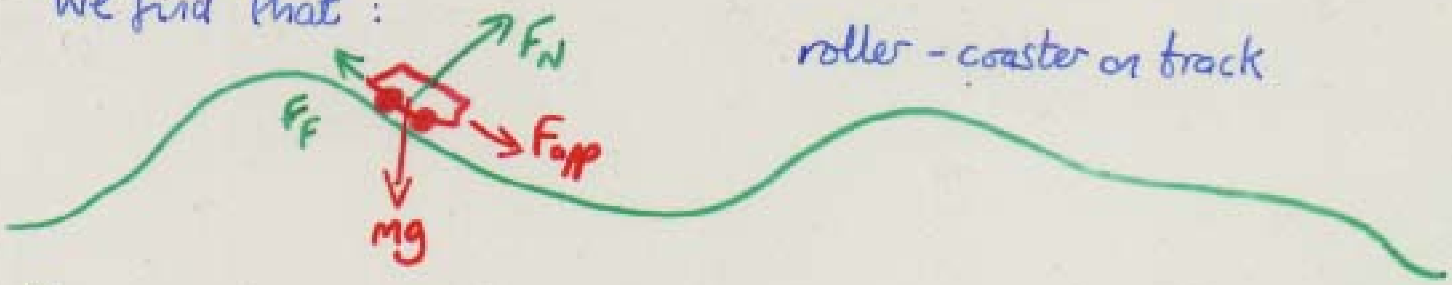
Work - Energy Theorem (Conservation of Energy)

We find that:



At any point: PE, KE are changing, forces applied + friction change motion

BUT Work input = $\Delta PE + \Delta KE$ − Work done against friction

i.e. adding all forms of energy E

• Be:n $\Delta E = 0$

⇒ general principle of Conservation of Energy.

- also found true for thermal, electrical, chemical processes.

Even in Special Relativity, cons. of energy applies.

Questions

- Force acting over distance changes energy via work

$$W = \int F.dx = \Delta KE$$

Same force acting over time changes momentum via **impulse**?

$$\int F.dt = \Delta mv \qquad F = \frac{d(mv)}{dt}$$

- When two objects collide, do we need to know $\vec{F}_{AB} = -\vec{F}_{BA}$ in detail during collision to predict result?

- Being lazy, from armchair I throw a ½kg mass at door to close it. Which projectile works best:
  (a) ½kg Playdoh (TM) (b) ½kg rubber ball?

- A truck hits parked car

or A car hits parked truck

- which collision caused most damage?

(KE converted to other forms)

Newton's 3rd Law and Impulse

e.g. rocket expels gas at rate $\frac{dm}{dt} = 5\,kg/s$

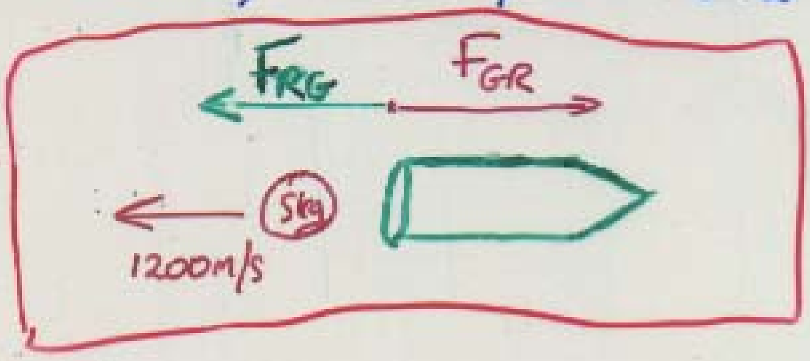
with exhaust speed $= 1200\,m/s$

In time $\Delta t = 1s$, impulse on 5kg of gas

$$F_{av}\Delta t = 5kg \times 1200\,m/s = 6000\,kg\,m/s$$

$\therefore$ From Newton III, reaction force on rocket $= |F_{av}| = 6000\,kg\,m/s^2$

