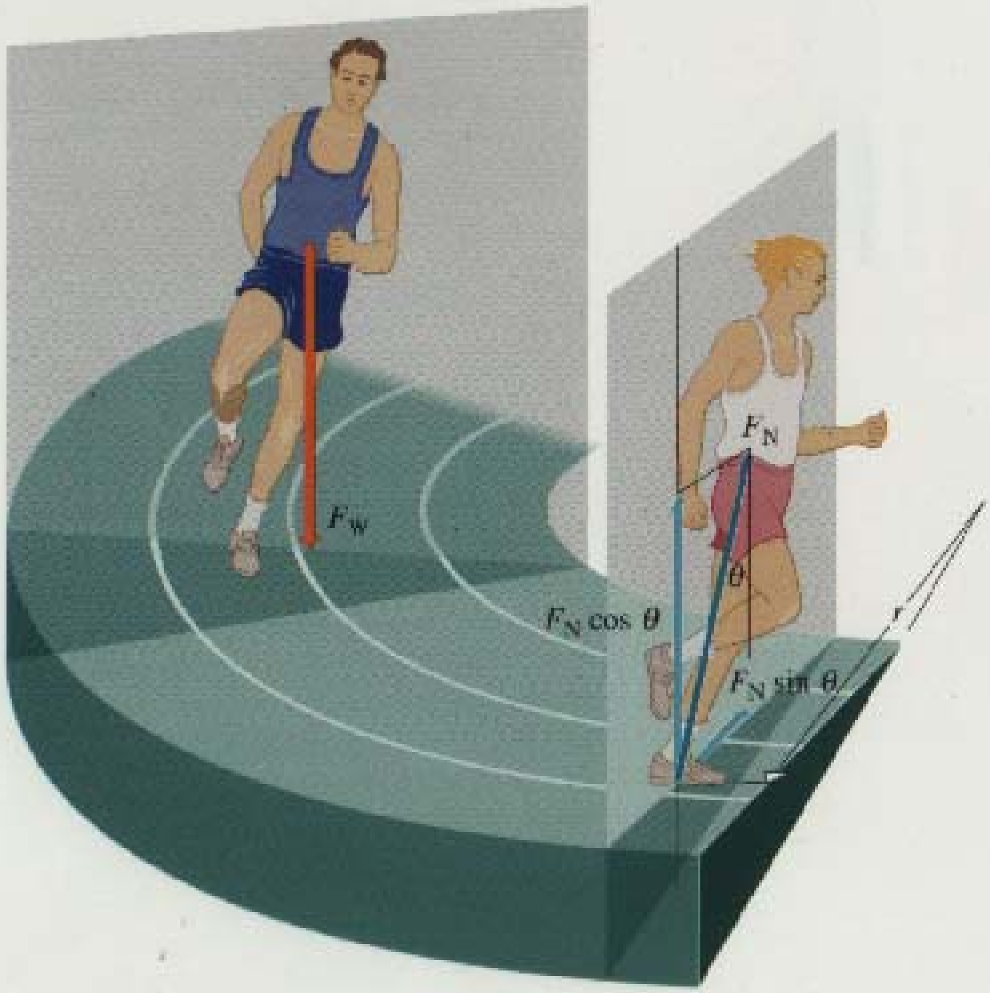


At surface, $\tan\theta = \frac{v^2}{rg}$ (no fluid friction), with $v = r\omega$

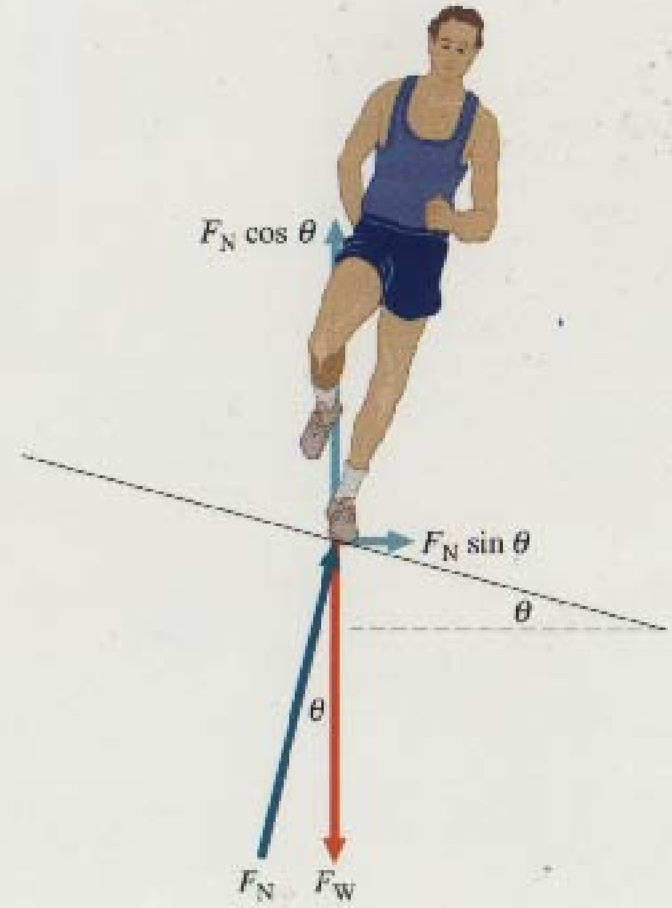
$$\Rightarrow \tan\theta = \frac{dy}{dr} = \frac{r^2\omega^2}{rg} = \frac{r\omega^2}{g} \Rightarrow y(r) = y_0 + \frac{\frac{1}{2}r^2\omega^2}{g}$$

i.e. surface forms a parabola.

Figure 5.5
A banked road



(a)



(b)

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