$= 6000\,N.$



$\therefore$ In 1s, impulse on rocket $=$ $-$ impulse on gas

$$(\Delta mv)_{rocket} = -(\Delta mv)_{gas}$$

$\Rightarrow$ total momentum change $(\Delta mv)_{rocket} + (\Delta mv)_{gas}$

$$= 0.$$

$\therefore$ Momentum of system (rocket + gas) unchanged

i.e. conserved

Impulse changes Momentum, Work changes K.E.

$$\boxed{\int F.dt = \Delta mv} \qquad \boxed{\int F.dx = \Delta(\tfrac{1}{2}mv^2)}$$

e.g. roller coaster with $m = 600kg$, $v = 22.6 m/s$ brought to rest by constant $F = 5600 N$

Force F must act over time to change momentum (Newton II)

$$F\Delta t = mv - 0 = \Delta p \quad (1)$$

Same force F acts over distance Δx to do work against K.E.

$$F\Delta x = \tfrac{1}{2}mv^2 - 0 \quad (2)$$

(Can see that if force $F \uparrow$, both Δt and $\Delta x \downarrow$)

But can same force F satisfy both eqn. (1) and (2)? $x - x_0 = \tfrac{1}{2}a\Delta t^2$

Yes! From (1) and (2) : $\Delta x = \frac{1}{2}\underbrace{\left(\frac{F}{m}\right)}_{\text{accel.!}}(\Delta t)^2$

- which is what we expect for constant force / accel.

For values above, we find $a = \frac{F}{m} = 9.33 m/s^2$

$\Delta t = 2.42s$, $\Delta x = 27.3m$ and

and $\Delta KE \approx 150 kJ$, converted to heat by brakes.

Momentum and Collisions (Ch 7)

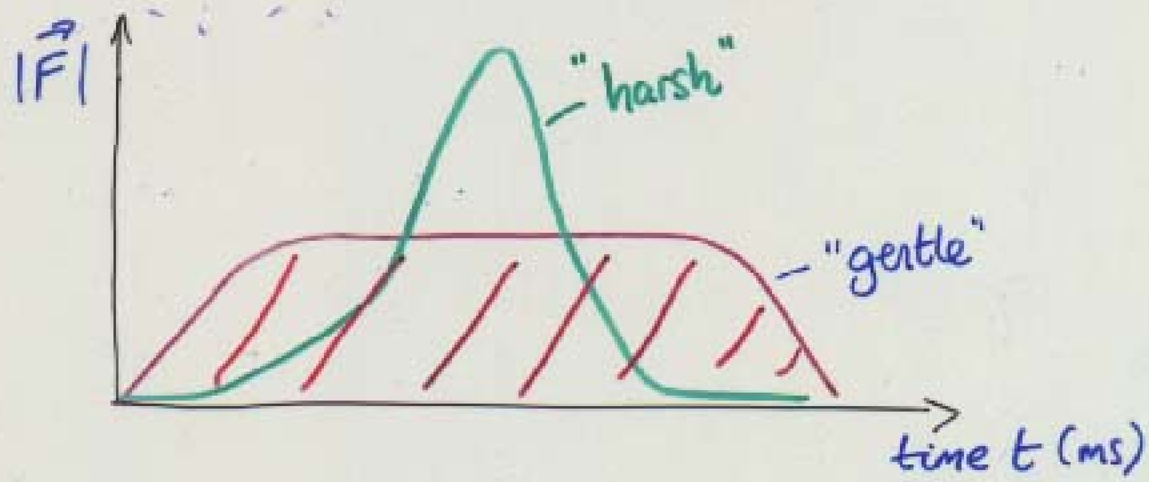
Impulse - change of momentum

From Newton II, $\vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$

So force acting over time changes momentum

i.e. The $\boxed{\text{impulse} \quad \Delta\vec{p} = \int \vec{F}.dt}$

e.g. many ways to accelerate golf ball ($m = 0.05$kg) up to $v = 60$m/s from rest:



Both cases $\Rightarrow$ impulse $\int F.dt = \Delta p = 0.05\text{kg} \times (60-0)$
$= 3\text{kg m/s}$

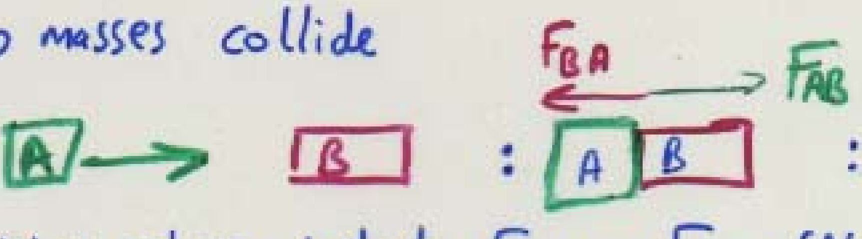
Other examples: catch baseball with hand on table.
For same impulse, if time $\Delta t \downarrow$, $F_{av} \uparrow$ (ouch!)

Conservation of Linear Momentum

Principle: With no external forces acting, the total momentum $\vec{p} = \vec{p}_1 + \vec{p}_2 + \dots$ of a system is conserved

i.e. $\Delta\vec{p} = 0$.

e.g. Two masses collide



During collision, at any instant $F_{AB} = -F_{BA}$ (Newton III)

and time intervals $\Delta t_A = \Delta t_B$

$\therefore$ Impulse on A $= \int F_{BA}\,dt = -$ Impulse on B $\int F_{AB}\,dt$

$\Delta(m_A v_A) \qquad = -(\Delta m_B v_B)$

$\Rightarrow \Delta P = \Delta(m_A v_A) + \Delta(m_B v_B) = 0$

Note: Add friction or gravity $\Rightarrow$ net external force on system $\Rightarrow$ momentum no longer conserved.

Example: McGuire's 70th Home Run (p.284)

Before:

Ball ($m_A = 0.4$ kg) pitched at $V_{Ai} = -40.25$ m/s

Bat ($m_B = 1$ kg) swung at $V_{Bi} = +38$ m/s

$\Rightarrow$ initial momenta $\quad p_{Ai} = m_A V_{Ai} = 0.4\,\text{kg} \times (-40.25)\,\text{m/s} = -16.1$ kg m/s

$\qquad\qquad\qquad\qquad\quad p_{Bi} = m_B V_{Bi} = 1.0\,\text{kg} \times 38\,\text{m/s} = 38$ "

$\therefore$ Total Momentum $p_i = p_A + p_B \quad = \underline{\underline{+21.9 \text{ kg m/s}}}$

After:

Use $p_f = p_i = 21.9$ kg m/s where $p_f = m_A V_{Af} + m_B V_{Bf}$

Given for ball: $p_{Af} = m_A V_{Af} = 0.4 \times 49.25 = +19.7$ kg m/s

$\Rightarrow$ bat: $p_{Bf} = 21.9 - 19.7 = 2.2$ kg m/s $\Rightarrow$ $\underline{V_{Bf} = 2.2 \text{ m/s}}$

Can now find <u>impulse</u> on ball (= -impulse on bat)

e.g. $\Delta p_A = p_{Af} - p_{Ai} \quad = 35.6$ kg m/s or $\underbrace{35.6 \text{ Ns}}_{\Delta mv}$

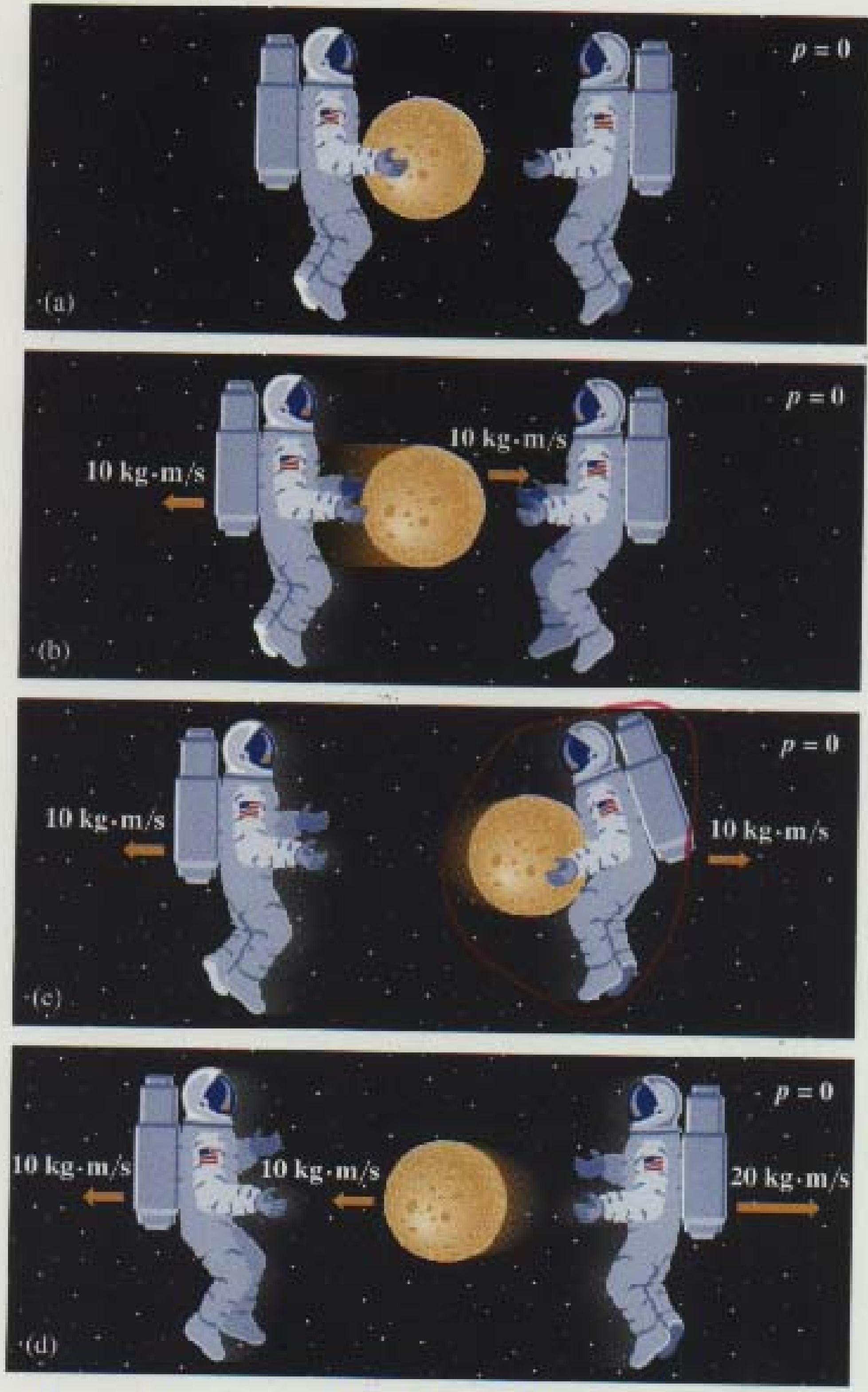
$= \int F.dt$ or $F_{av}.\Delta t$

So collision time $\Delta t = 1$ ms $\Rightarrow$ $F_{av} = \dfrac{\overbrace{35.6\text{Ns}}}{\underset{\Delta t}{10^{-3}\text{s}}} = 35.6$ kN! (shattering!)

Can also find $(KE)_f$ vs. $(KE)_i$ .....

Figure 7.9

Conservation of linear momentum

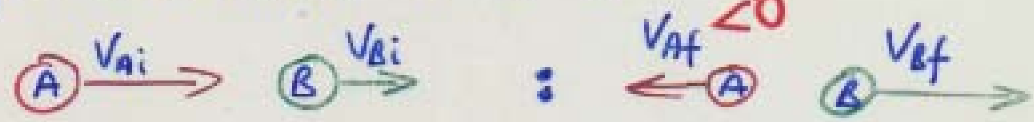


Same result as long as $\int F.dt = 10\,kg\,m/s$
- "gentle" or "harsh" throw or catch makes no difference.

Collisions and Conservation of Momentum

- "Collision" $\equiv$ any interaction where momentum transfered.
- Total momentum (before, during, after) = constant if no external forces, i.e. $\Delta\vec{P} = 0$.

e.g for two masses A and B



Conservation $\Rightarrow$ $\underline{m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}}$ (1)

Note: $\vec{p} = m\vec{v}$ is a vector, so use correct signs for v_A, v_B

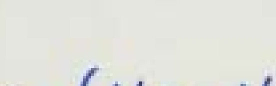
However: 1 equation, 2 unknowns (v_{Af}, v_{Bf})

So need to specify type of collision to determine

- Elastic : momentum and KE are conserved

$$\rightarrow \tfrac{1}{2} m_A v_{Ai}^2 + \tfrac{1}{2} m_B v_{Bi}^2 = \tfrac{1}{2} m_A v_{Af}^2 + \tfrac{1}{2} m_B v_{Bf}^2 \quad (2)$$

- Inelastic ("sticky") - A and B stick together $v_{Af} = v_{Bf} = v_f$



$\rightarrow$ only one unknown v_f.

$(KE)_f < (KE)_i$: energy lost to heat, deformation, sound energy.

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Elastic Collisions

In 1-D: billiard balls, tennis racket + ball etc.

(Let B be stationary at first, $v_{Bi} = 0$)

$\Delta \vec{P} = 0$ (momentum) : $m_A v_{Ai} + 0 = m_A \underline{v_{Af}} + m_B \underline{v_{Bf}}$

$\Delta KE = 0$ (kinetic energy) : $\frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2$

Can solve for final velocities v_{Af}, v_{Bf}.

We find (eq. 7.9, 7.10, 7.11)

$$v_{Af} = \frac{(m_A - m_B)}{(m_A + m_B)} v_{Ai} \qquad v_{Bf} = \frac{2 m_A}{(m_A + m_B)} v_{Ai}$$

Note: $v_{Bf} > 0$ always (B knocked forwards), but v_{Af} can be < 0 if $m_A < m_B$ (rebounds backwards)

Also find $v_{Bf} - v_{Af} = v_{Ai}$: "relative speeds" are the same before/after collision.

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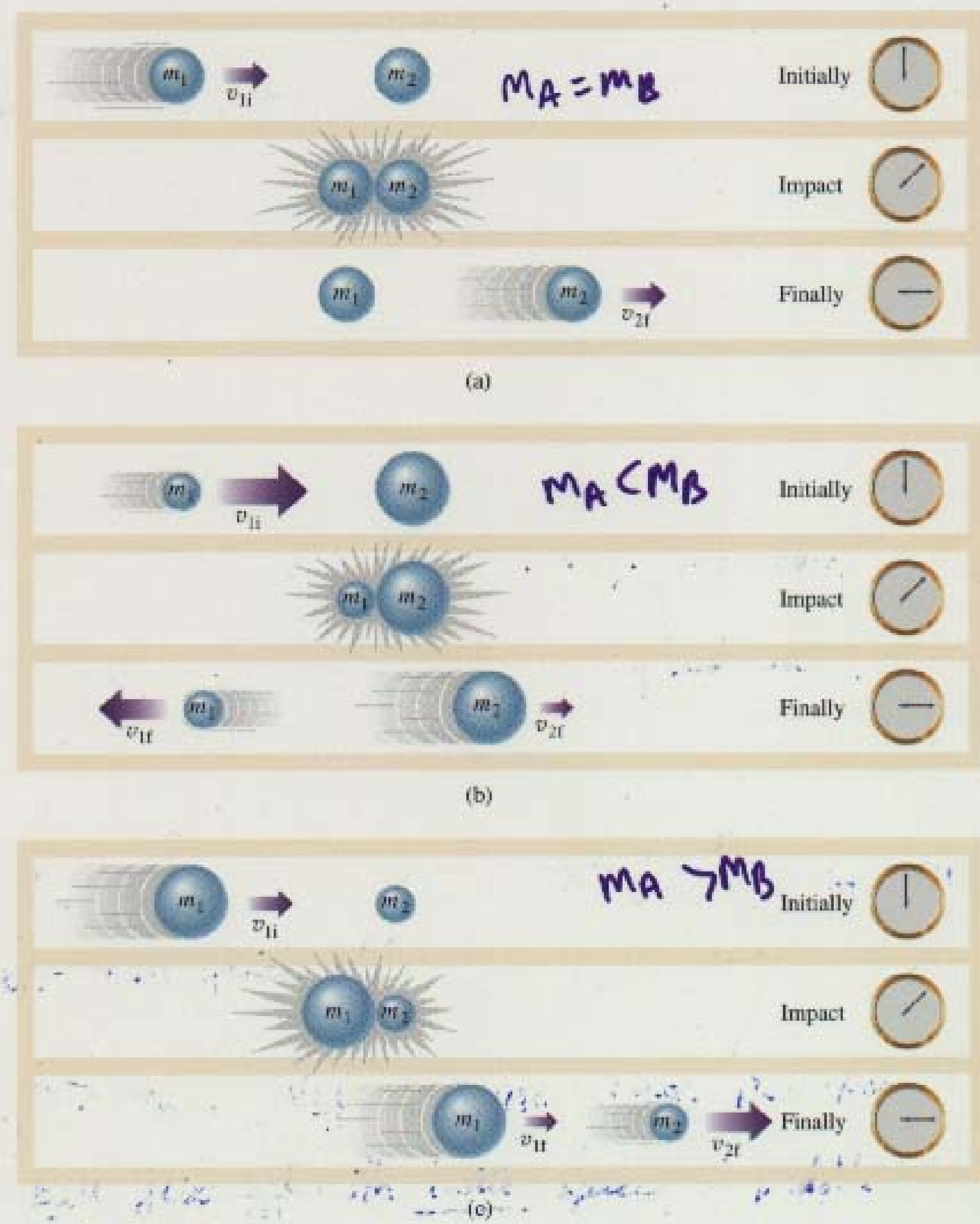
Figure 7.13
Three elastic collisions



Elastic Collision Features:

1) $m_A = m_B$: Then $v_{Af} = 0$, $v_{Bf} = v_{Ai}$

i.e. all momentum (+ k.e.) transfered $A \rightarrow B$, A is "stopped"

– most efficient for k.e. transfer

(e.g. club-head/golf ball)

2) $m_A < m_B$: Then $v_{Af} < 0$ (rebounds backwards)

and momentum transfer to B $m_B v_{Bf} = m_A (v_{Ai} - v_{Af})$

Note: if $m_A << m_B$, $v_{Af} = -v_{Ai}$ – perfect rebound

(e.g. throw ball at a ship, hit bowling ball with golf club!).

3) $m_A > m_B$: Then $v_{Af} > 0$ ("follows through"), $v_{Bf} > v_{Af}$

- lighter mass flies off

For $m_A >> m_B$, $v_{Bf} \approx 2 v_{Ai}$ (e.g. ping-pong ball flies off with twice speed of paddle).